Name: ______________________________________
Student ID: ______________________________________

CSE 322 Spring 2010: Take-Home Final Exam

Total: 150 points, 8 questions

Due: Before 4:30pm, Monday, June 7, 2010

Where: CSE Front Desk

Instructions:

1. Write your name and student ID on the first sheet and your last name on all sheets.
2. Write or mark your answers in the space provided. If you need more space, make sure you write down the question number and your name on any additional sheets, and staple these to the exam.
3. If you don't know the answer to a question, don't omit it - do the best you can. You may still get partial credit for whatever you wrote down.
4. Collaboration policy is the same as for the homework assignments.

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1. (25 points: 5 each) Circle True (T) or False (F) below. Very briefly justify your answers (e.g., by contradiction or an example/counter-example, by citing a theorem or result we proved in class, or by *briefly* sketching a construction).

a. For any two sets A and B, if A is uncountably infinite and B is countably infinite, then $A \cap B$ is countably infinite ………………………….....................… T F
Why/Why not?

b. If R is any regular language and L is any context free language, then $L \circ R$ is context-free …………………..………………………..………………….. T F
Why/Why not?

c. The language $\{a^m b^n c^n d^m \mid m, n \geq 0\}$ over $\Sigma = \{a,b,c,d\}$ is not context free... T F
Why/Why not?

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1. (cont.)

d. For any two languages A and B, if $A \subseteq B$, then A is reducible to B……..… T F
Why/Why not?

e. If language A is reducible to language B and B is undecidable, then A must be undecidable……………………………....…………………………..……..… T F
Why/Why not?

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2. (18 points: 6 each)

Let L = {w | $w \in$ {0,1}* and w contains neither 00 nor 11 as a substring}. Give:

a. The state diagram of a finite automaton (DFA or NFA) accepting L.

b. A regular expression denoting L.

c. A context-free grammar generating L.

You can either follow the constructions given in the lectures/book for converting one of these forms to the other or you can just give a direct answer for each part.

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3. (15 points)
A language is prefix-closed if the prefix of any string in the language is also in the language. Show that every *infinite prefix-closed context free language* contains an *infinite regular* subset. (Hint: Go over the proof of the pumping lemma for context free languages and see what it implies if the language is also prefix-closed).

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4. (12 points)
A k-PDA is a pushdown automaton with k stacks. Show that 2-PDAs are more powerful than conventional PDAs with only 1 stack. (Hint: Give a language that you can show is recognizable by a 2-PDA but not by 1-PDAs. An implementation level description is sufficient – no need to formally define the 2-PDA).

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5. (15 points)

Show that a 2-PDA is in fact as powerful as a Turing machine (TM). Describe how a 2-PDA can simulate an arbitrary TM. How is a particular configuration uq_iv of the TM, where $u, v \in \Gamma^*$, represented by the 2-PDA using its two stacks? How does the 2-PDA simulate the TM transitions $\delta(q_i,a) = (q_j,b,L)$ and $\delta(q_i,a) = (q_j,c,R)$? An implementation level description is sufficient – no need to formally define the 2-PDA.

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6. (20 points: 10 each)

a. Let L_1 and L_2 be any two decidable languages, decided by TMs M_1 and M_2 respectively. Construct a decider TM M for the language L_1 - L_2. Give a high level description, starting with M = "On input w: …".

b. Let L_1 and L_2 be any two Turing-recognizable languages, recognized by TMs M_1 and M_2 respectively. Prove or Disprove: L_1 - L_2 is Turing-recognizable. Either sketch a proof or give a specific counterexample.

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7. (30 points: 15, 15 points) For the following proofs, you may use high-level descriptions of the required TMs.

a. Show that for any infinite language L, L is decidable iff some enumerator TM enumerates L in lexicographic order.

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7. (cont.)

b. Show that every infinite Turing-recognizable language has an infinite decidable subset. (Hint: Use the result from (a) above and the result you know regarding Turing-recognizable languages and enumerator TMs (Theorem 3.21 in the text)).

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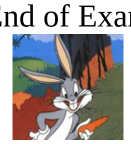
8. (15 points)
After proposing a toast to "Touring machines," an already toasted CS major from [name-deleted] university claims that the problem of figuring out whether a given TM accepts a finite number of strings is simple enough to be decidable. Having taken CSE 322, you know better, so you define the language $FINITE_{TM} = \{ \langle M \rangle \mid M$ is a TM and $L(M)$ is finite$\}$. Show that $FINITE_{TM}$ is undecidable by giving a reduction from a known undecidable language to $FINITE_{TM}$. For your reduction, you may use any of the languages shown to be undecidable in Section 5.1 in the textbook (up to Theorem 5.4 and its proof). (Hint: Use a reduction roughly along the lines of the one used in Theorem 5.2 and discussed in class.)

[End of Exam]



Have a great summer!