Name: ______________________________
Student ID: ______________________________

CSE 322 Autumn 2001: Sample Final Exam

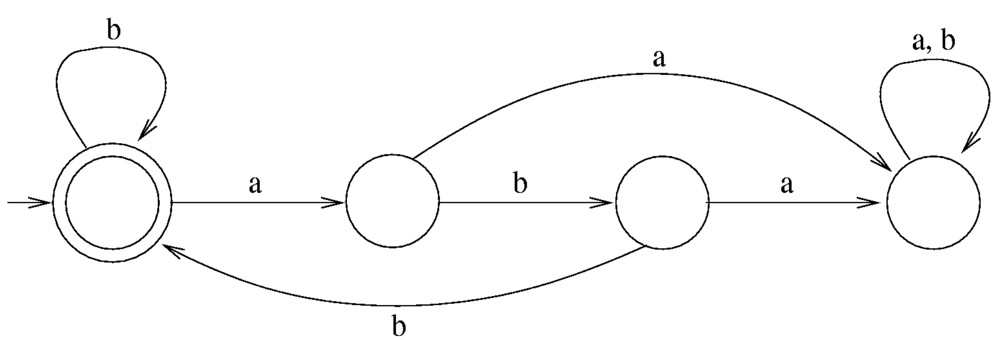
(some parts based on L. Ruzzo's sample final, Autumn 2000)
(closed book, closed notes except for 1-page summary)
Total: 150 points, 6 questions. Time: 1 hour and 50 minutes

Instructions:

1. Write your name and student ID on each sheet. Write or mark your answers in the space provided. If you need more space or scratch paper, you can get additional sheets from the instructor. Make sure you write down the question number and your name/id on any additional sheets.
2. Read all questions carefully before answering them. Feel free to come to the front to ask for clarifications.
3. *Hint 1*: You may answer the questions in any order, so if you find that you're having trouble with one of them, move on to another one that seems easier.
4. *Hint 2*: If you don't know the answer to a question, don't omit it - do the best you can! You may still get partial credit for whatever you wrote down. Good luck!

1. (25 points)
Let L be the set of strings in {a,b}* such that each a, if any, has two b's immediately to its right. Give:

(a) a finite automaton accepting L.



(b) a regular expression denoting L.

$(b \cup abb)^*$

(c) a context-free grammar generating L.

$S \rightarrow bS \mid abbS \mid \varepsilon$

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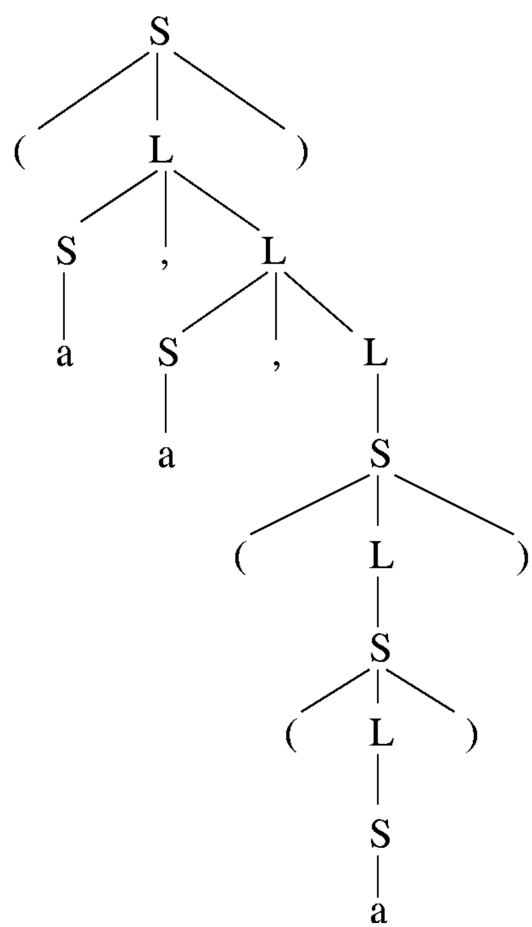
2. (25 points)
Define a well-formed list to be either the single "atom" a, or a sequence of one or more well-formed lists separated by commas and enclosed in parentheses. For example, "a" is a list, as are "(a)" and "(a,a,((a)))" but "((),aa)" is not (for two reasons).

(a) Give a context-free grammar that generates all such lists.

S → a | (L)
L → S | S, L

(b) Give a parse tree for the list "(a,a,((a)))" with respect to your grammar.



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3. (25 points)
Let G = (V,Σ,R,S) be a context-free grammar, where Σ = {w, c, {, }, ;, a}, V = Σ ∪ {S} and R is given by:

S → wcS
S → {S}
S → S;S
S → a

(This grammar models some statements in C-like languages, where w represents **whil**e, c represents a Boolean condition, {…} represents a compound statement, and a represents a non-**while** statement.)

(a) Show two distinct leftmost derivations, and the corresponding parse trees, for the string: wca;a

Derivation 1: S ⇒ wcS ⇒ wcS;S ⇒ wca;S ⇒ wca;a
Derivation 2: S ⇒ S;S ⇒ wcS;S ⇒ wca;S ⇒ wca;a

(Draw the two parse trees according to the above two derivations (each level of the tree corresponds to 1 step in the derivation). See page 98 in the text for an example).

(b) Is *G* ambiguous? Why or why not?

Yes, because there exists a string in L(G) that has 2 distinct parse trees (2 distinct leftmost derivations).

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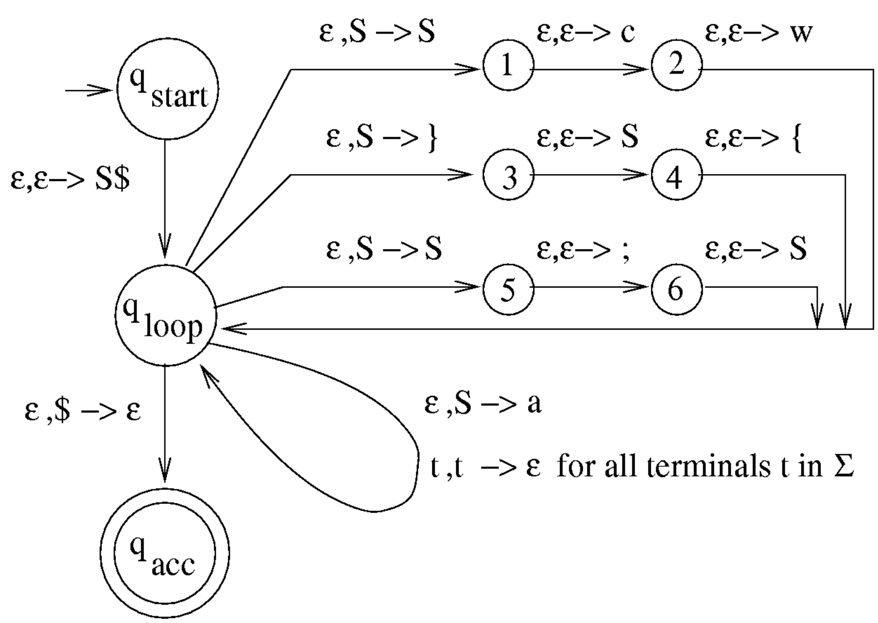
(c) Convert the CFG G above to a PDA accepting the same language.



(d) Show an accepting computation of your PDA for the input: wc{a}. Be sure to show the full configuration at each step — state, portion of input string that remains to be read, and contents of stack. If your PDA is nondeterministic, there may be more than one possible accepting computation. Show only *one* of them. [*Clearly indicate the top of the stack.*]

(Hint: Might be helpful to construct the parse tree first)

	State	Input to be read	Stack
1.	q_{start}	wc{a}	ε
2.	q_{loop}	wc{a}	$S [top]
3.	1	wc{a}	$S
4.	2	wc{a}	$Sc
5.	q_{loop}	wc{a}	$Scw
6.	q_{loop}	c{a}	$Sc
7.	q_{loop}	{a}	$S
8.	3	{a}	$}
9.	4	{a}	$}S
10.	q_{loop}	{a}	$}S{
11.	q_{loop}	a}	$}S
12.	q_{loop}	a}	$}a
13.	q_{loop}	}	$}
14.	q_{loop}	ε	$
15.	q_{acc}	ε	ε

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4. (25 points)

a. Show that the language L = {ww| $w \in$ {0,1}*} is not a context-free language.

See Example 2.22 on page 119 in the text.

b. Show that L is decidable. (Give an implementation-level description of a Turing machine that decides L).

The solution is identical to the example on pages 126-127 in the text, except add code for finding the middle of the input string (verify it has even length) and insert a #, before following the procedure given in the example.

5. (25 points)

a. Show that the class of context-free languages is *not closed* under intersection.

See Lecture slides for a solution.

b. Show that the class of decidable languages is closed under complementation.

Let L be a decidable language and let M be the decider for L. Then, the following TM M' decides the complement of L:
"On input w:

1. **Simulate M on w.**
2. **If M accepts w, then REJECT; otherwise, ACCEPT."**

(M is a decider, so M always halts, either in accept state or reject state)

M' accepts w iff M rejects w i.e. M' accepts complement of L(M) = L.

c. Show that the class of Turing-recognizable languages is closed under concatenation.

See solutions for the last homework assignment.

6. (25 points)

Prove that the language $EMPTY_{TM}$ = {<M> | M is a TM and L(M) = $\emptyset$} is undecidable. Use a reduction from A_{TM} = {<M,w> | M is a TM that accepts w} to $EMPTY_{TM}$.

See pages 173-174 in the text.