4: Sorting II

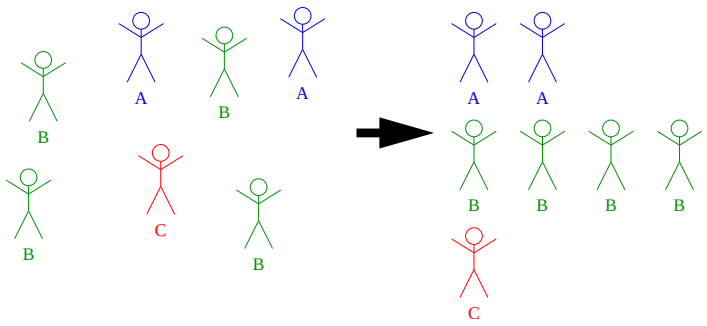
CSE326 Spring 2002

April 8, 2002

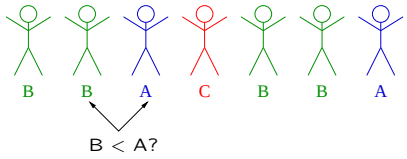
Digital Sorting

- *Comparison* sorting uses only $<, >, =$
- *Digital* sorting takes advantage of item representation.
  - strings are sequences of characters
  - numbers are sequences of bits
  - sorting key comes from a limited range

Sorting Students by Grade

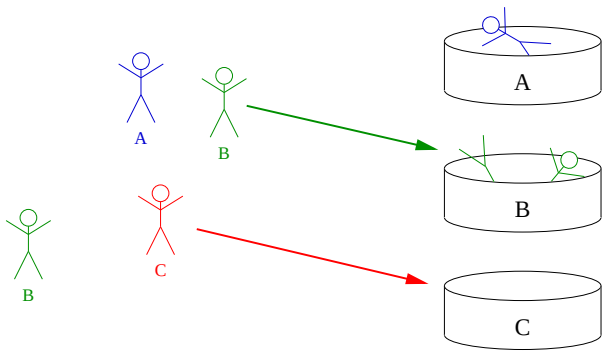


Sorting Students by Grade



Could use one of our sorting algorithms

Bucket Sort



Easier to just group by grade

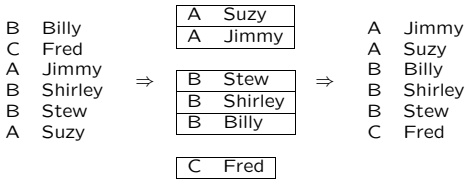
Bucket Sort

```
// elements in array have key values 0... N−1
void BucketSort(Elt *array, int n)
{
  for (i = 0 to N−1) Q[i] = new EltQueue;

  for (j = 0 to n−1;)
    Q[array[j].key].enqueue(array[j]);

  j = 0;
  for (i = 0 to N−1) {
    while (!Q[i].isEmpty()) {
      array[j] = Q[i].dequeue();
      j++;
  }
}
```

Bucket Sort



We use queues to keep students alphabetized by name within each grade

(*i.e.* with queues, bucket sort is *stable*)

Bucket Sort Analysis

Putting the *Digit* in *Digit*al

7857420978	4570980954
4395790423	1908523934
0934713547	8057523458
8504854743	7438209545
7587597589	4857875089
8970509824	7809238571

Suppose we want to sort a bunch of 10-digit numbers

- Range too large for bucket sort
- But each *digit* has small range.
- Can we iterate bucket sort?

Radix Sort

```
// Integer is a class for our D-digit numbers
void RadixSort(Integer *array, int n)
{
  for (i = 0 to 9) Q[i] = new IntegerQueue;
  for (k = 0 to D-1) {
    for (j = 0 to n-1)
      Q[array[j].Digit[k]].enqueue(array[j]);
    j = 0;
    for (i = 0 to 9) {
      while (!Q[i].isEmpty()) {
        array[j] = Q[i].dequeue();
        j++;
      }
    }
  }
}
```

This works for the same reason bucket sort on grades keeps names alphabatized: *stability*

Radix Sort Example

259 168 249 368 287

$\overbrace{247}^{7}\ \overbrace{158\ 368}^{8}\ \overbrace{259\ 249}^{9}$

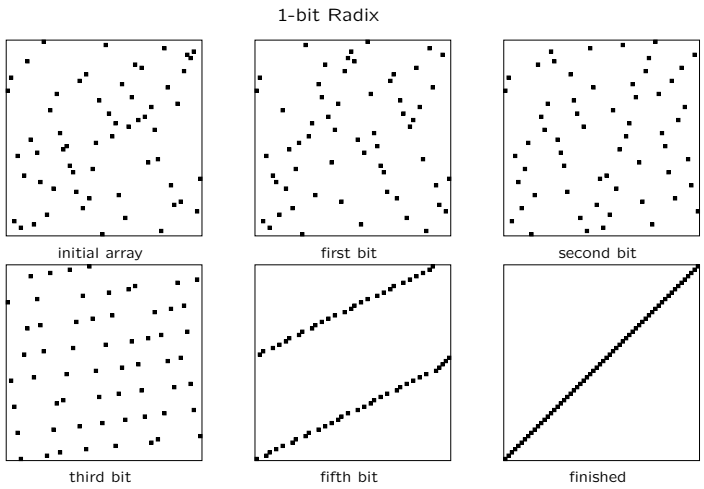
$\overbrace{247\ 249}^{4}\ \overbrace{158\ 259}^{5}\ \overbrace{368}^{6}$

$\overbrace{158}^{1}\ \overbrace{247\ 249\ 259}^{2}\ \overbrace{368}^{3}$

Radix Sort Analysis

Assume n items of D "digits" of range $0 \ldots N-1$.

Radix Sort: What It Looks Like



Extending Radix Sort

- Don't need to use decimal digits
  - binary, ASCII (8-bit), . . .
- Doesn't need to be numbers
  - Strings. Variable-length?

Radix Sort Question

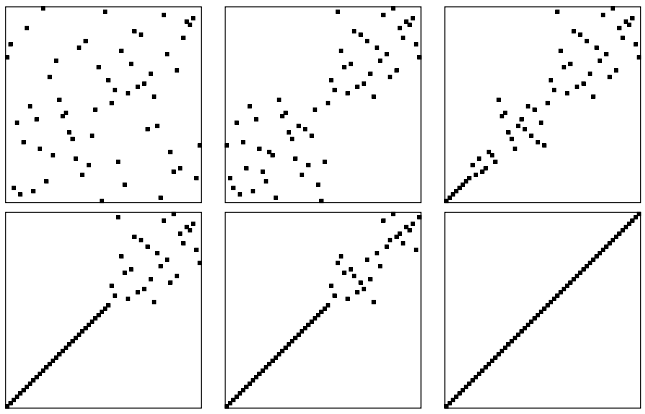
Why do we sort from right to left and not left to right?

Radix Exchange Sort

```
// sort rg[low ... high-1] on bits k ...0
void RadixExchangeSort(int *rg, int low, int high, int k)
{
  if (k >= 0 && low < high - 1) {
    i = low; j = high-1;
    while (i < j) {
      while (i < j && Bit(rg[i],k) == 0) i++;
      while (i < j && Bit(rg[j],k) == 1) j--;
      if (i < j)
        swap(rg[i], rg[j]), i++, j--;
    }
    if (Bit(rg[i]k) == 0) i++;
    RadixExchangeSort(rg, low, i, k-1);
    RadixExchangeSort(rg, i, high, k-1);
  }
}
```

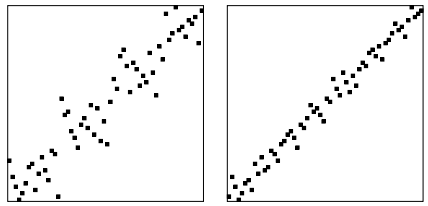
A Digital Version of Quicksort

Radix Exchange Sort: What it Looks Like



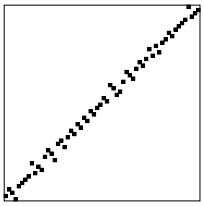
Radix Exchange vs. Quicksort

Lazy Radix Exchange

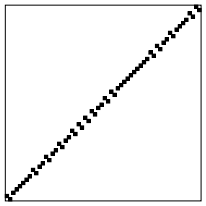


stop at bit 4

stop at bit 3



stop at bit 2



stop at bit 1

- Array mostly sorted by last few bits
- Is there an easier way to finish a mostly sorted array?