CSE 326: Data Structures
Cool Graphs n' Pretty Pictures

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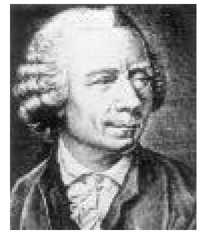
Summer Quarter 2002

Famous Dead Guy

- Number theory
- Numerical Analysis
- Graph Theory

- See the History of Mathematics biography on Euler:
  - http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Euler.html

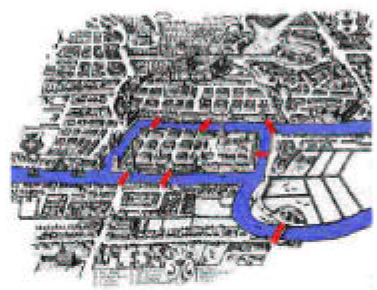


Leonhard Euler 1707-1783

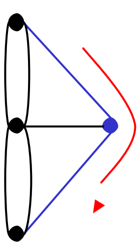
The Bridges of Königsberg

Can we walk around Konigsberg, crossing each bridge exactly once?

The (Graffitied) Bridges of Königsberg



The Bridges of Königsberg, Formally

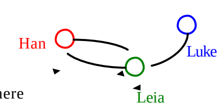


- Each bridge is an edge
- Each part of town is a vertex
- Is there a path that crosses each edge exactly once?

Graph… ADT?

Graphs are a formalism useful for representing relationships between things

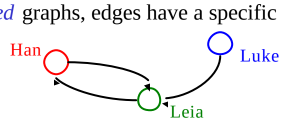
- A graph **G** is represented as **G = (V, E)**
  - **V** is a set of vertices: $\{v_1, v_2, \ldots, v_n\}$
  - **E** is a set of edges: $\{e_1, e_2, \ldots, e_m\}$ where each e_i connects two vertices (v_{i1}, v_{i2})
- Operations include:
  - Iterating over vertices
  - Iterating over edges
  - Iterating over vertices adjacent to a specific vertex
  - Asking whether two vertices are connected via an edge



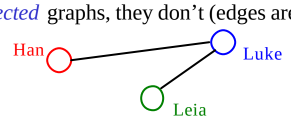

```
V = {Han, Leia, Luke}
E = {(Luke, Leia),
     (Han, Leia),
     (Leia, Han)}
```

Graph Definitions

In *directed* graphs, edges have a specific direction:



In *undirected* graphs, they don't (edges are two-way):



Vertices **u** and **v** are *adjacent* if **(u, v)** ∈ **E**

More Definitions: Simple Paths and Cycles

A *simple path* repeats no vertices (except that the first can be the last):

p = {Seattle, Salt Lake City, San Francisco, Dallas}

p = {Seattle, Salt Lake City, Dallas, San Francisco, Seattle}

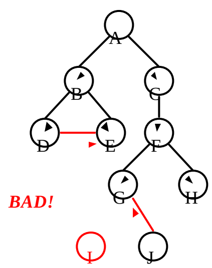
A *cycle* is a path that starts and ends at the same node:

p = {Seattle, Salt Lake City, Dallas, San Francisco, Seattle}

A *simple cycle* is a cycle that repeats no vertices except that the first vertex is also the last (in undirected graphs, no edge can be repeated)

Trees as Graphs

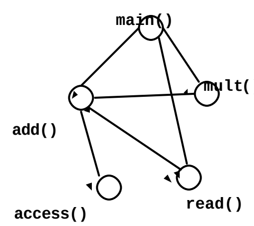
- Every tree is a graph with some restrictions:
  - The tree is *directed*
  - There are *no cycles* (directed or undirected)
  - There is a *directed path* from the root *to every node*



Directed Acyclic Graphs (DAGs)

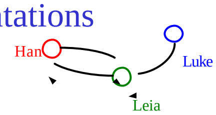
DAGs are directed graphs with no cycles.

If program call-graph is a DAG, then all procedure calls can be in-lined



Trees ⊂ DAGs ⊂ Graphs

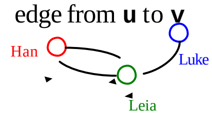
Graph Representations



- List of vertices + list of edges
- 2-D matrix of vertices (marking edges in the cells)
  "adjacency matrix"
- List of vertices each with a list of adjacent vertices
  "adjacency list"

Representation 1: Adjacency Matrix

A **|V| x |V|** array in which an element **(u, v)** is true if and only if there is an edge from **u** to **v**

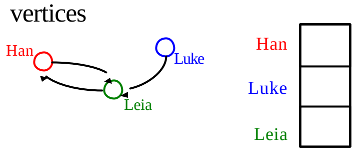


	Han	Luke	Leia
Han			
Luke			
Leia			

runtime: space requirements:

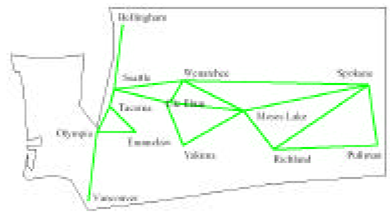
Representation 2: Adjacency List

A **|V|**-ary list (array) in which each entry stores a list (linked list) of all adjacent vertices



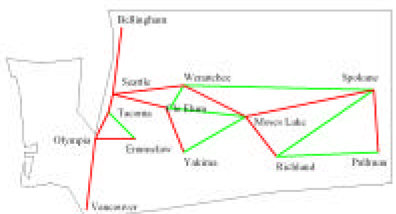
runtime: space requirements:

Some Applications: Moving Around Washington



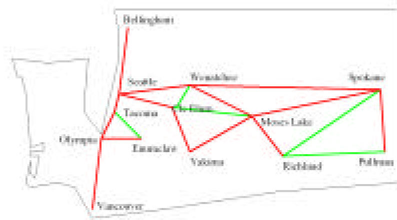
What's the fastest way from Seattle to Spokane?

Some Applications: Communication in Washington



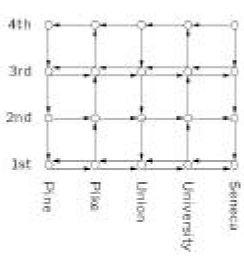
What's the cheapest inter-city network?

Some Applications: Reliability of Communication



If we lose Wenatchee, can Seattle still talk to Spokane?

Some Applications: Bus Routes in Downtown Seattle

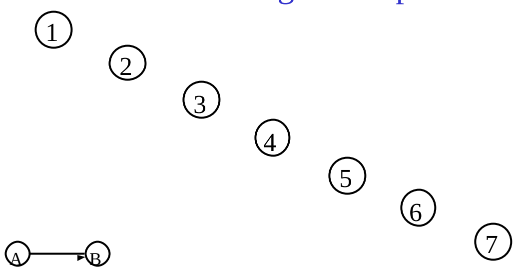


If we're at 3rd and Pine, can we get to 1st and Union?

Some Applications: Orderings and Determining Dependancies

Okay, everybody, get up and stretch!

Total Ordering on Graphs



means A must go before B

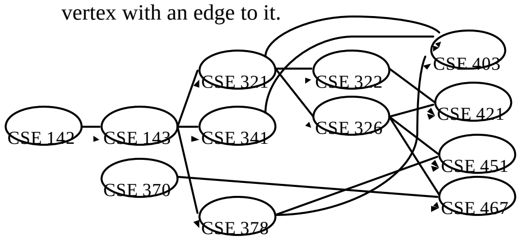
Does it make sense to define an ordering on an undirected graph?

Partial Order: Taking a Break in Class

Okay, everybody, stand up and stretch!

Some Applications: Topological Sort

Given a graph, **`G = (V, E)`**, output all the vertices in **`V`** such that no vertex is output before any other vertex with an edge to it.



Topo-Sort (Take One)

Label each vertex's *in-degree* (# of inbound edges)
While there are vertices remaining
  Pick a vertex with in-degree of zero and output it
  Reduce the in-degree of all vertices adjacent to it
  Remove it from the list of vertices

Runtime:

Topo-Sort (Take Two)

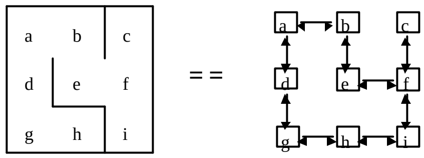
Label each vertex's in-degree
Initialize a queue to contain all in-degree zero vertices
While there are vertices remaining in the queue
  Pick a vertex *v* with in-degree of zero and output it
  Reduce the in-degree of all vertices adjacent to *v*
  Put any of these with new in-degree zero on the queue
  Remove *v* from the queue

Runtime:

Other Graph Applications?

Mazes == Graphs

- Cells are vertices
- Edges are doors from one cell to another



Breadth-First Search

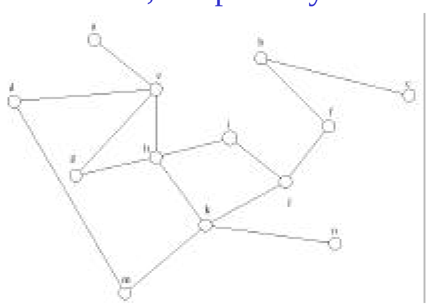
BFS characteristics:

– Nodes being worked on maintained in a FIFO Queue, not a stack (like DFS)

– Iterative style procedures sometimes easier to design than recursive procedures

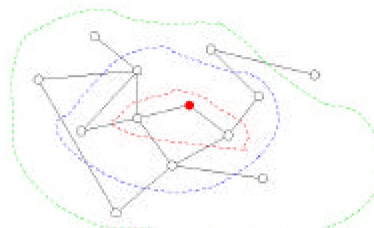
```
Put root in a Queue
Repeat until Queue is empty:
   Dequeue a node
   Process it
   Add its children to queue
```

BFS, Graphically



Explore vertices in order of distance from start

More BFS pictures



Using BFS

How do we …

– Determine if G is connected?

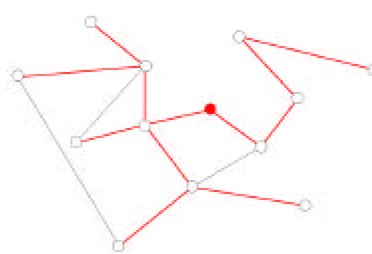
– Find the distance from the root to a node?

– Determine if G has any cycles?

– Determine if G is a tree?

– Find a path from the root to a node?

Introducing the BFS tree

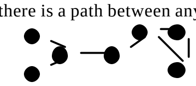


Graph Traversals

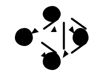
- Breadth-first search (and depth-first search) work for arbitrary (directed or undirected) graphs - not just mazes!
  - Must mark visited vertices so you do not go into an infinite loop!
- Either can be used to determine connectivity:
  - Is there a path between two given vertices?
  - Is the graph (weakly) connected?
- Important difference: Breadth-first search always finds a shortest path from the start vertex to any other (for unweighted graphs)
  - Depth first search may not!

"Weakly connected": A detour into connectivity

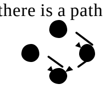
Undirected graphs are *connected* if there is a path between any two vertices



Directed graphs are *strongly connected* if there is a path from any one vertex to any other



Directed graphs are *weakly connected* if there is a path between any two vertices, *ignoring direction*



A *complete* graph has an edge between every pair of vertices