CSE 326: Data Structures
O Vertex, Where Art Thou?

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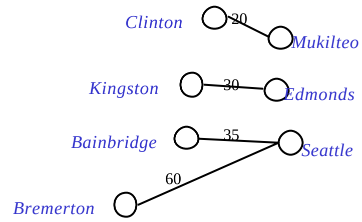
Summer Quarter 2002

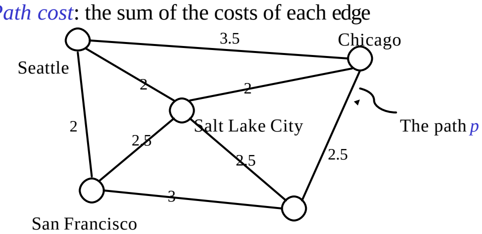
A Slight Searching Wrinkle: Weighted Graphs

Each edge has an associated weight or cost.



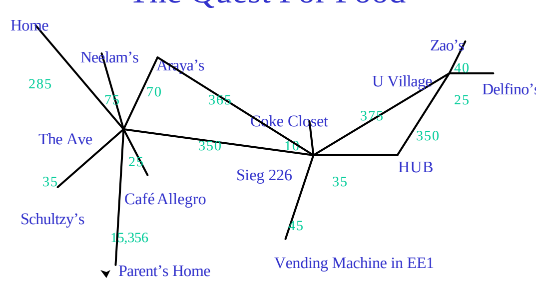
Diffentiating Between *Path Length* and *Path Cost*

Path length : the number of edges in the path
Path cost: the sum of the costs of each edge



length(*p*) = 5
cost(*p*) = 11.5

The Quest For Food



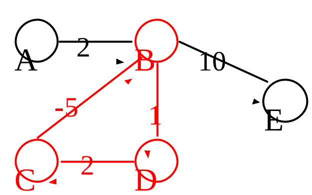
Can we calculate shortest distance to all nodes from Sieg 226?

Formally speaking …

Given a graph **`G = (V, E)`** and a vertex **`s`** Î **`V`**,
find the shortest path from s to every vertex in **`V`**

Many variations:
- Weighted vs. unweighted
- Cyclic vs. acyclic
- Positive weights only vs. negative weights allowed
- Multiple weight types to optimize
- Directed vs undirected graph

The Trouble with Negative Weighted Cycles



What's the shortest path from A to E?
(or to B, C, or D, for that matter)

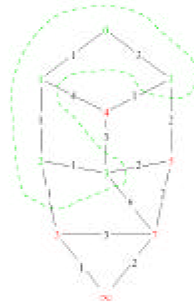
Dijkstra, Edsger Wybe

Legendary figure in computer science; now a professor at University of Texas.

Supports teaching introductory computer courses without computers (pencil and paper programming)

Supposedly wouldn't (until recently) read his e-mail; so, his staff had to print out messages and put them in his box.

Dijkstra's Idea

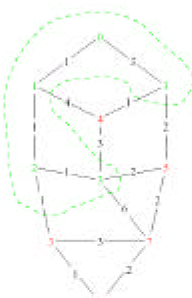


Adapt BFS to handle weighted graphs

Two kinds of vertices:

- Finished vertices
  - Shortest distance is computed
- Unknown vertices
  - Have tentative distance

Dijkstra's Idea



At each step:

1) Pick closest unknown vertex
2) Add it to finished vertices
3) Update distances

Dijkstra's vs BFS

At each step:

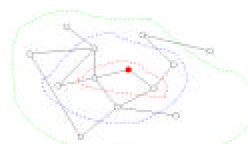
1) Pick closest unknown vertex
2) Add it to finished vertices
3) Update distances

Dijkstra's Algorithm

At each step:

1) Pick vertex from queue
2) Add it to visited vertices
3) Update queue with neighbours

Breadth-first Search



- Finished vertices in the middle
- Vertices from the fringe added at each step

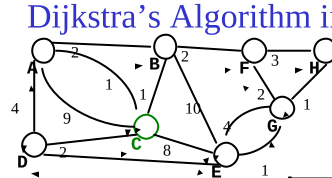
Dijkstra Pseudocode

Initialize the cost of each node to ∞

Initialize the cost of the source to 0

While there are unknown nodes left in the graph
- Select the unknown node with the lowest cost: *n*
- Mark *n* as known
- For each node *a* which is adjacent to *n*
  - *a*'s cost = min(*a*'s old cost, *n*'s cost + cost of (*n*, *a*))

Dijkstra's Algorithm in Action

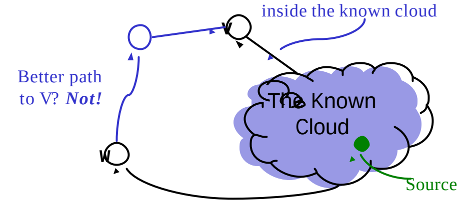


vertex	known	cost
A		
B		
C		
D		
E		
F		
G		
H		

Dijkstra's Algorithm
for Single Source, Shortest Path

- Classic algorithm for solving shortest path in weighted graphs without negative weights
- A greedy algorithm (irrevocably makes decisions without considering future consequences)
- Intuition:
  - shortest path from source vertex to itself is 0
  - cost of going to adjacent nodes is at most edge weights
  - cheapest of these must be shortest path to that node
  - update paths for new node and continue picking cheapest path

The Cloud Proof



- But, if path to **V** is shortest, path to **W** must be at least as long.
- So, how can the path through **W** to **V** be shorter?

Inside the Cloud (Proof)

Prove by induction on # of nodes in the cloud:

Initial cloud is just the source with shortest path 0

<u>Assume</u>: Everything inside the cloud has the correct shortest path

<u>Inductive step</u>: Once we prove the shortest path to some node ***V*** (which is not in the cloud) is correct, we add it to the cloud

When does Dijkstra's algorithm not work?

Data Structures
for Dijkstra's Algorithm

|V| times:

Select the unknown node with the lowest cost

`findMin/deleteMin`

|E| times:

a's cost = min(*a*'s old cost, …)

`decreaseKey`

`find by name`

Data structure(s)?
Runtime?

Graphs are *Really Important!*