CSE 326: Data Structures
AVL Trees

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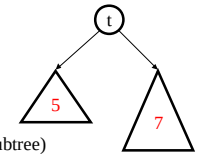
Beauty is Only Θ(log n) Deep

- Binary Search Trees are fast if they're shallow:
  - perfectly complete
  - perfectly complete except the one level fringe (like a heap)
  - anything else?

What *matters* here?

Problems occur when one branch is much longer than the other!

Balance

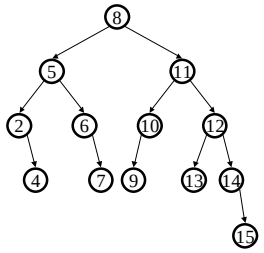


- Balance
  - height(left subtree) - height(right subtree)
  - zero everywhere ⇒ perfectly balanced
  - small everywhere ⇒ balanced enough

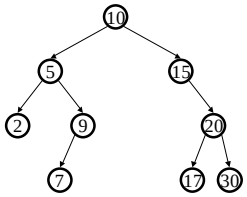
Balance between -1 and 1 everywhere ⇒ maximum height of 1.44 log n

AVL Tree
Dictionary Data Structure

- Binary search tree properties
  - binary tree property
  - search tree property
- Balance property
  - balance of every node is: `-1 ≤ b ≤ 1`
  - result:
    - depth is `Θ(log n)`

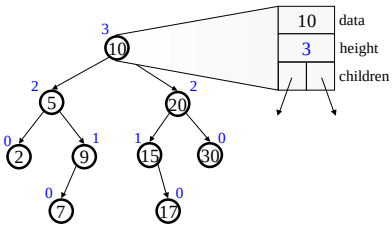


Testing the Balance Property



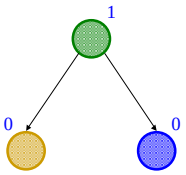
`NULL`s have height `-1`

An AVL Tree



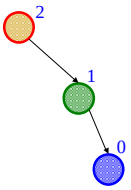
Beautiful Balance

Insert(middle)
Insert(small)
Insert(tall)

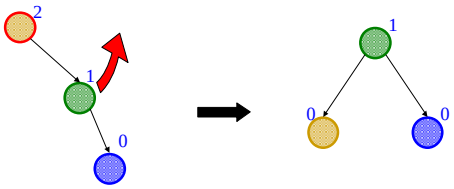


Bad Case #1

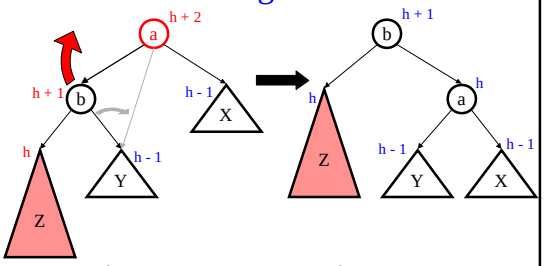
Insert(small)
Insert(middle)
Insert(tall)



Single Rotation



General Single Rotation

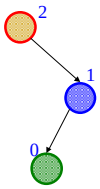


- Height of subtree same as it was before insert!
- Height of all ancestors unchanged.

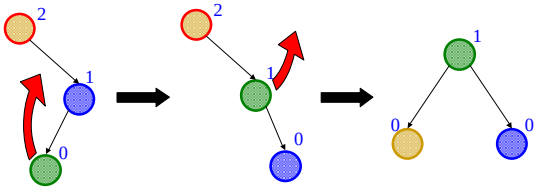
So?

Bad Case #2

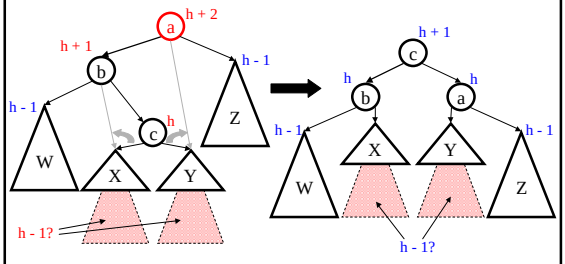
Insert(small)
Insert(tall)
Insert(middle)



Double Rotation



General Double Rotation



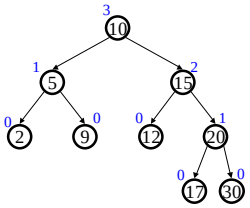
- Height of subtree **still** the same as it was before insert!
- Height of all ancestors unchanged.

Insert Algorithm

- Find spot for value
- Hang new node
- Search back up for imbalance
- If there is an imbalance:
  - case #1: Perform single rotation and exit
  - case #2: Perform double rotation and exit

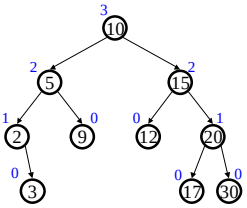
Easy Insert

Insert(3)

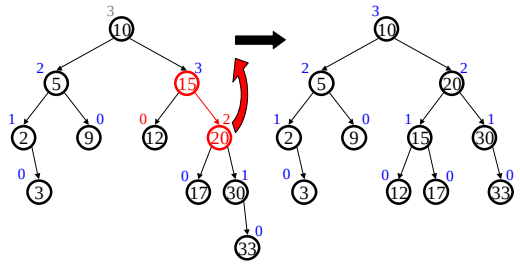


Hard Insert (Bad Case #1)

Insert(33)

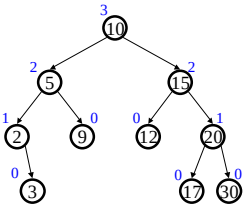


Single Rotation

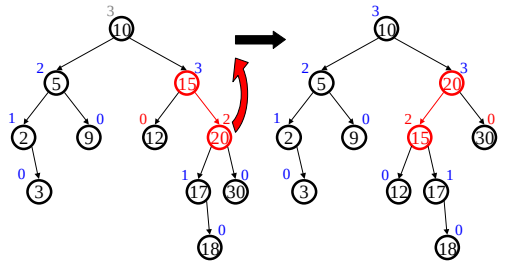


Hard Insert (Bad Case #2)

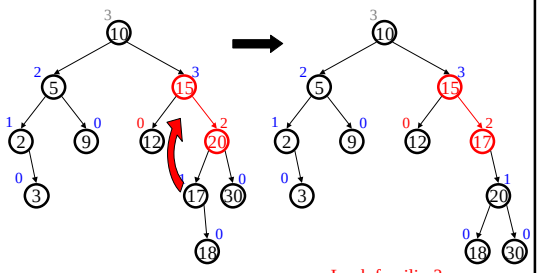
Insert(18)



Single Rotation (oops!)

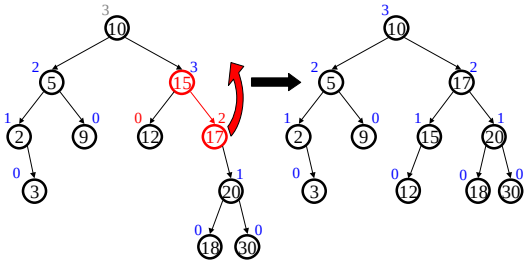


Double Rotation (Step #1)



Look familiar?

Double Rotation (Step #2)



AVL Algorithm Revisited

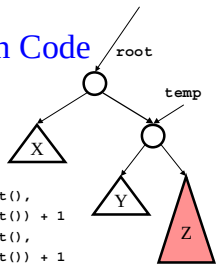
- Recursive

1. Search downward for spot
2. Insert node
3. Unwind stack, correcting heights
   a. If imbalance #1, single rotate
   b. If imbalance #2, double rotate

- Iterative

1. Search downward for spot, stacking parent nodes
2. Insert node
3. Unwind stack, correcting heights
   a. If imbalance #1, single rotate and exit
   b. If imbalance #2, double rotate and exit

Single Rotation Code

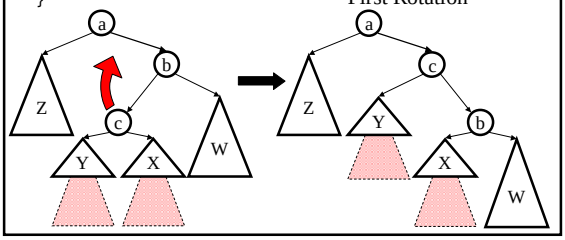



```
void RotateRight(Node root) {
  Node temp = root.right
  root.right = temp.left
  temp.left = root
  root.height = max(root.right.height(),
                    root.left.height()) + 1
  temp.height = max(temp.right.height(),
                    temp.left.height()) + 1
  root = temp
}
```

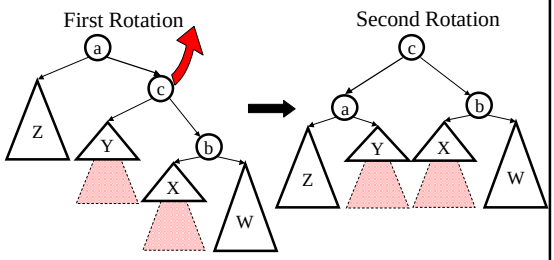
Double Rotation Code

```
void DoubleRotateRight(Node root) {
  RotateLeft(root.right)
  RotateRight(root)
}
```

First Rotation



Double Rotation Completed

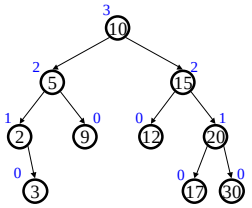


AVL

- Automatically Virtually Leveled
- Architecture for inVisible Leveling (the "in" is inVisible)
- All Very Low
- Absolut Vodka Logarithms
- Amazingly Vexing Letters

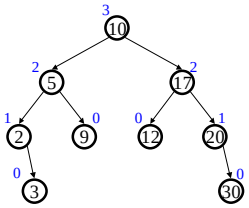
Bonus: Deletion (Easy Case)

Delete(15)

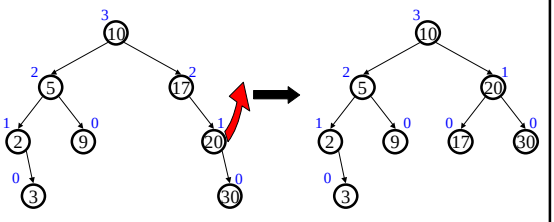


Deletion (Hard Case #1)

Delete(12)



Single Rotation on Deletion



Something *very* bad happened!

Coming Up

- Splay trees
- B-trees
- and even more trees!