CSE 326: Data Structures
Don't Sweat It - Splay It

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AVL Trees: Are They Worth It?

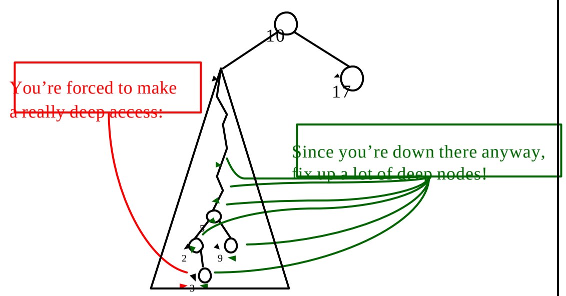
Advantages
- Rotations are cool!

Disadvantages
- Wouldn't want to meet one in a dark alley at night

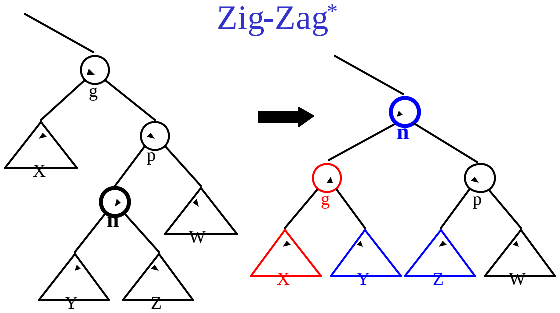
Splay What?

- Blind adjusting version of AVL trees
  - Why worry about balances? Just rotate anyway!
- *Amortized* time for all operations is O(log n)
- Worst case time is O(n)
- Insert/Find always rotates node *to the root*!

Idea

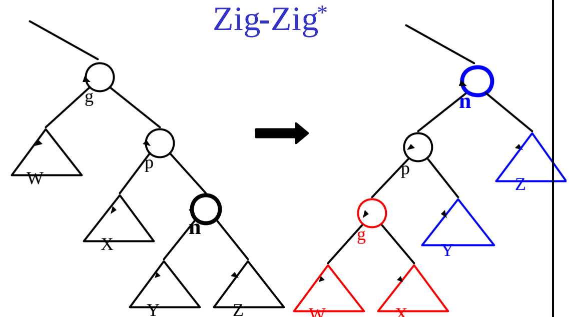


Zig-Zag*



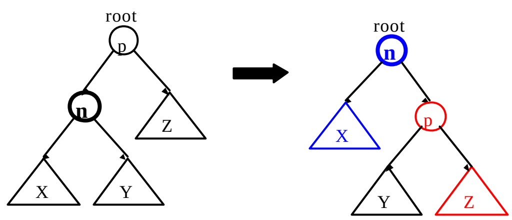
*This is just an AVL double rotation

Zig-Zig*

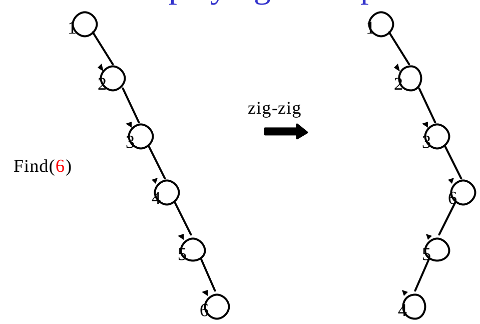


*Yes, the original 1985 paper actually uses this terminology!

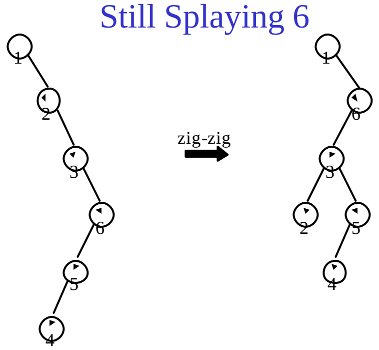
Zig



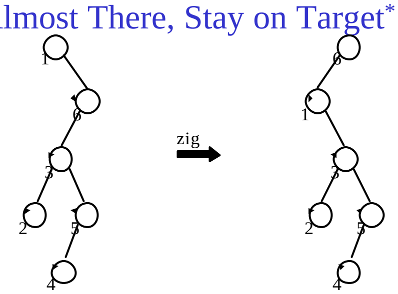
Splaying Example



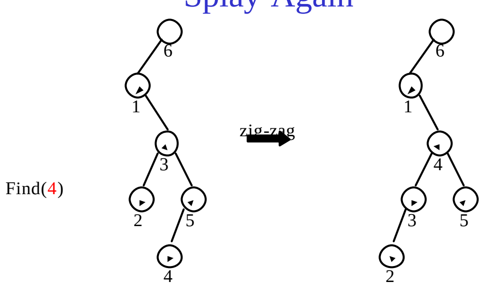
Still Splaying 6



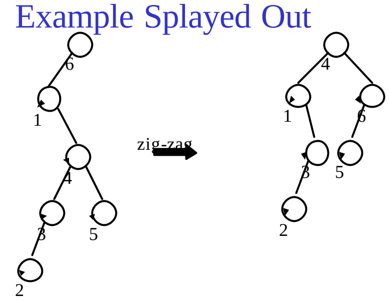
Almost There, Stay on Target*



Splay Again



Example Splayed Out



Why Splaying Helps

- If a node *n* on the access path is at depth *d* before the splay, it's at about depth *d/2* after the splay
  - Exceptions are the root, the child of the root, and the node splayed
- Overall, nodes which are below nodes on the access path tend to move closer to the root
- Splaying gets amortized O(log n) performance.

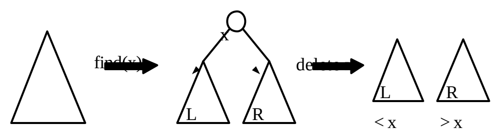
Splay Operations: Find

- Find the node in normal BST manner
- Splay the node to the root

Splay Operations: Insert

- Insert the node in normal BST manner
- Splay the node to the root

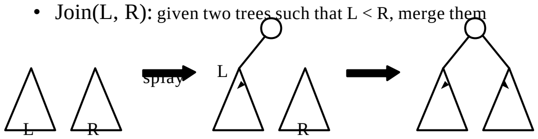
Splay Operations: Remove



Now what?

Join

- Join(L, R): given two trees such that L < R, merge them

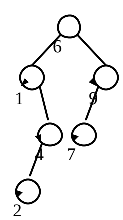


- Splay on the maximum element in L then attach R

Proof by analogy is ***not*** *a valid proof technique!*

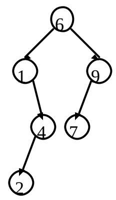
Insert Example

Insert(5)



Delete Example

Delete(4)



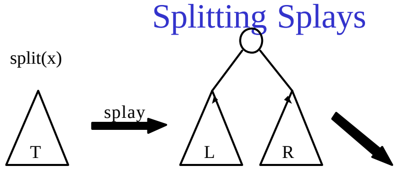
Nifty Splay Operation: Splitting

- Split(T, x) creates two BSTs L and R:
  - all elements of T are in either L or R (**T = L È R**)
  - all elements in L are ≤ x
  - all elements in R are ≥ x
  - L and R share no elements (**L Ç R = Æ**)

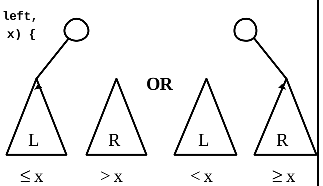
How do we split a splay tree?

Splitting Splays

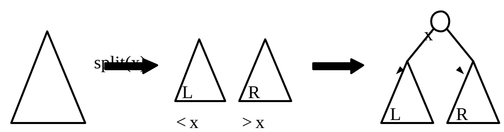
split(x)




```
void split(Node * root, Node *& left,
           Node *& right, Object x) {
  Node * target = root->find(x);
  splay(target);
  if (target < x) {
    left = target->left;
    target->left = NULL;
    right = target;
  }
  ...
}
```



Pssstt: Another Way to Insert




```
void insert(Node *& root, Object x) {
  Node * left, * right;
  split(root, left, right, x);
  root = new Node(x, left, right);
}
```

Interesting note: split-and-insert was the original algorithm. But insert-and-splay has better constants

Splay Tree Summary

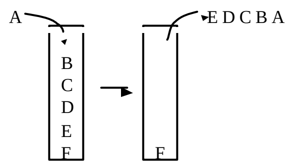
- All operations are in amortized O(log n) time
- Splaying can be done top-down; better because:
  - only one pass
  - no recursion or parent pointers necessary
- Splay trees are *very* effective search trees
  - Relatively simple
  - No extra fields required
  - Excellent *locality* properties: frequently accessed keys are cheap to find

Interlude: Amortized Analysis

- Consider *any* sequence of operations applied to a data structure
  - *Your worst enemy could choose the sequence!*
- Some operations may be fast, others slow
- Goal:
  - Show that the average time per operation is still good

$$\frac{\text{total time for n operations}}{n}$$

Stack ADT



- Stack operations
  - push
  - pop
  - isEmpty
- Stack property: if x is on the stack before y is pushed, then x will be popped after y is popped

What is biggest problem with an array implementation?

Stretchy Stack Implementation

```
int * data;
int maxsize;
int top;

Push(e){
  if (top == maxsize){
     temp = new int[2*maxsize];
     for (i=0;i<maxsize;i++) temp[i]=data[i]; ;
     delete data;
     data = temp;
     maxsize = 2*maxsize; }
  else { data[++top] = e; }
```

Best case Push $\in$ O( )

Worst case Push $\in$ O( )

Stretchy Stack Amortized Analysis

- Consider sequence of n operations

  push(3); push(19); push(2); …
- What is the max number of stretches?
- What is the total time?
  - let's say a regular push takes time a, and stretching an array contain k elements takes time bk.
- Amortized time =

Series

- Arithmetic series: $\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$
- Geometric series: $\sum_{i=0}^{N} A^i = \frac{A^{N+1}-1}{A-1}$

$$\sum_{i=0}^{n} 2^i = \frac{2^{n+1}-1}{2-1} = 2^{n+1}-1$$

$$\sum_{i=0}^{\log n} 2^i = \frac{2^{\log n+1}-1}{2-1} = (2^{\log n})2^1 - 1 = 2n-1$$

Moral of the Story

- For more complicated analyses, this procedure is formalized with the idea of *credit*
- We said that a splay was O(log n)
  - Always *invest* log n per splay
  - For an easy splay, *bank* the leftover money
  - For a hard splay, *use* money from the bank
  - Prove there's *always* enough money in the bank for any operation

Bug Brian or Hannah for more references on amortized analysis!