CSE 326: Data Structures
Sorting It All Out

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Calendar

- **Today: Finish Sorting**
  - **Read Weiss Ch 7 (skip 7.8)**
- Friday, Feb. 15th: Disjoint Sets & Union Find
  - Read Weiss Ch 8
  - Some written homework problems to be due Wednesday, Feb. 20th
- Monday, Feb. 18th: President's Day, no class
- Wednesday, Feb. 20th: Graph Algorithms
  - Weiss Ch 9 + additional material from lecture notes
  - Several lectures
- Monday, Feb 25th: Word-counting project due
- Various specialized data structures & algorithms
  - Mergeable heaps, quad-trees, Huffman codes, …
- Friday, March 8th: final written homework due
- Friday, March 15th: Last day of class
  - Final programming project – building and solving mazes – due

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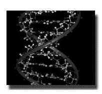
Sorting HUGE Data Sets

- *US Telephone Directory:*
  - 300,000,000 records
    - 64-bytes per record
      - Name: 32 characters
      - Address: 54 characters
      - Telephone number: 10 characters
  - About 2 gigabytes of data
  - Sort this on a machine with 128 MB RAM…
- Other examples?



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MergeSort Good for Something!

- Basis for most external sorting routines
- Can sort any number of records using a tiny amount of main memory
  - in extreme case, only need to keep 2 records in memory at any one time!

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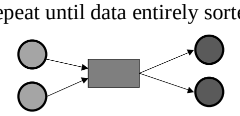
External MergeSort

- Split input into two "tapes" (or areas of disk)
- Merge tapes so that each group of 2 records is sorted
- Split again
- Merge tapes so that each group of 4 records is sorted
- Repeat until data entirely sorted



log N passes

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Better External MergeSort

- Suppose main memory can hold M records.
- Initially read in groups of M records and sort them (*e.g.* with QuickSort).
- Number of passes reduced to log(N/M)

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Sorting by Comparison: Summary

- Sorting algorithms that only compare adjacent elements are $\Theta(N^2)$ worst case – but may be $\Theta(N)$ best case
- HeapSort and MergeSort - $\Theta(N \log N)$ both best and worst case
- QuickSort $\Theta(N^2)$ worst case but $\Theta(N \log N)$ best and average case
- *Any* comparison-based sorting algorithm is $\Omega(N \log N)$ worst case
- External sorting: MergeSort with $\Theta(\log N/M)$ passes

but not quite the end of the story…

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BucketSort

- If all keys are 1…K
- Have array of K buckets (linked lists)
- Put keys into correct bucket of array
  – linear time!
- BucketSort is a *stable* sorting algorithm:
  – Items in input with the same key end up in the same order as when they began
- Impractical for large K…

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RadixSort

- Radix = "The base of a number system" (Webster's dictionary)
  – *alternate terminology: radix is number of bits needed to represent 0 to base-1; can say "base 8" or "radix 3"*
- Used in 1890 U.S. census by Hollerith
- Idea: BucketSort on each digit, bottom up.

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The Magic of RadixSort

- Input list:
  126, 328, 636, 341, 416, 131, 328
- BucketSort on lower digit:
  341, 131, 126, 636, 416, 328, 328
- BucketSort result on next-higher digit:
  416, 126, 328, 328, 131, 636, 341
- BucketSort that result on highest digit:
  126, 131, 328, 328, 341, 416, 636

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Inductive Proof that RadixSort Works

- Keys: K-digit numbers, base B
  – (that wasn't hard!)
- Claim: after i^{th} BucketSort, least significant i digits are sorted.
  – Base case: i=0. 0 digits are sorted.
  – Inductive step: Assume for i, prove for i+1.
    Consider two numbers: X, Y. Say X_i is i^{th} digit of X:
    - $X_{i+1} < Y_{i+1}$ then $i+1^{th}$ BucketSort will put them in order
    - $X_{i+1} > Y_{i+1}$, same thing
    - $X_{i+1} = Y_{i+1}$, order depends on last i digits. Induction hypothesis says already sorted for these digits because BucketSort is **stable**

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Running time of Radixsort

- N items, K digit keys in base B
- How many passes?
- How much work per pass?
- Total time?

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Running time of Radixsort

- N items, K digit keys in base B
- How many passes? K
- How much work per pass? N + B
  - just in case B>N, need to account for time to empty out buckets between passes
- Total time? O(K(N+B))

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RadixSorting Strings example

	5th pass	4th pass	3rd pass	2nd pass	1st pass
String 1	z	i	p	p	y
String 2	z	a	p		
String 3	a	n	t	s	
String 4	f	l	a	p	s

NULLs are just like fake characters

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Evaluating Sorting Algorithms

- What factors other than asymptotic complexity could affect performance?
- Suppose two algorithms perform exactly the same number of instructions. Could one be better than the other?

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Example Memory Hierarchy Statistics

Name	Extra CPU cycles used to access	Size
L1 (on chip) cache	0	32 KB
L2 cache	8	512 KB
RAM	35	256 MB
Hard Drive	500,000	8 GB

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The Memory Hierarchy Exploits Locality of Reference

- Idea: *small* amount of *fast* memory
- Keep *frequently* used data in the *fast* memory
- LRU replacement policy
  - Keep recently used data in cache
  - To free space, remove Least Recently Used data

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So what?

- Optimizing use of cache can make programs way faster
- One TA made RadixSort 2x faster, rewriting to use cache better!
- Not just for sorting

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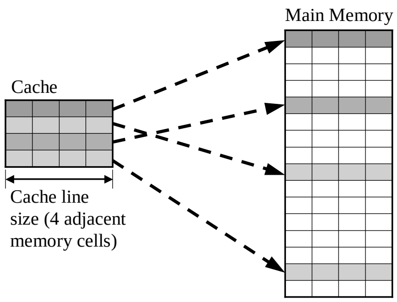
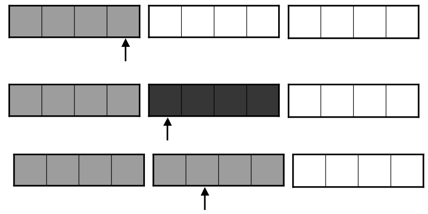
Cache Details (simplified)

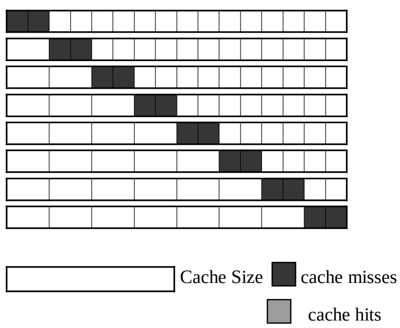


Traversing an Array

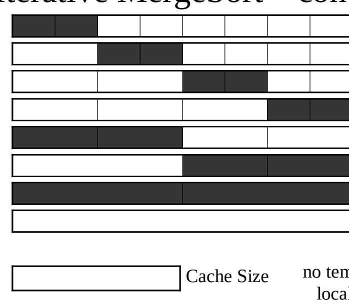


- One miss for every 4 accesses in a traversal

Iterative MergeSort

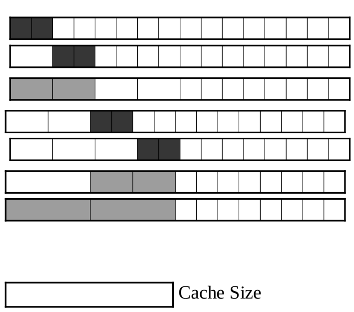


Iterative MergeSort – cont'd

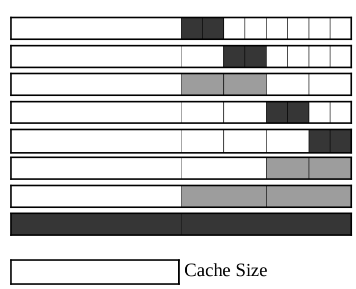


no temporal locality!

"Tiled" MergeSort – better



"Tiled" MergeSort – cont'd



QuickSort

- Initial partition causes a lot of cache misses
- As subproblems become smaller, they fit into cache
- Good cache performance

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Radix Sort – Very Naughty

- On each BucketSort
  - Sweep through input list – cache misses along the way (bad!)
  - Append to output list – indexed by pseudo-random digit (ouch!)

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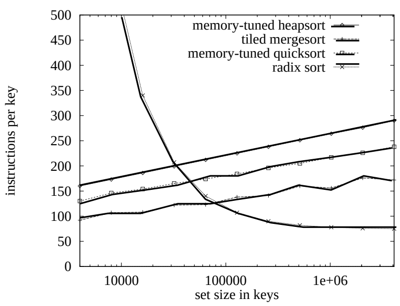
Instruction Count



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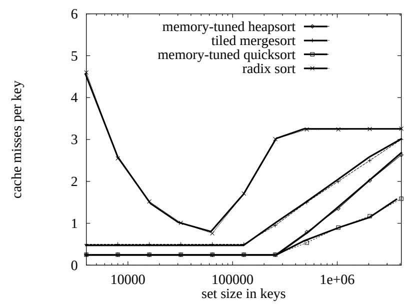
Cache Misses



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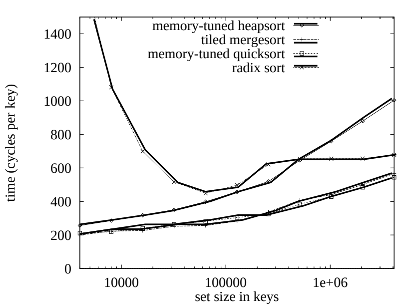
Sorting Execution Time



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Conclusions

- Speed of cache, RAM, and external memory has a huge impact on sorting (and other algorithms as well)
- Algorithms with same asymptotic complexity may be best for different kinds of memory
- Tuning algorithm to improve cache performance can offer large improvements (iterative vs. tiled mergesort)

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