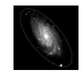
CSE 326: Data Structures
Lecture #17
Heuristic Graph Search

Henry Kautz

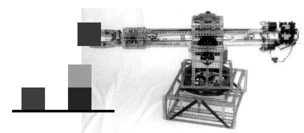
Winter Quarter 2002

Huge Graphs



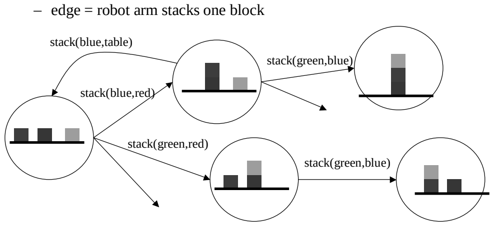
- Consider some really huge graphs…
  - All cities and towns in the World Atlas
  - All stars in the Galaxy
  - All ways 10 blocks can be stacked

Huh???



Implicitly Generated Graphs

- A huge graph may be implicitly specified by rules for generating it on-the-fly
- Blocks world:
  - vertex = relative positions of all blocks
  - edge = robot arm stacks one block



Blocks World

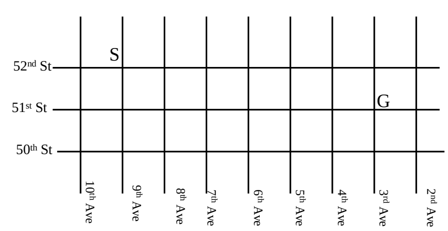
- Source = initial state of the blocks
- Goal = desired state of the blocks
- Path source to goal = sequence of actions (program) for robot arm!
- n blocks ≈ n^n vertices
- 10 blocks ≈ 10 billion vertices!

Problem: Branching Factor

- Cannot search such huge graphs exhaustively. Suppose we know that goal is only *d* steps away.
- Dijkstra's algorithm is basically breadth-first search (modified to handle arc weights)
- Breadth-first search (or for weighted graphs, Dijkstra's algorithm) – If out-degree of each node is 10, potentially visits 10^d vertices
  - 10 step plan = 10 billion vertices visited!

An Easier Case

- Suppose you live in Manhattan; what do you do?

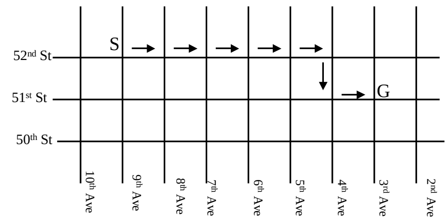


Best-First Search

- The *Manhattan distance* (Δ x+ Δ y) is an estimate of the distance to the goal
  - a heuristic value
- Best-First Search
  - Order nodes in priority to minimize estimated distance to the goal h(n)
- Compare: BFS / Dijkstra
  - Order nodes in priority to minimize distance from the start

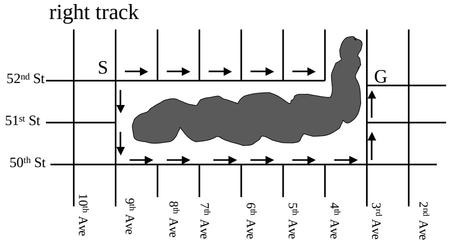
Best First in Action

- Suppose you live in Manhattan; what do you do?



Problem 1: Led Astray

- Eventually will expand vertex to get back on the right track

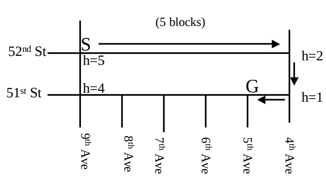


Problem 2: Optimality

- With Best-First Search, are you *guaranteed* a shortest path is found when
  - goal is first seen?
  - when goal is removed from priority queue (as with Dijkstra?)

Sub-Optimal Solution

- No! Goal is by definition at distance 0: will be removed from priority queue immediately, even if a shorter path exists!



Synergy?

- Dijkstra / Breadth First guaranteed to find *optimal* solution
- Best First often visits *far fewer* vertices, but may not provide optimal solution
  - *Can we get the best of both?*

A* ("A star")

- Order vertices in priority queue to minimize (distance from start) + (estimated distance to goal)

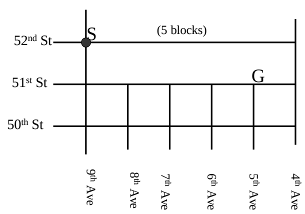
$f(n) = g(n) + h(n)$

$f(n)$ = priority of a node
$g(n)$ = true distance from start
$h(n)$ = heuristic distance to goal

Optimality

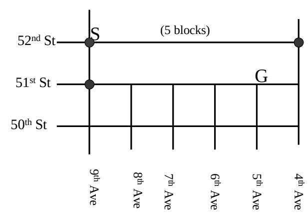
- Suppose the estimated distance (h) is *always* less than or equal to the true distance to the goal
  - heuristic is a *lower bound on true distance*
- Then: when the goal is removed from the priority queue, we are guaranteed to have found a shortest path!

Problem 2 Revisited



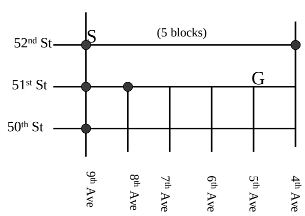
vertex	g(n)	h(n)	f(n)
52nd & 9th	0	5	5

Problem 2 Revisited



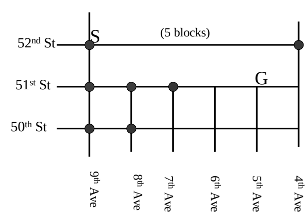
vertex	g(n)	h(n)	f(n)
52nd & 4th	5	2	7
51st & 9th	1	4	5

Problem 2 Revisited



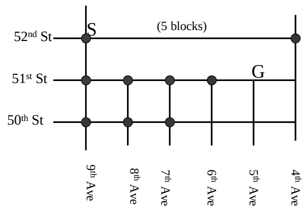
vertex	g(n)	h(n)	f(n)
52nd & 4th	5	2	7
51st & 8th	2	3	5
50th & 9th	2	5	7

Problem 2 Revisited



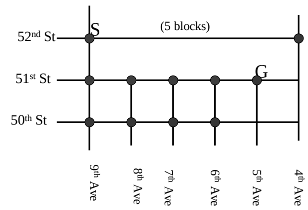
vertex	g(n)	h(n)	f(n)
52nd & 4th	5	2	7
51st & 7th	3	2	5
50th & 9th	2	5	7
50th & 8th	3	4	7

Problem 2 Revisited



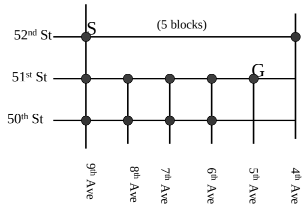
vertex	g(n)	h(n)	f(n)
52^{nd} & 4^{th}	5	2	7
51^{st} & 6^{th}	4	1	5
50^{th} & 9^{th}	2	5	7
50^{th} & 8^{th}	3	4	7
50^{th} & 7^{th}	4	3	7

Problem 2 Revisited



vertex	g(n)	h(n)	f(n)
52^{nd} & 4^{th}	5	2	7
51^{st} & 5^{th}	5	0	5
50^{th} & 9^{th}	2	5	7
50^{th} & 8^{th}	3	4	7
50^{th} & 7^{th}	4	3	7

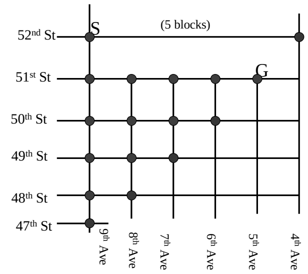
Problem 2 Revisited



vertex	g(n)	h(n)	f(n)
52^{nd} & 4^{th}	5	2	7
50^{th} & 9^{th}	2	5	7
50^{th} & 8^{th}	3	4	7
50^{th} & 7^{th}	4	3	7

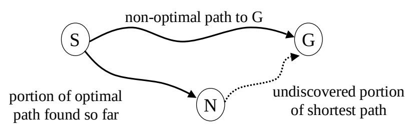
DONE!

What Would Dijkstra Have Done?



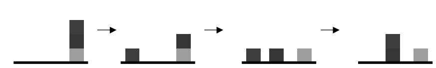
Proof of A* Optimality

- A* terminates when G is popped from the heap.
- Suppose G is popped but the path found isn't optimal:
  priority(G) > optimal path length c
- Let P be an optimal path from S to G, and let N be the last vertex on that path that has been *visited but not yet popped.*
  There must be such an N, otherwise the optimal path would have been found.
  priority(N) = g(N) + h(N) ≤ c
- So N should have popped before G can pop. Contradiction.



What About Those Blocks?

- "Distance to goal" is not always physical distance
- Blocks world:
  - distance = number of stacks to perform
  - heuristic lower bound = number of blocks out of place



out of place = 2, true distance to goal = 3

Other Real-World Applications

- Routing finding – computer networks, airline route planning
- VLSI layout – cell layout and channel routing
- Production planning – “just in time” optimization
- Protein sequence alignment
- Many other “NP-Hard” problems
  - A class of problems for which no exact polynomial time algorithms exist – so heuristic search is the best we can hope for

Coming Up

- How to make Depth First Search optimal
- Other graph problems
  - Connected components
  - Spanning trees
  - Max-Flow
- Other cool data structures & algorithms
  - Search trees for graphical data
  - Huffman codes
  - Mergeable heaps