CSE 326: Data Structures
Lecture #6
From Lists to Trees

Henry Kautz

Winter 2002

Questions...

1. What is a call stack?

2. Could you write a compiler that did **not** use one?

3. What data structure does a printer queue use?

Sparse Matrices

- ➢ Sparse matrices

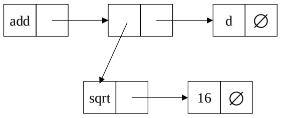
18	0	33	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	99	0	0
0	0	0	0	0	0
0	0	0	0	0	27

  – what does this remind us of?
  – how could we represent it?

Lists of Lists

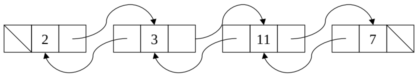
- ➢ LISP
  – programming used in AI, math, functional programing
  – lists (of lists)$^\infty$

(add (sqrt 16) d)

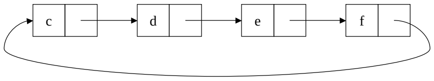


Other Data Structures for Lists

- ➢ Doubly Linked List - *when useful?*



- ➢ Circular List - *when useful?*

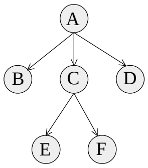


Why Do We Need Trees?

- ➢ Lists, Stacks, and Queues represent linear sequences
- ➢ Data often contain hierarchical relationships that cannot be expressed as a linear ordering
  – File directories or folders on your computer
  – Moves in a game
  – Employee hierarchies in organizations and companies
  – Family trees
  – Classification hierarchies (e.g. phylum, family, genus, species)

Tree Jargon

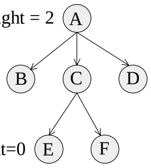
- Basic terminology:
  - nodes and edges
  - root
  - subtrees
  - parent
  - children, siblings
  - leaves
  - path
  - ancestors
  - descendants
  - path length



Note: Arrows denote directed edges
Trees always contain directed edges
but arrows are often omitted.

More Tree Jargon

- Length of a path = number of edges
- Depth of a node N = length of path from root to N
- Height of node N = length of longest path from N to a leaf
- Depth and height of tree = ?



depth=0, height = 2

depth = 2, height=0

Definition and Tree Trivia

Recursive Definition of a Tree:
A tree is a set of nodes that is
a. an empty set of nodes, or
b. has one node called the root from which zero or more trees (subtrees) descend.

- A tree with N nodes always has ___ edges
- Two nodes in a tree have at most how many paths between them?
- Can a non-zero path from node N reach node N again?
- Does depth of nodes in a non-zero path increase or decrease?

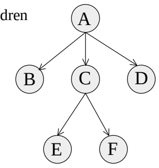
Definition and Tree Trivia

Recursive Definition of a Tree:
A tree is a set of nodes that is
a. an empty set of nodes, or
b. has one node called the root from which zero or more trees (subtrees) descend.

- A tree with N nodes always has N-1 edges
- Two nodes in a tree have at most one path between them
- Can a non-zero path from node N reach node N again?
  - No! Trees can never have cycles.
- Does depth of nodes in a non-zero path increase or decrease?
  - Depth always increases in a non-zero path

Implementation of Trees

- Obvious Pointer-Based Implementation: Node with value and pointers to children
  - Problem: Do not usually know number of children for a node in advance. How many pointers should we allocate space for?
- Better Implementation: 1st Child/Next Sibling Representation
  - Each node has 2 pointers: one to its first child and one to next sibling
  - Can handle arbitrary number of children
  - Exercise: Draw the representation for this tree…



Application: Arithmetic Expression Trees

Example Arithmetic Expression:

A + (B * (C / D))

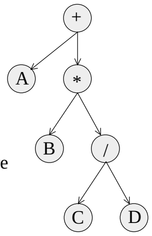
How would you express this as a tree?

Application: Arithmetic Expression Trees

Example Arithmetic Expression:

A + (B * (C / D))

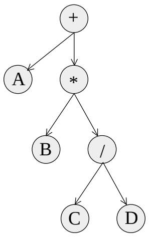
Tree for the above expression:



- Used in most compilers
- No parenthesis need – use tree structure
- Can speed up calculations e.g. replace / node with C/D if C and D are known
- Calculate by traversing tree (how?)

Traversing Trees

- ➢ Preorder: Root, then Children
  - + A * B / C D
- ➢ Postorder: Children, then Root
  - A B C D / * +
- ➢ Inorder: Left child, Root, Right child
  - A + B * C / D



Example Code for Recursive Preorder

```
void print_preorder ( TreeNode * T)
{ TreeNode * P;
  if  ( T == NULL ) return;
  else  {  print_element(T-> Element);
           P = T -> FirstChild;
           while (P != NULL) {
               print_preorder ( P );
               P = P -> NextSibling; }
         }
}
```

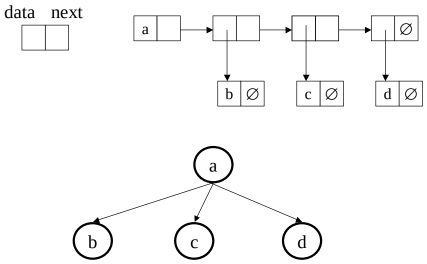
What is the running time for a tree with N nodes?

Preorder Traversal with a Stack

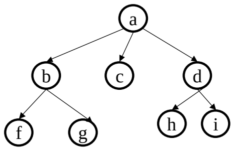
```
void Stack_Preorder (TreeNode * T, Stack S)
{
if (T == NULL) return; else push(T,S);
while (!isempty(S)) {
   T = pop(S);
   print_element(T -> Element);
   if  (T -> Right !=  NULL) push(T -> Right, S);
   if  (T -> Left !=  NULL)  push(T -> Left, S);
   }
}
```

What is the running time for a tree with N nodes?

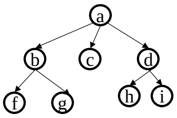
Alternative: Nested List Implementation of a Tree

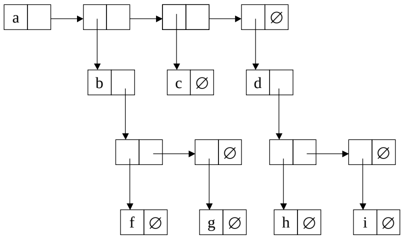


How To Represent?



How To Represent?





Recursive Preorder for Nested List Implementation

```
void print_preorder ( Node * T)
{     Node * P;

      if ( T == NULL ) return;
      print_element(T-> data);
      P = T -> next;
      while (P != NULL) {
            if (type(P->data)!= (Node*))
                  signal error;
            print_preorder ( P->data );
            P = P->next;
      }
}
```

Determining Type of a Node

```
class node {
public: enum Tag { I, P };
private:
 union { int i; node * p; };
 Tag tag;
 void check(Tag t){ if (tag!=t) error();}
public:
 Tag get_tag() { return tag; }
 int & ival() { check(I); return i; }
 node * & pval() { check(P); return p; }
```

Creating and Setting Nodes

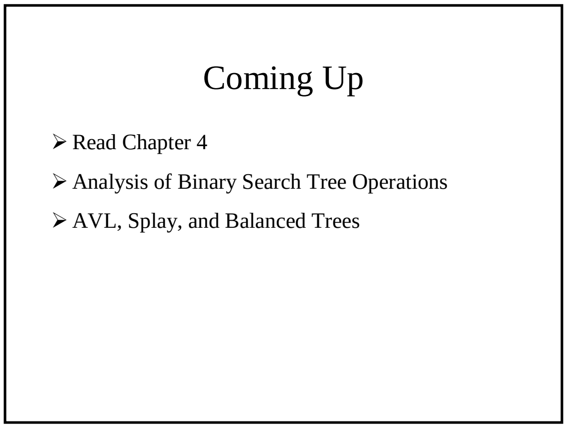
```
class node {
...
public:
 // Creating a new node
 node(int ii) { i=ii; tag=I; }
 node(node * pp) { p=pp; tag=P; }
 // Changing the value in a node
 void set(int ii) { i=ii; tag=I; }
 void set(node * pp) { p=pp; tag=P; }
};
```

Binary Trees

- Every node has at most two children
  - Most popular tree in computer science
- Given N nodes, what is the minimum depth of a binary tree?
- What is the maximum depth of a binary tree?

Binary Trees

- Every node has at most two children
  - Most popular tree in computer science
- Given N nodes, what is the minimum depth of a binary tree?
  - At depth d, you can have $N = 2^d$ to $2^{d+1}-1$ nodes (a full tree)
  - So, minimum depth d is: $\log N \leq d \leq \log(N+1)-1$ or $\Theta(\log N)$
- What is the maximum depth of a binary tree?
  - Degenerate case: Tree is a linked list!
  - Maximum depth = N-1
- Goal: Would like to keep depth at around log N to get better performance than linked list for operations like Find.



Coming Up

- Read Chapter 4
- Analysis of Binary Search Tree Operations
- AVL, Splay, and Balanced Trees