CSE 326: Data Structures

Topic #17: Let's get connected… minimally!

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Today's Outline

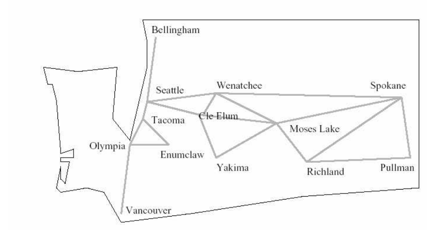
- Discuss Quiz #5
- Finish Shortest Path Problems
- **Minimum Spanning Trees**

Before we move on…

- Dijkstra's algorithm, as we saw, gives the minimum distance between *s* and *t*.
- Can we modify it to output the shortest path between *s* and *t*?

node	known	cost	
A			

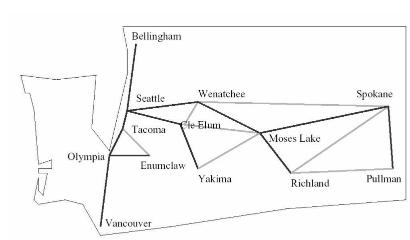
An Application: Moving Around Washington



What's the fastest way from Seattle to Pullman?

Answer:

A Different Application: Communication in Washington



What's the cheapest inter-city network?

Is This Problem Really Different?

- Is knowing Dijkstra's algorithm enough to solve the latter application?

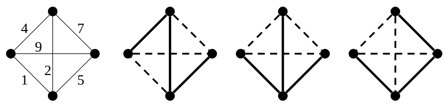
Yes? Then how?

No? Then why?

Spanning Tree, MST

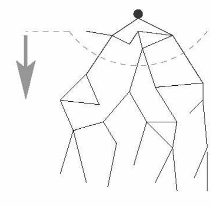
Spanning tree: a *subgraph* of a connected, undirected graph that

1. touches all vertices in the graph (*spans* the graph)
2. forms a tree (is connected and contains no cycles)

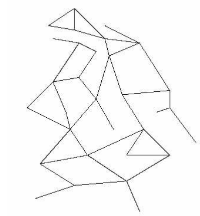


Minimum spanning tree: the spanning tree with the least total edge cost.

Two Different Approaches



Prim's Algorithm
Almost identical to Dijkstra's



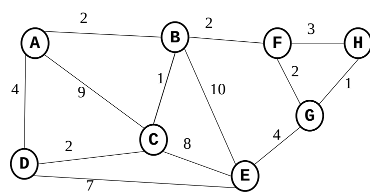
Kruskals's Algorithm
Completely different!

Prim's Algorithm for MST

A *node-based* greedy algorithm
Builds MST by greedily adding nodes

1. Select a node to be the "root"
   - mark it as known
   - Update cost of all its neighbors
2. While there are unknown nodes left in the graph
   a. Select an unknown <u>node *b* with the smallest cost</u> from some known node *a*
   b. Mark *b* as known
   c. Add (*a*, *b*) to MST
   d. Update cost of all nodes adjacent to *b*

Prim's Algorithm: Example



node	known	cost
A		
B		
C		
D		
E		
F		
G		
H		

Prim's Algorithm: Complexity

- Depends on what?

- How long does each step take?

Runtime:

Prim's Algorithm: Correctness

- A proof very similar to that of Dijkstra's algorithm works!

(left as exercise)

Kruskal's Algorithm for MST

An *edge-based* greedy algorithm

Builds MST by greedily adding edges

1. Initialize with
   - empty MST
   - all vertices marked unconnected
   - all edges unmarked
2. While there are still unmarked edges
   a. Pick the <u>lowest cost edge</u> **`(u,v)`** and mark it
   b. If **`u`** and **`v`** are not already connected, add **`(u,v)`** to the MST and mark **`u`** and **`v`** as connected to each other

Doesn't it sound familiar?

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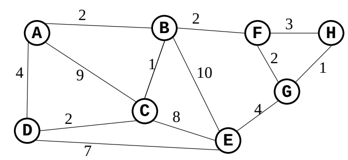
Kruskal's Algorithm: Example



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Kruskal's Algorithm: Complexity

- Depends, of course, on the data structures/ADT used. What *should* we use?

- How long does each step take?

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Kruskal's Algorithm: Correctness

It clearly generates a spanning tree. Call it T_K.

Suppose T_K is *not* minimum:

Pick another spanning tree T_{min} with *lower cost* than T_K

Pick the smallest edge $e_1=(u,v)$ in T_K that is not in T_{min}

T_{min} already has a path p in T_{min} from u to v

$\Rightarrow$ Adding e_1 to T_{min} will create a cycle in T_{min}

Pick an edge e_2 in p that Kruskal's algorithm considered *after* adding e_1 (must exist: u and v unconnected when e_1 considered)

$\Rightarrow$ $\text{cost}(e_2) \geq \text{cost}(e_1)$

$\Rightarrow$ can replace e_2 with e_1 in T_{min} without increasing cost!

Keep doing this until T_{min} is identical to T_K

$\Rightarrow$ T_K must also be minimal – contradiction!

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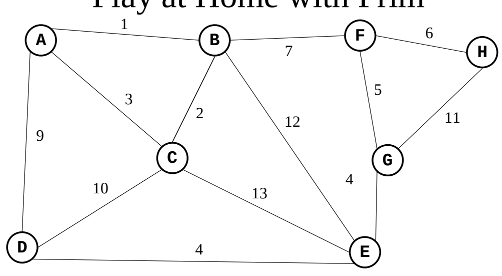
Play at Home with Prim



1. Starting at node A, find the MST using Prim's method.
(continue on next slide)

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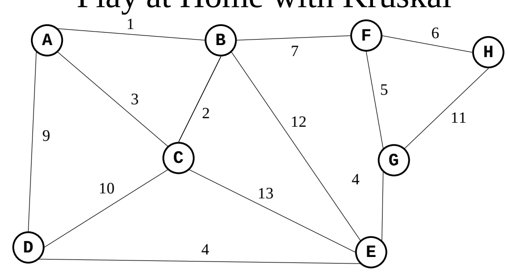
Play at Home with Kruskal



2. Now find the MST using Kruskal's method.
3. Under what conditions will these methods give the same result?
4. What data structures should be used for Kruskal's? Running time?

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To Do

- Read sections 9.1 – 9.3, 9.5

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