CSE 326: Data Structures
Mind Your Priority Queues

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Today's Outline

- Finish Asymptotic Analysis
- Questions
- Trees Review
- Priority Queues
- Heaps
- d-Heaps

Simplifying Recurrences

1. Given some equation for the running time:
   e.g. T(n) = log[floor(n/2)]
2. Solve the recursive equation
   - For an upper-bound analysis, you can optionally simplify the equation to something larger
     e.g. T(n) = T(floor(n/2)) + 1 ✍ T(n) < T(n/2) + 1
   - For a lower-bound analysis, you can optionally simplify the equation to something smaller
     e.g. T(n) = 2T(N/2 + 1) + 1 ✍ T(n) > 2T(N/2) + 1

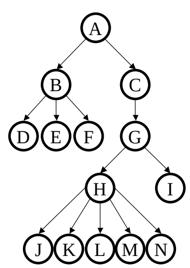
The One Page Cheat Sheet

- **Calculating series:**
  e.g. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
  1. Brute force (Section 1.2.3)
  2. Induction (Section 1.2.5)
  3. Memorize simple ones!
- **Solving recurrences:**
  e.g. T(n) = T(N/2) + 1
  1. Expansion (example in class)
  2. Induction (Section 1.2.5, slides)
  3. Telescoping (later…)
- **General proofs (Section 1.2.5)**
  e.g. How many edges in a binary tree?
  1. Counterexample
  2. Induction
  3. Contradiction

(we'll see more examples coming up)

Tree Review

root:
leaf:
child:
parent:
sibling:
ancestor:
descendent:
subtree:



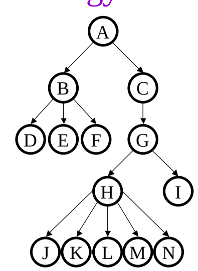
More Tree Terminology

depth:

height:

degree:

branching factor:

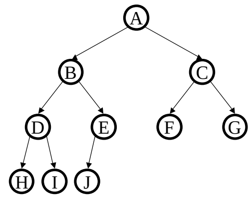


One More Tree Terminology Slide

binary:

n-ary:

complete:



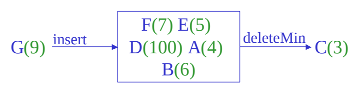
Back to Queues

- Some applications
  - ordering CPU jobs
  - simulating events
  - picking the next search site
- Problems?
  - short jobs **should go first**
  - earliest (simulated time) events **should go first**
  - most promising sites **should be searched first**

Remember ADTs?

Priority Queue ADT

- Priority Queue operations
  - create
  - destroy
  - insert
  - deleteMin
  - is_empty



- Priority Queue property: for two elements in the queue, *x* and *y*, if *x* has a lower priority value than *y*, *x* will be deleted before *y*

Applications of the Priority Q

- Hold jobs for a printer in order of length
- Store packets on network routers in order of urgency
- Simulate events
- Select symbols for compression
- Sort numbers
- Anything *greedy*

Naïve Priority Q Data Structures

- Unsorted array:
  - *insert:*
  - *deleteMin:*
- Sorted array:
  - *insert:*
  - *deleteMin:*

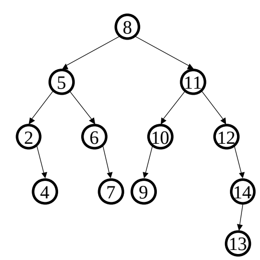
Binary Search Tree Priority Q Data Structure (that's a mouthful)

Average performance
insert:

deleteMin:

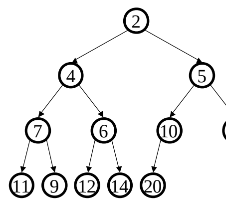
Problems
1.

2.



Binary Heap Priority Q Data Structure

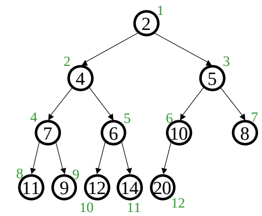
- Heap-order property
  - parent's key is less than children's keys
  - result: minimum is always at the top
- Structure property
  - complete tree with fringe nodes packed to the left
  - result: depth is always O(log n); next open location always known



How do we find the minimum?

Nifty Storage Trick

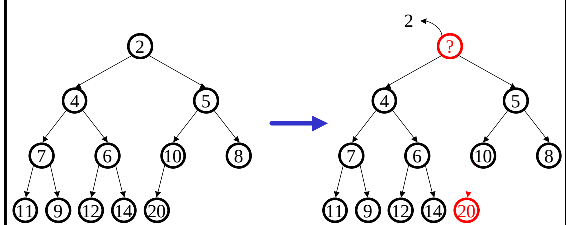
- Calculations:
  - child:
  - parent:
  - root:
  - next free:



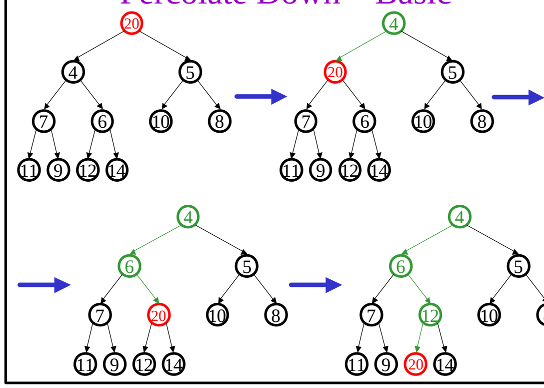
0	1	2	3	4	5	6	7	8	9	10	11	12	
12	2	4	5	7	6	10	8	11	9	12	14	20	

DeleteMin

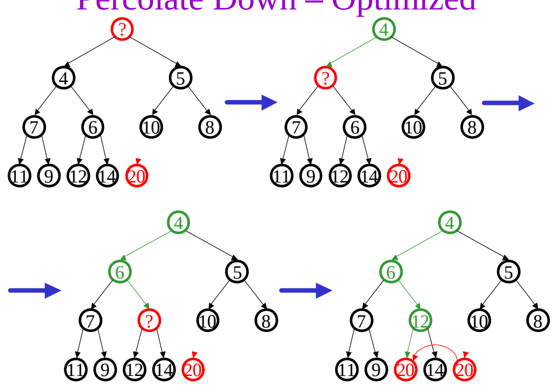
pqueue.deleteMin()



Percolate Down – Basic



Percolate Down – Optimized



DeleteMin Code (Optimized)

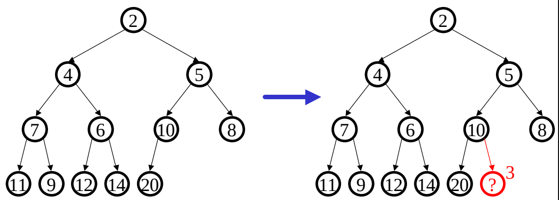
```
Object deleteMin() {
  assert(!isEmpty());
  returnVal = Heap[1];
  size--;
  newPos =
    percolateDown(1,
        Heap[size+1]);
  Heap[newPos] =
    Heap[size + 1];
  return returnVal;
}
```

runtime:

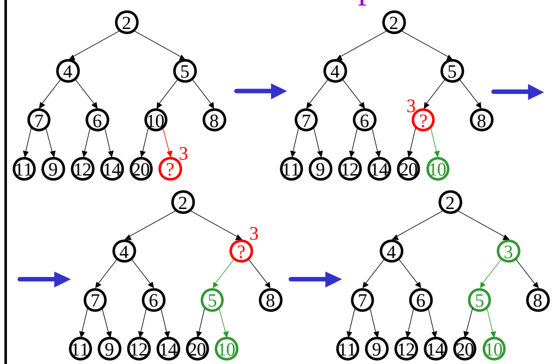
```
int percolateDown(int hole,
                  Object val) {
while (2*hole <= size) {
    left = 2*hole;
    right = left + 1;
    if (right <= size &&
        Heap[right] < Heap[left])
      target = right;
    else
      target = left;

    if (Heap[target] < val) {
      Heap[hole] = Heap[target];
      hole = target;
    }
    else
      break;
  }
  return hole;
}
```

Insert

`pqueue.insert(3)`



Percolate Up



Insert Code

```
void insert(Object o) {
  assert(!isFull());
  size++;
  newPos =
    percolateUp(size,o);
  Heap[newPos] = o;
}
```

```
int percolateUp(int hole,
                Object val) {
  while (hole > 1 &&
         val < Heap[hole/2])
    Heap[hole] = Heap[hole/2];
    hole /= 2;
  }
  return hole;
}
```

runtime:

Other Priority Queue Operations

- **decreaseKey**
  - given a pointer to an object in the queue, reduce its priority value

  Solution: change priority and ___________________________

- **increaseKey**
  - given a pointer to an object in the queue, increase its priority value

  Solution: change priority and ____________________________

Still More Priority Queue Operations

- **remove**
  - given a pointer to an object in the queue, remove it

  Solution: set priority to negative infinity, percolate up to root and deleteMin

- **buildHeap**

  Naïve solution:

  Running time:

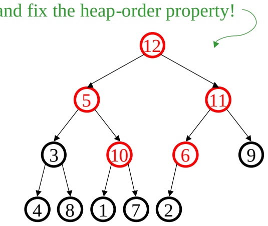
  Can we do better?

BuildHeap

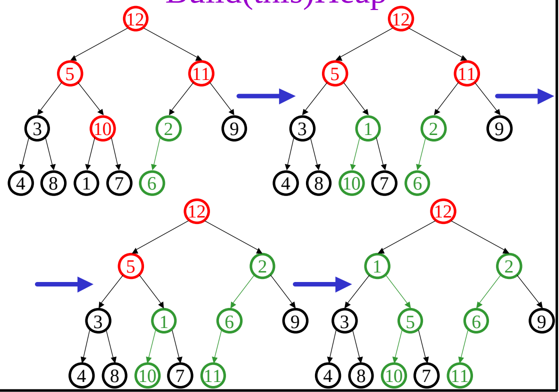
Floyd's Method. Thank you, Floyd.

12	5	11	3	10	6	9	4	8	1	7	2

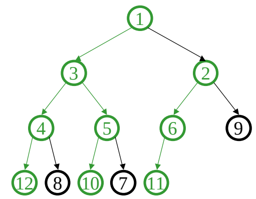
pretend it's a heap and fix the heap-order property!



Build(this)Heap



Finally…



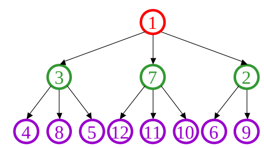
runtime:

Thinking about Heaps

- Observations
  - finding a child/parent index is a multiply/divide by two
  - operations jump widely through the heap
  - each operation looks at only two new nodes
  - inserts are at least as common as deleteMins
- Realities
  - division and multiplication by powers of two are **fast**
  - looking at one new piece of data sucks in a cache line
  - with **huge** data sets, disk accesses dominate

Solution: d-Heaps

- Each node has *d* children
- Still representable by array
- Good choices for *d*:
  - optimize performance based on # of inserts/removes
  - choose a power of two for efficiency
  - fit one set of children in a cache line
  - fit one set of children on a memory page/disk block



12	1	3	7	2	4	8	5	12	11	10	6	9

Does this help **insert** or **deleteMin** more?

One More Operation

- Merge two heaps. Ideas?

To Do

- Finish Homework #1
  - Start Homework #2 if you've already finished
- Read chapter 6 in the book

Coming Up

- Mergable Priority Q's
- Leftist heaps
- Skew heaps

- No class on July 4!!!