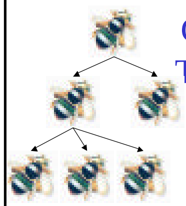

CSE 326: Data Structures
Topic 8: Big, Bad B-Trees

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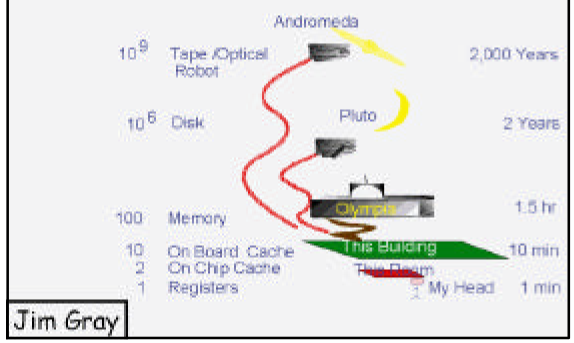
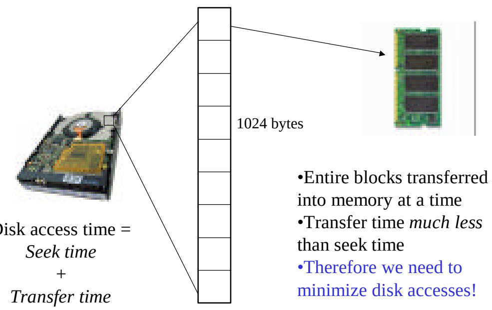
Summer Quarter 2003

Something We Forgot: Disk Acesses



We Want To Minimize Disk Accesses!

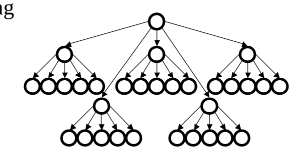


Disk access time =
Seek time
+
Transfer time

- Entire blocks transferred into memory at a time
- Transfer time *much less* than seek time
- Therefore we need to minimize disk accesses!

M-ary Search Tree

- Maximum branching factor of ***M***
- Complete tree has depth = $\log_M N$

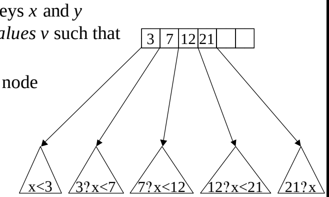


runtime:

Problems with *M*-ary Search Trees

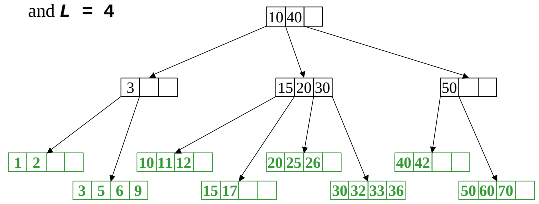
B-Trees

- B-Trees are specialized *M*-ary search trees
- Each node has many keys (max M-1)
  - subtree between two keys *x* and *y* contains leaves with *values v* such that *x* ? *v* < *y*
  - binary search within a node to find correct subtree
- Each node takes one full {*page*, *block*} of memory



Example

B-Tree with **M = 4**
and **L = 4**



B-Tree Properties‡

- Properties
  - maximum branching factor of **M**
  - the root has between 2 and **M** children *or* at most **L** keys
  - other internal nodes have between ?*M/2*? and *M* children
  - internal nodes contain only search keys (no data)
  - All values are stored at the leaves
  - smallest datum between search keys *x* and *y* equals *x*
  - each (non-root) leaf contains between ?*L/2*? and *L* keys
  - all leaves are at the same depth

‡These are technically B^+-Trees

B-Tree Properties

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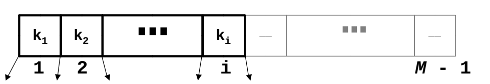
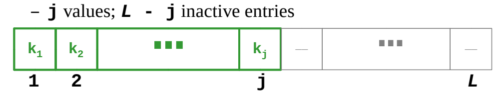
  - each (non-root) leaf contains between ?*L/2*? and *L* keys
  - all leaves are at the same depth
- Result
  - tree is ? $(\log_M n)$ deep
  - all operations run in ? $(\log_M n)$ time
  - operations pull in about **M/2** or **L/2** items at a time

B-Tree Nodes

- Internal node
  - **i** search keys; **i+1** subtrees; **M - i - 1** inactive entries

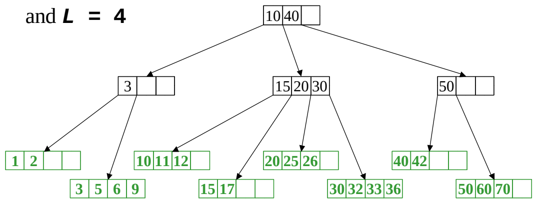


- Leaf
  - **j** values; **L - j** inactive entries

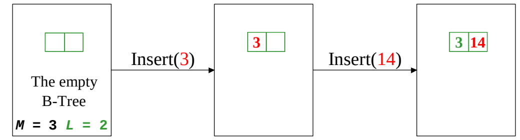


Example Redux

B-Tree with **M = 4**
and **L = 4**

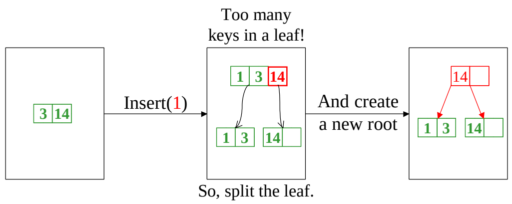


Making a B-Tree

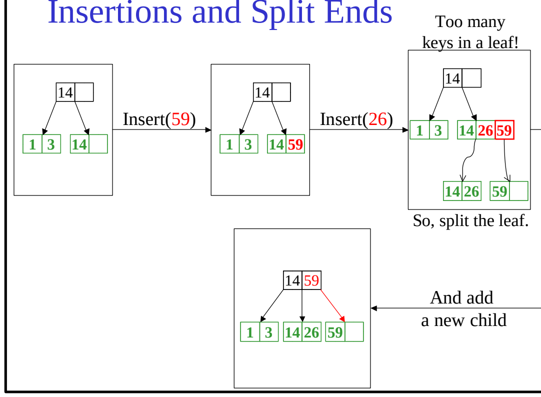


Now, Insert(1)?

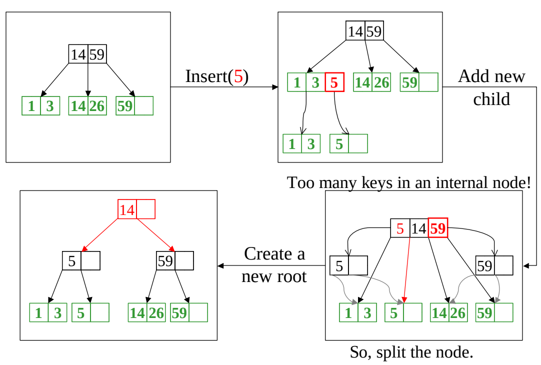
Splitting the Root



Insertions and Split Ends



Propagating Splits

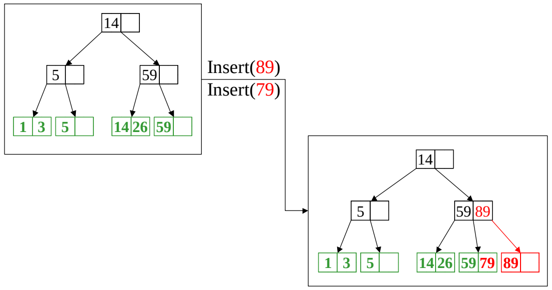


Insertion in Boring Text

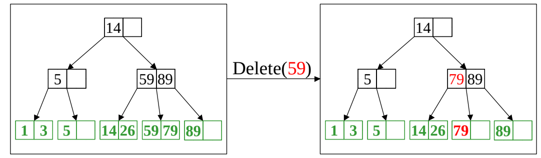
- Insert the key in its leaf
- If the leaf ends up with L+1 items, **overflow**!
  - Split the leaf into two nodes:
    - original with ⌈**(L+1)/2**⌉ items
    - new one with ⌊**(L+1)/2**⌋ items
  - Add the new child to the parent
  - If the parent ends up with **M+1** items, **overflow**!
- If an internal node ends up with M+1 items, **overflow**!
  - Split the node into two nodes:
    - original with ⌈**(M+1)/2**⌉ items
    - new one with ⌊**(M+1)/2**⌋ items
  - Add the new child to the parent
  - If the parent ends up with **M+1** items, **overflow**!
- Split an overflowed root in two and hang the new nodes under a new root

This makes the tree deeper!

After More Routine Inserts

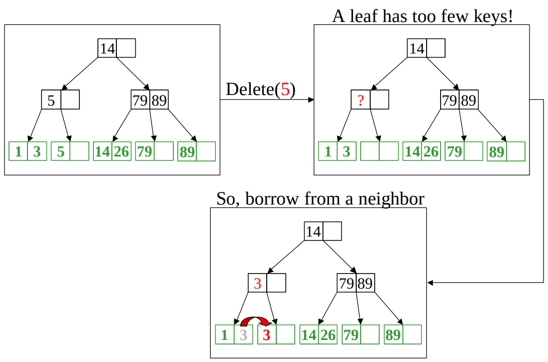


Deletion

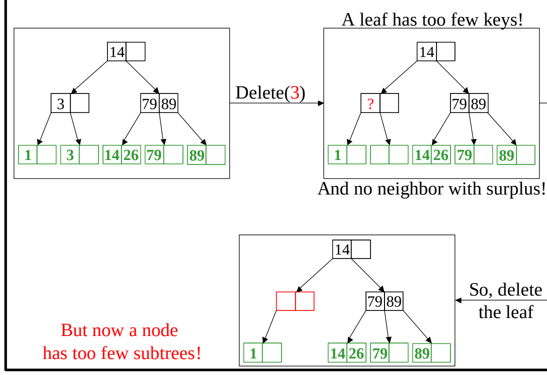


What could go wrong?

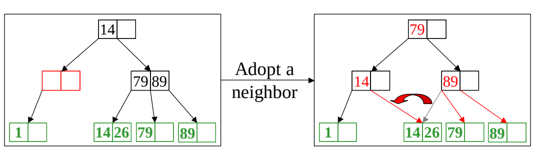
Deletion and Adoption



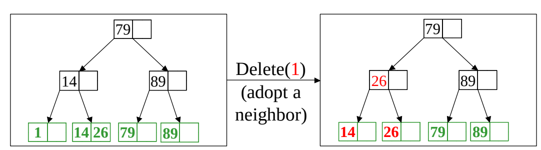
Deletion with Propagation



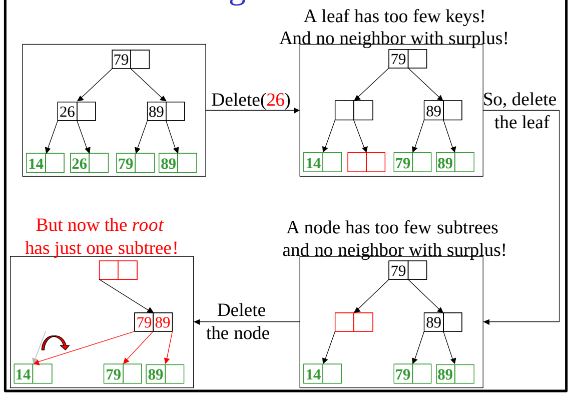
Finishing the Propagation (More Adoption)



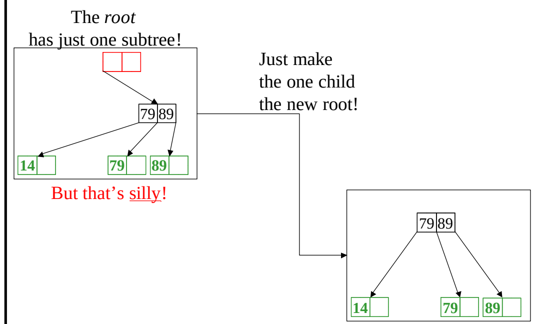
A Bit More Adoption



Pulling out the Root



Pulling out the Root (continued)



Deletion in *Two* Boring Slides of Text

- Remove the key from its leaf
- If the leaf ends up with fewer than ?***L/2***? items, **underflow**!
  - Adopt data from a neighbor; update the parent
  - If borrowing won't work, delete node and divide keys between neighbors
  - If the parent ends up with fewer than ?***M/2***? items, **underflow**!

Why will dumping keys always work if borrowing doesn't?

Deletion Slide Two

- If a node ends up with fewer than ?***M/2***? items, **underflow**!
  - Adopt subtrees from a neighbor; update the parent
  - If borrowing won't work, delete node and divide subtrees between neighbors
  - If the parent ends up with fewer than ?***M/2***? items, **underflow**!
- If the root ends up with only one child, make the child the new root of the tree

This reduces the height of the tree!

B-trees vs AVL trees

We have a database* with 100 million items (100,000,000):

- Depth of AVL Tree
- Depth of B+ Tree with B = 128, L = 64

** A very simple type of database, called "Berkeley Database" is basically a B+-tree*

Thinking about B-Trees

- B-Tree insertion can cause (expensive) splitting and propagation
- B-Tree deletion can cause (cheap) borrowing or (expensive) deletion and propagation
- Propagation is rare if ***M*** and ***L*** are large *(Why?)*
- Repeated insertions and deletion can cause thrashing
- If ***M* = *L* = 128**, then a B-Tree of height 4 will store at least 30,000,000 items

A Tree with Any Other Name

FYI:

- B-Trees with `M = 3`, `L = x` are called 2-3 trees
  - Nodes can have 2 or 3 keys
- B-Trees with `M = 4`, `L = x` are called 2-3-4 trees
  - Nodes can have 2, 3, or 4 keys

Why would we ever use these?

To Do

- Finish Homework #3
  - Don’t forget contest submission!
- Read Chapter 5