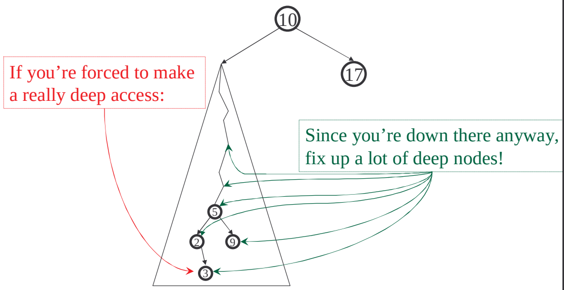
## The Splay Tree Idea

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## Find/Insert in Splay Trees

1. Find or insert a node ***k***
2. **Splay *k* to the root using:**
   zig-zag, zig-zig, or plain old zig rotation

Why could this be good??

1. Helps the new root, ***k***
   - *Great if k is accessed again*
2. And helps many others!
   - *Great if many others on the path are accessed*

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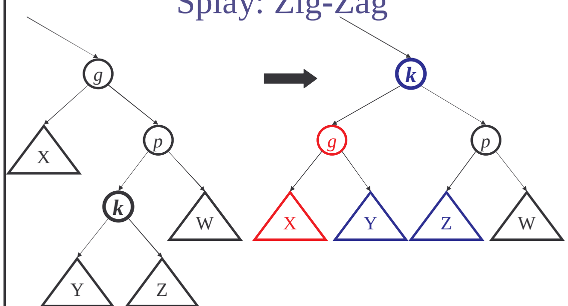
## Splay: Zig-Zag*




*Just like an…

Which nodes improve depth?

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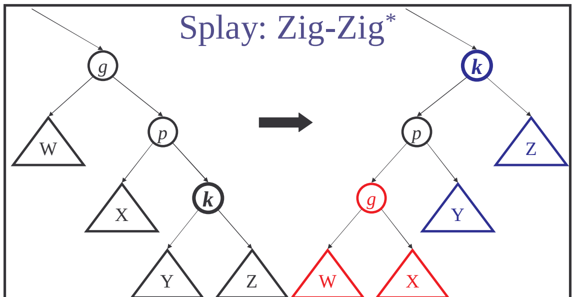
## Splay: Zig-Zig*




*Is this just two AVL single rotations in a row?

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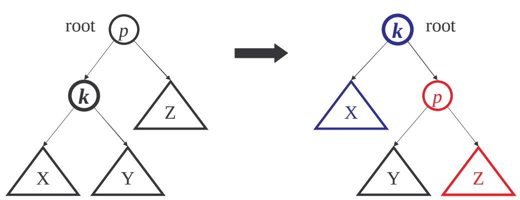
## Special Case for Root: Zig




Relative depth of *p*, Y, Z?

Relative depth of everyone else?

Why not drop zig-zig and just zig all the way?

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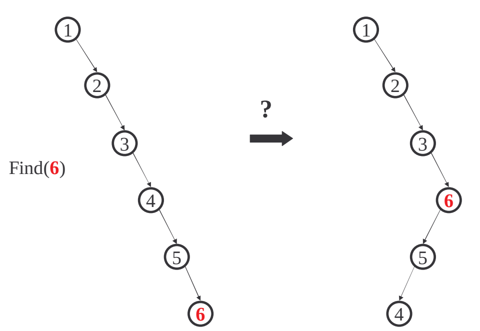
## Splaying Example: Find(6)

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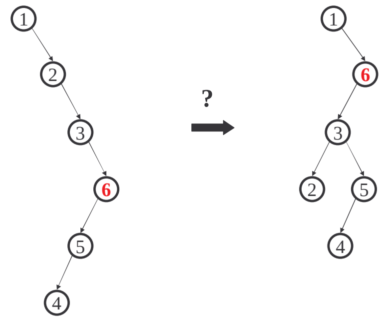
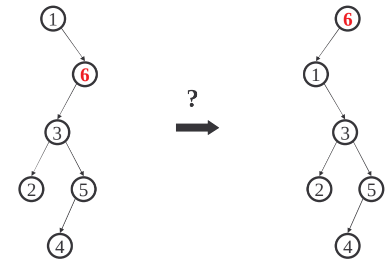
## Still Splaying 6





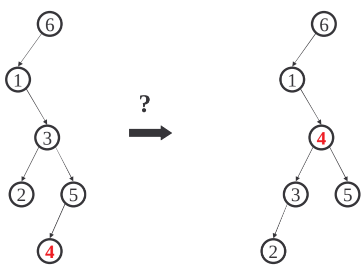

## Finally…





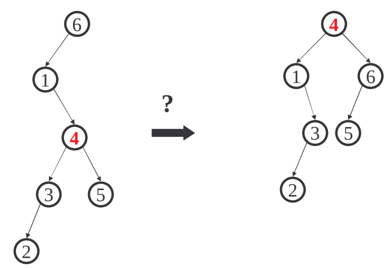

## Another Splay: Find(4)

Find(4)



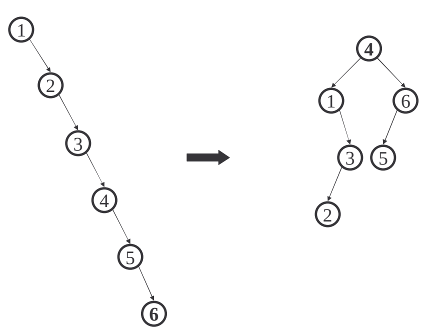



## Example Splayed Out






## Original Tree    Tree after two finds





## But Wait…

What happened here?

Didn't *two* find operations take linear time instead of logarithmic?

What about the amortized O(log *n*) guarantee?

## Why Splaying Helps

- If a node *n* on the access path is at depth *d* before the splay, it's at about depth *d/2* after the splay
- Overall, nodes which are low on the access path tend to move closer to the root
- Splaying gets amortized O(log n) performance.
  (Maybe not now, but soon, and for the rest of the operations.)

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## Practical Benefit of Splaying

- No heights to maintain, no imbalance to check for
  - Less storage per node, easier to code
- Often data that is accessed once, is soon accessed again!
  - Splaying does implicit *caching* by bringing it to the root

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## Splay Operations: Find

- Find the node in normal BST manner
- Splay the node to the root
  - if node <u>not</u> found, splay what would have been its parent

What if we didn't splay?

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## Splay Operations: Insert

- Insert the node in normal BST manner
- Splay the node to the root

What if we didn't splay?

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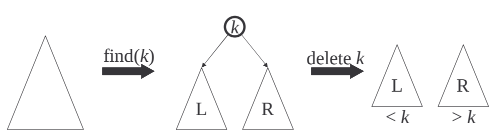
## Splay Operations: Remove




Now what?

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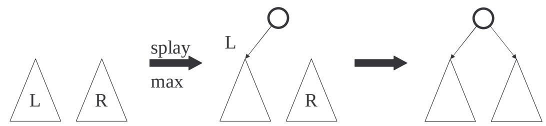
## Join

Join(L, R):
given two trees such that (stuff in L) < (stuff in R), merge them:




**Splay on the maximum element in L, then attach R**

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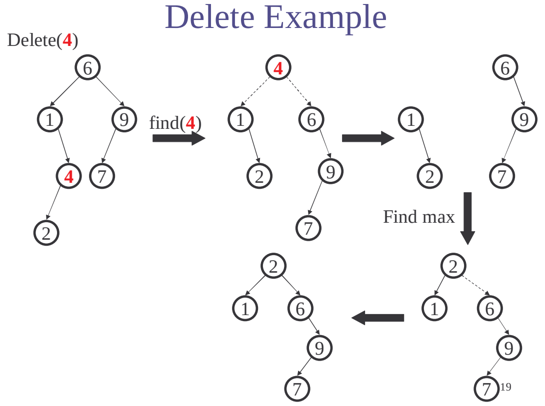
## Delete Example

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## Splay Tree Summary

- All operations are in amortized O(log *n*) time
- Splaying can be done top-down; this may be better because:
  - only one pass
  - no recursion or parent pointers necessary
  - *we didn't cover top-down in class*
- Splay trees are *very* effective search trees
  - Relatively simple
  - No extra fields required
  - Excellent *locality* properties: frequently accessed keys are cheap to find

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## The Memory Hierarchy & Locality

## Why do we need to know about the memory hierarchy/locality?

- One of the assumptions that Big-Oh makes is that all operations take the same amount of time.
- Is that really true?

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## Definitions

**Cycle** – (for our purposes) the time it takes to execute a single simple instruction. (ex. Add 2 registers together)

**Memory Latency** – time it takes to access memory

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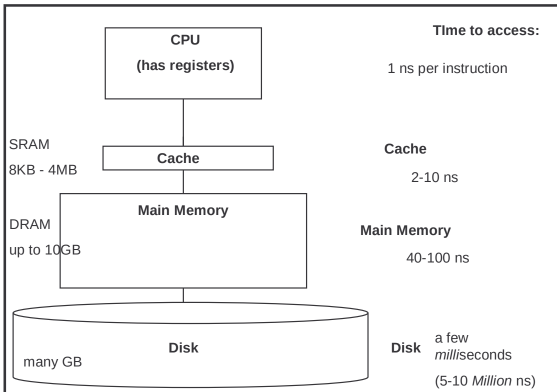


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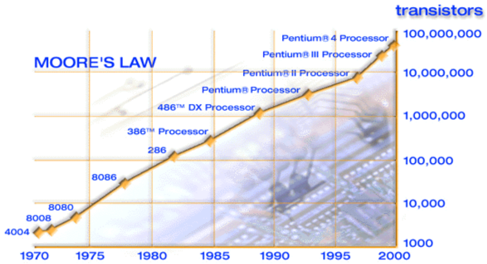
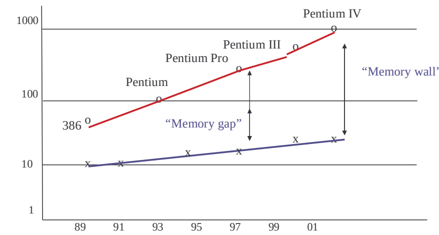
## Moore's Law






## Processor-Memory Performance Gap

- x86 CPU speed (100x over 10 years)






## What can be done?

- **Goal**: Attempt to reduce the number of accesses to the slower levels.
- How?



## Locality

**Temporal Locality** (locality in time) – If an item is referenced, it will tend to be referenced again soon.

**Spatial Locality** (locality in space) – If an item is referenced, items whose addresses are close by will tend to be referenced soon.



## Caches

- Each level is a **sub-set** of the level below.

**Cache Hit** – address requested is in cache

**Cache Miss** – address requested is NOT in cache

**Cache line size** (chunk size) – the number of contiguous bytes that are moved into the cache at one time



## Examples

```
x = a + 6;
y = a + 5;
z = 8 * a;
```

```
x = a[0] + 6;
y = a[1] + 5;
z = 8 * a[2];
```