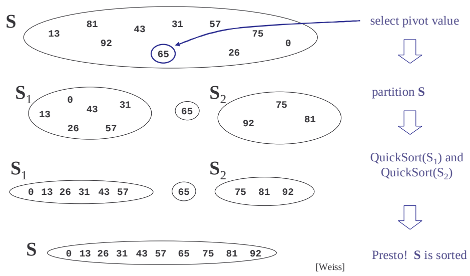
Sorting: *The Big Picture*

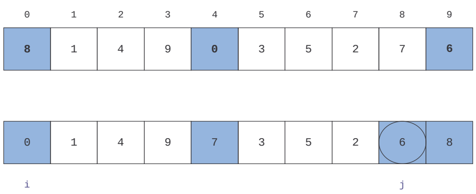
Given *n* comparable elements in an array, sort them in an increasing (or decreasing) order.

Simple algorithms: $O(n^2)$	Fancier algorithms: $O(n \log n)$	Comparison lower bound: $\Omega(n \log n)$	Specialized algorithms: $O(n)$	Handling huge data sets
Insertion sort Selection sort Bubble sort Shell sort …	Heap sort Merge sort Quick sort …		Bucket sort Radix sort	External sorting

The steps of QuickSort

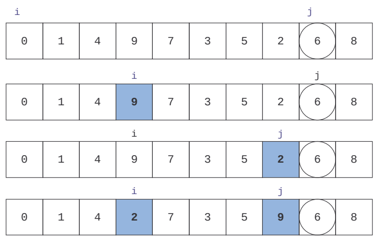


QuickSort Example



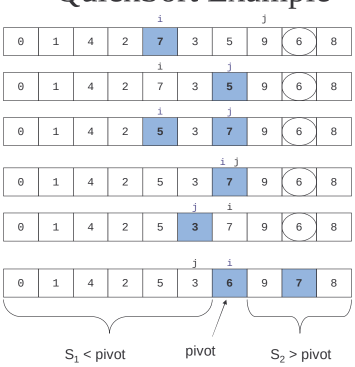
- Choose the pivot as the median of three.
- Place the pivot and the largest at the right and the smallest at the left

QuickSort Example



- Move i to the right to be larger than pivot.
- Move j to the left to be smaller than pivot.
- Swap

QuickSort Example



Recursive Quicksort

```
Quicksort(A[]: integer array, left,right : integer): {
pivotindex : integer;
if left + CUTOFF ≤ right then
  pivot := median3(A,left,right);
  pivotindex := Partition(A,left,right-1,pivot);
  Quicksort(A, left, pivotindex – 1);
  Quicksort(A, pivotindex + 1, right);
else
  Insertionsort(A,left,right);
}
```

Don't use quicksort for small arrays.
CUTOFF = 10 is reasonable.

QuickSort:
Best case complexity

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QuickSort:
Worst case complexity

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QuickSort:
Average case complexity

Turns out to be O(n log n)

See Section 7.7.5 for an idea of the proof.
Don't need to know proof details for this course.

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Features of Sorting Algorithms

- In-place
  - Sorted items occupy the same space as the original items. (No copying required, only O(1) extra space if any.)
- Stable
  - Items in input with the same value end up in the same order as when they began.

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Sort Properties

Are the following:	**stable?**		**in-place?**	
Insertion Sort?	No	Yes	Can Be No	Yes
Selection Sort?	No	Yes	Can Be No	Yes
MergeSort?	No	Yes	Can Be No	Yes
QuickSort?	No	Yes	Can Be No	Yes

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How fast can we sort?

- Heapsort, Mergesort, and Quicksort all run in O(N log N) best case running time
- Can we do any better?
- No, if the basic action is a comparison.

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Sorting Model

- Recall our basic assumption: we can <u>only compare two elements at a time</u>
  - we can only reduce the possible solution space by half each time we make a comparison
- Suppose you are given N elements
  - Assume no duplicates
- How many possible orderings can you get?
  - This is the number of potential inputs the algorithm must separate

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Permutations

- How many possible orderings can you get?
  - Example: a, b, c (N = 3)
  - (a b c), (a c b), (b a c), (b c a), (c a b), (c b a)
  - 6 orderings = 3·2·1 = 3!
  - All the possible permutations of a set of 3 elements
- For N elements
  - N choices for the first position, (N-1) choices for the second position, …, (2) choices, 1 choice
  - N(N-1)(N-2)□(2)(1)= <u>N! possible orderings</u>

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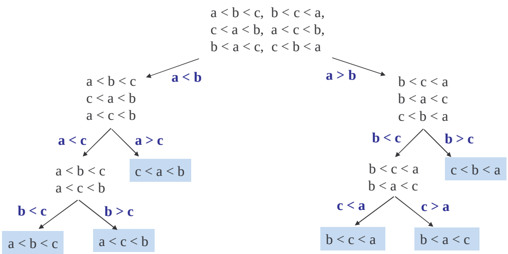
Decision Tree



The leaves contain all the possible orderings of a, b, c

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Lower bound on Height

- A binary tree of height h has **at most** *how many* leaves?

L

- A binary tree with L leaves has height **at least**:

h

- The decision tree has how many leaves:
- So the decision tree has height:

h

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log(*N*!) is Ω(*N*log*N*)

$$\log(N!) = \log\big(N\cdot(N-1)\cdot(N-2)\cdots(2)\cdot(1)\big)$$
$$= \log N + \log(N-1) + \log(N-2) + \cdots + \log 2 + \log 1$$

select just the first N/2 terms

$$\geq \log N + \log(N-1) + \log(N-2) + \cdots + \log\frac{N}{2}$$

each of the selected terms is ≥ logN/2

$$\geq \frac{N}{2}\log\frac{N}{2}$$
$$\geq \frac{N}{2}(\log N - \log 2) = \frac{N}{2}\log N - \frac{N}{2}$$
$$= \Omega(N\log N)$$

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Ω(N log N)

- Run time of any comparison-based sorting algorithm is **Ω(N log N)**
- Can we do better if we don't use comparisons?

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BucketSort (aka BinSort)

If all values to be sorted are *known* to be between 1 and *K*, create an array `count` of size *K*, **increment** counts while traversing the input, and finally output the result.

Example *K*=5. Input = (5,1,3,4,3,2,1,1,5,4,5)

`count` array	
1	
2	
3	
4	
5	

Running time to sort n items?

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BucketSort Complexity: O(*n*+*K*)

- Case 1: *K* is a constant
  – BinSort is linear time
- Case 2: *K* is variable
  – Not simply linear time
- Case 3: *K* is constant but large (e.g. 2^{32})
  – ???

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Fixing impracticality: RadixSort

- Radix = "The base of a number system"
  – We'll use 10 for convenience, but could be anything

- Idea: BucketSort on each **digit**,
  least significant to most significant
  (lsd to msd)

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Radix Sort Example (1st pass)

Input data: 478, 537, 9, 721, 3, 38, 123, 67

Bucket sort by 1's digit

0	1	2	3	4	5	6	7	8	9
	721		3 123				537 67	478 38	9

After 1st pass: 721, 3, 123, 537, 67, 478, 38, 9

This example uses B=10 and base 10 digits for simplicity of demonstration. Larger bucket counts should be used in an actual implementation.

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Radix Sort Example (2nd pass)

After 1st pass: 721, 3, 123, 537, 67, 478, 38, 9

Bucket sort by 10's digit

0	1	2	3	4	5	6	7	8	9
03 09		721 123	537 38			67	478		

After 2nd pass: 3, 9, 721, 123, 537, 38, 67, 478

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Radix Sort Example (3rd pass)

After 2nd pass: 3, 9, 721, 123, 537, 38, 67, 478

Bucket sort by 100's digit

0	1	2	3	4	5	6	7	8	9
003 009 038 067	123			478	537		721		

After 3rd pass: 3, 9, 38, 67, 123, 478, 537, 721

Invariant: after k passes the low order k digits are sorted.

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Radixsort: Complexity

- How many passes?
- How much work per pass?
- Total time?
- Conclusion?
- In practice
  - RadixSort only good for large number of elements with relatively small values
  - Hard on the cache compared to MergeSort/QuickSort

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Internal versus External Sorting

- Need sorting algorithms that minimize disk/tape access time
- **External sorting –** Basic Idea:
  - Load chunk of data into RAM, sort, store this “run” on disk/tape
  - Use the Merge routine from Mergesort to merge runs
  - Repeat until you have only one run (one sorted chunk)
  - Text gives some examples

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