BucketSort (aka BinSort)

If all values to be sorted are *known* to be between 1 and *K*, create an array `count` of size *K*, **increment** counts while traversing the input, and finally output the result.

Example *K*=5. Input = (5,1,3,4,3,2,1,1,5,4,5)

`count` array	
1	
2	
3	
4	
5	

Running time to sort n items?

BucketSort Complexity: O(*n*+*K*)

- Case 1: *K* is a constant
  - BinSort is linear time
- Case 2: *K* is variable
  - Not simply linear time
- Case 3: *K* is constant but large (e.g. 2^{32})
  - ???

Fixing impracticality: RadixSort

- Radix = "The base of a number system"
  - We'll use 10 for convenience, but could be anything
- Idea: BucketSort on each **digit**, least significant to most significant (lsd to msd)

Radix Sort Example (1st pass)

Input data: 478, 537, 9, 721, 3, 38, 123, 67

Bucket sort by 1's digit

0	1	2	3	4	5	6	7	8	9
	721		3 123				537 67	478 38	9

After 1st pass: 721, 3, 123, 537, 67, 478, 38, 9

This example uses B=10 and base 10 digits for simplicity of demonstration. Larger bucket counts should be used in an actual implementation.

Radix Sort Example (2nd pass)

After 1st pass: 721, 3, 123, 537, 67, 478, 38, 9

Bucket sort by 10's digit

0	1	2	3	4	5	6	7	8	9
03 09		721 123	537 38			67	478		

After 2nd pass: 3, 9, 721, 123, 537, 38, 67, 478

Radix Sort Example (3rd pass)

After 2nd pass: 3, 9, 721, 123, 537, 38, 67, 478

Bucket sort by 100's digit

0	1	2	3	4	5	6	7	8	9
003 009 038 067	123			478	537		721		

After 3rd pass: 3, 9, 38, 67, 123, 478, 537, 721

Invariant: after k passes the low order k digits are sorted.

Radixsort: Complexity

- How many passes?
- How much work per pass?
- Total time?
- Conclusion?
- In practice
  - RadixSort only good for large number of elements with relatively small values
  - Hard on the cache compared to MergeSort/QuickSort

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Internal versus External Sorting

- Need sorting algorithms that minimize disk/tape access time
- **External sorting** – Basic Idea:
  - Load chunk of data into RAM, sort, store this "run" on disk/tape
  - Use the Merge routine from Mergesort to merge runs
  - Repeat until you have only one run (one sorted chunk)
  - Text gives some examples

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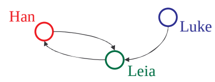
Graphs

Chapter 9 in Weiss

Graph… ADT?

- Not quite an ADT… operations not clear
- A formalism for representing relationships between objects

  Graph `G = (V,E)`
  - *Set of vertices*: $V = \{v_1, v_2, \ldots, v_n\}$
  - *Set of edges*: $E = \{e_1, e_2, \ldots, e_m\}$ where each e_i connects two vertices (v_{i1}, v_{i2})




```
V = {Han, Leia, Luke}
E = {(Luke, Leia),
     (Han, Leia),
     (Leia, Han)}
```

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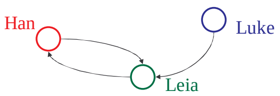
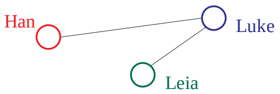
Graph Definitions

In *directed* graphs, edges have a specific direction:



In *undirected* graphs, they don't (edges are two-way):



v is *adjacent* to **u** if $(u,v) \in E$

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More Definitions: Simple Paths and Cycles

A *simple path* repeats no vertices (except that the first can be the last):
p = {Seattle, Salt Lake City, San Francisco, Dallas}
p = {Seattle, Salt Lake City, Dallas, San Francisco, Seattle}

A *cycle* is a path that starts and ends at the same node:
p = {Seattle, Salt Lake City, Dallas, San Francisco, Seattle}
p = {Seattle, Salt Lake City, Seattle, San Francisco, Seattle}

A *simple cycle* is a cycle that repeats no vertices except that the first vertex is also the last (in undirected graphs, no edge can be repeated)

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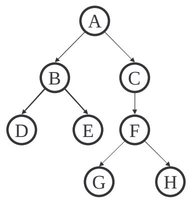
Trees as Graphs

- Every tree is a graph!
- Not all graphs are trees!

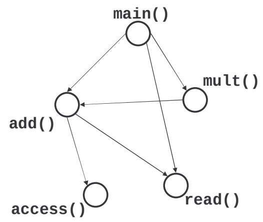
A graph is a tree if
- There are *no cycles* (directed or undirected)
- There is a *path* from the root *to every node*



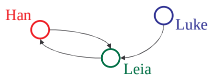
Directed Acyclic Graphs (DAGs)

DAGs are directed graphs with no (directed) cycles.

Aside: *If program call-graph is a DAG, then all procedure calls can be in-lined*



Graph Representations



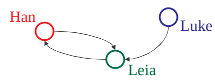
0. List of vertices + list of edges
1. 2-D matrix of vertices (marking edges in the cells)
   "adjacency matrix"
2. List of vertices each with a list of adjacent vertices
   "adjacency list"

Vertices and edges may be labeled

Things we might want to do:
- iterate over vertices
- iterate over edges
- iterate over vertices adj. to a vertex
- check whether an edge exists

Representation 1: Adjacency Matrix

A **|V| x |V|** array in which an element **(u,v)** is true if and only if there is an edge from **u** to **v**



	Han	Luke	Leia
Han			
Luke			
Leia			

space requirements:

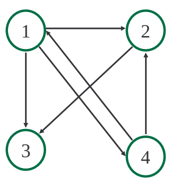
runtime:

Representation

- adjacency **matrix**:

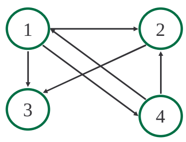
$$A[u][v] = \begin{cases} \text{weight} & \text{, if } (u, v) \in E \\ 0 & \text{, if } (u, v) \notin E \end{cases}$$

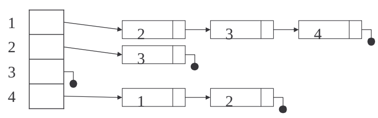
	1	**2**	**3**	**4**
1				
2				
3				
4				



Representation

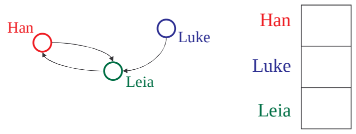
- adjacency **list:**





Representation 2: Adjacency List

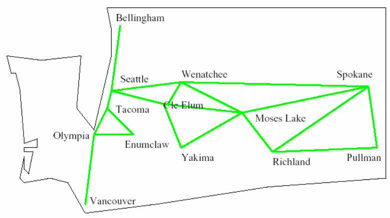
A **|V|**-ary list (array) in which each entry stores a list (linked list) of all adjacent vertices



space requirements:

runtime:

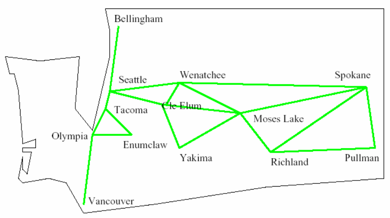
Some Applications: Moving Around Washington



What's the *shortest way* to get from Seattle to Pullman?

Edge labels:

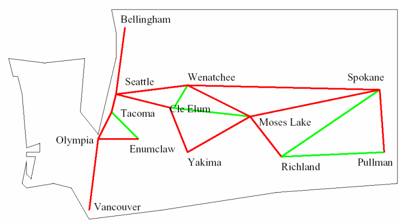
Some Applications: Moving Around Washington



What's the *fastest way* to get from Seattle to Pullman?

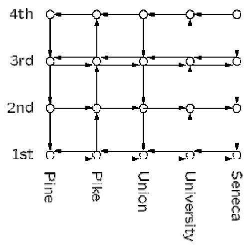
Edge labels:

Some Applications: Reliability of Communication



If Wenatchee's phone exchange *goes down*, can Seattle still talk to Pullman?

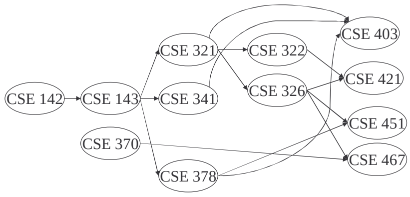
Some Applications: Bus Routes in Downtown Seattle



If we're at 3rd and Pine, how can we get to 1st and University using Metro?

Application: Topological Sort

Given a directed graph, **G = (V,E)**, output all the vertices in **V** such that no vertex is output before any other vertex with an edge to it.



Is the output unique?

Topological Sort: Take One

1. Label each vertex with its *in-degree* (# of inbound edges)
2. **While** there are vertices remaining:
    a. Choose a vertex *v* of *in-degree zero*; output *v*
    b. Reduce the in-degree of all vertices adjacent to *v*
    c. Remove *v* from the list of vertices

Runtime:


```
void Graph::topsort(){
  Vertex v, w;

  labelEachVertexWithItsIn-degree();

  for (int counter=0; counter < NUM_VERTICES;
                                    counter++){
    v = findNewVertexOfDegreeZero();

    v.topologicalNum = counter;
    for each w adjacent to v
      w.indegree--;
  }
}
```