

CSE 326 Summer 2006

Assignment 1

Due: Wednesday, June 28

For all algorithm and data structure design problems, please provide elegant pseudocode and an adequate explanation of your methods. It is often helpful to include small examples demonstrating the method. Put your name at the top of each sheet of paper that you turn in.

1. Weiss, 1.8a, 1.8b, 1.12a, 2.1.
2. The classic way to evaluate a polynomial is called Horner's rule. It can be stated recursively as follows. Let $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$. To compute $p(c)$ for some constant c , first evaluate $q(c)$ where $q(x) = a_1 + a_2x + \dots + a_nx^{n-1}$ recursively; then $p(c) = a_0 + cq(c)$.
 - (a) Provide a base case for this method, and show the method works mathematically.
 - (b) For a polynomial of degree n , as a function of n , how many additions and how many multiplications are used to evaluate the polynomial in Horner's rule?
 - (c) Give pseudocode for a recursive function implementing Horner's rule. Assume that the coefficients of the polynomial are stored in an array A with $A[i] = a_i$, $i = 0 \dots k$. You will need to think about the parameters of your function carefully. Keep in mind that the subarray $A[i \dots k]$ can be thought of as a polynomial of degree $k - i$.
3. Consider the following general recurrence $T(1) \leq d$ and $T(n) \leq aT(n/b) + cn$ for $n \geq 1$. Show that:
 - (a) If $a < b$ then $T(n) = O(n)$.
 - (b) If $a = b$ then $T(n) = O(n \log n)$.
 - (c) If $a > b$ then $T(n) = O(n^{\log_b a})$, i.e. n to the power \log base b of a .

You may assume that n is a power of b in your argument. A very good strategy would be to determine the constants and low order terms hidden by the big O notation.

4. Consider the following algorithm for counting the number of 1s in the binary representation of an integer:

```
countOnes(integer x)
  if x = 0
    return 0
  else
    return (x mod 2) + countOnes(x/2)
```

Prove using induction that `countOnes` correctly returns the number of 1s in the binary representation of x .