CSE 326: Data Structures

Binary Search Trees

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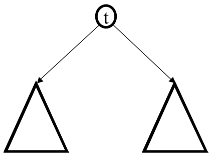
Tree Calculations

Recall: height is max number of edges from root to a leaf

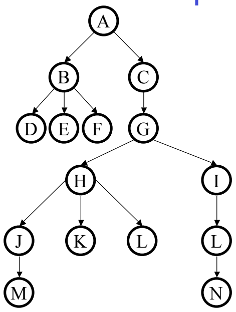


Find the height of the tree...

runtime:

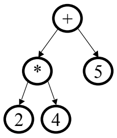
Tree Calculations Example

How high is this tree?



More Recursive Tree Calculations: Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree



(an expression tree)

Three types:

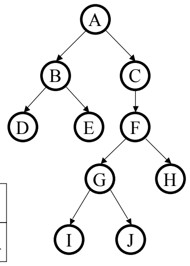
- Pre-order: Root, left subtree, right subtree
- In-order: Left subtree, root, right subtree
- Post-order: Left subtree, right subtree, root

Traversals

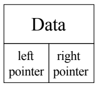
```
void traverse(BNode t){
  if (t != NULL)
    traverse (t.left);
    print t.element;
    traverse (t.right);
  }
}
```

Binary Trees

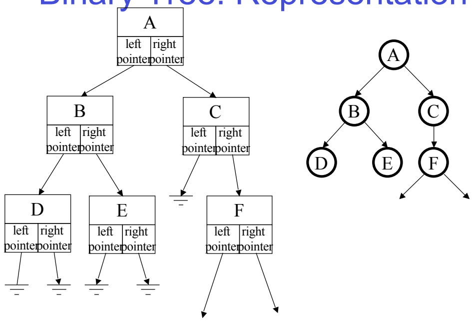
- Binary tree is
  - a root
  - left subtree *(maybe empty)*
  - right subtree *(maybe empty)*



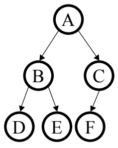
- Representation:



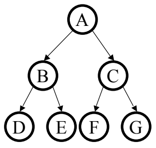
Binary Tree: Representation



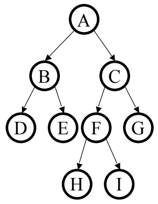
Binary Tree: Special Cases



Complete Tree



Perfect Tree



Full Tree

Binary Tree: Some Numbers!

For binary tree of height *h*:

- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

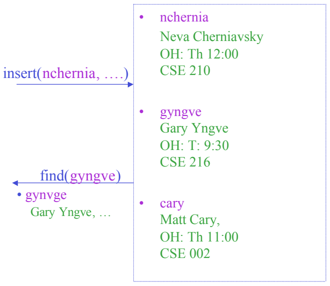
ADTs Seen So Far

- Stack
  - Push
  - Pop
- Queue
  - Enqueue
  - Dequeue
- Priority Queue
  - Insert
  - DeleteMin

What about decreaseKey?

The Dictionary ADT

- Data:
  - a set of (key, value) pairs
- Operations:
  - Insert (key, value)
  - Find (key)
  - Remove (key)



insert(nchernia, ….)

find(gyngve)

- gynvge
  Gary Yngve, …

- nchernia
  Neva Cherniavsky
  OH: Th 12:00
  CSE 210
- gyngve
  Gary Yngve
  OH: T: 9:30
  CSE 216
- cary
  Matt Cary,
  OH: Th 11:00
  CSE 002

*The Dictionary ADT is sometimes called the "**Map ADT**"*

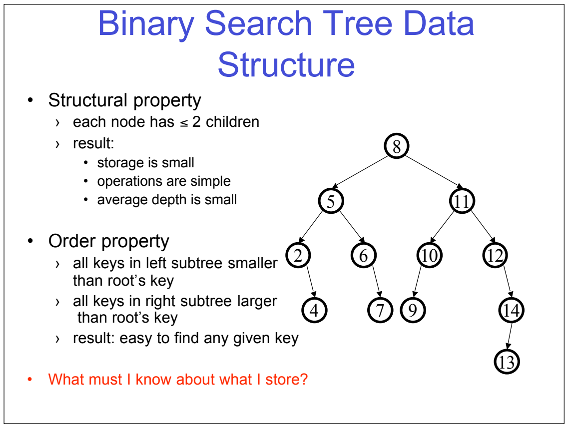
A Modest Few Uses

- Sets
- Dictionaries
- Networks : Router tables
- Operating systems : Page tables
- Compilers : Symbol tables

Probably the most widely used ADT!

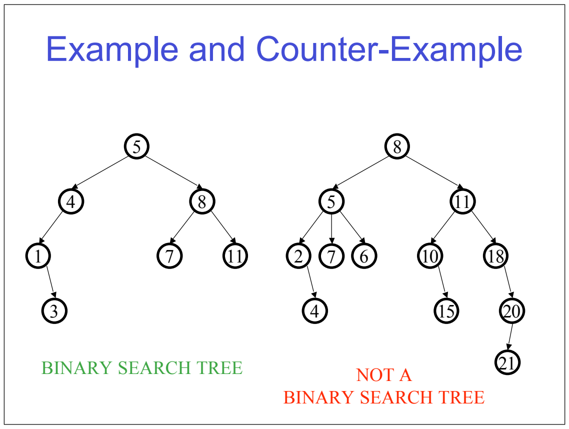
Implementations

	insert	find	delete
Unsorted Linked-list			
Unsorted array			
Sorted array			



Binary Search Tree Data Structure

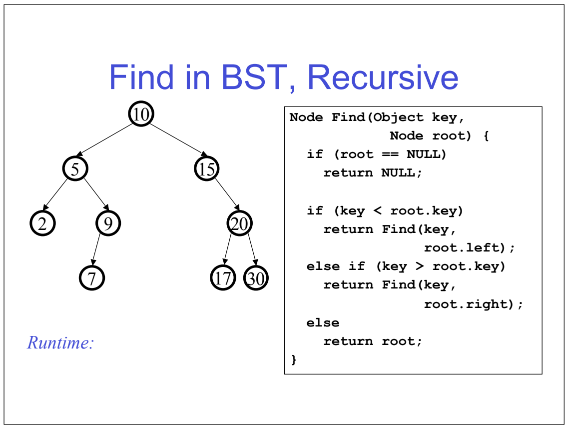
- Structural property
  - each node has ≤ 2 children
  - result:
    - storage is small
    - operations are simple
    - average depth is small
- Order property
  - all keys in left subtree smaller than root's key
  - all keys in right subtree larger than root's key
  - result: easy to find any given key
- What must I know about what I store?



Example and Counter-Example

BINARY SEARCH TREE

NOT A
BINARY SEARCH TREE

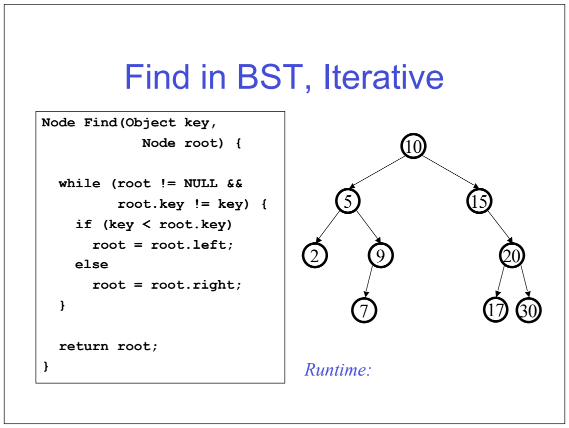


Find in BST, Recursive

```
Node Find(Object key,
            Node root) {
  if (root == NULL)
    return NULL;

  if (key < root.key)
    return Find(key,
                root.left);
  else if (key > root.key)
    return Find(key,
                root.right);
  else
    return root;
}
```

Runtime:



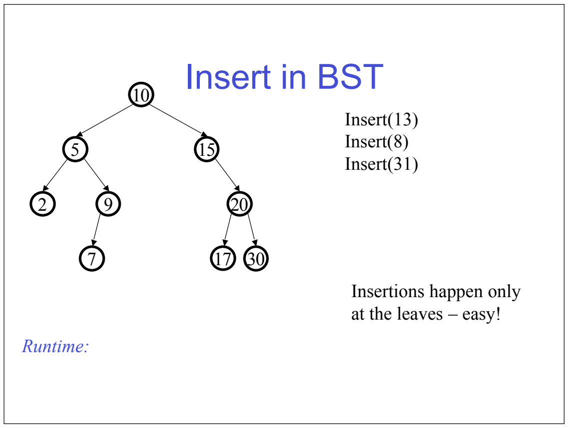
Find in BST, Iterative

```
Node Find(Object key,
            Node root) {

  while (root != NULL &&
         root.key != key) {
    if (key < root.key)
      root = root.left;
    else
      root = root.right;
  }

  return root;
}
```

Runtime:



Insert in BST

Insert(13)
Insert(8)
Insert(31)

Insertions happen only
at the leaves – easy!

Runtime:

BuildTree for BST

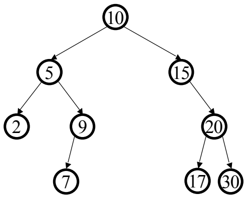
- Suppose keys 1, 2, 3, 4, 5, 6, 7, 8, 9 are inserted into an initially empty BST.

Runtime depends on the order!

  - in given order
  - in reverse order
  - median first, then left median, right median, etc.

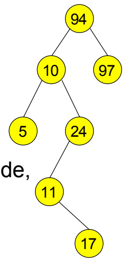
Bonus: FindMin/FindMax

- Find minimum
- Find maximum



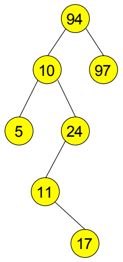
Delete Operation

- Delete is a bit trickier…Why?
- Suppose you want to delete 10
- Strategy:
  - Find 10
  - Delete the node containing 10
- Problem: When you delete a node, what do you replace it by?



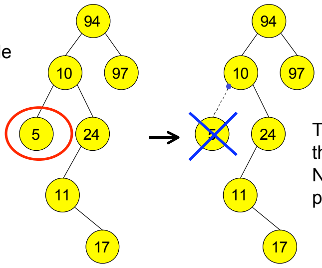
Delete Operation

- Problem: When you delete a node, what do you replace it by?
- Solution:
  - If it has no children, by NULL
  - If it has 1 child, by that child
  - If it has 2 children, by the node with the smallest value in its right subtree (the successor of the node)



Delete “5” - No children

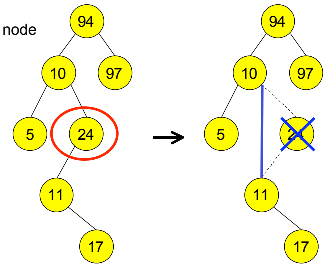
Find 5 node



Then Free the 5 node and NULL the pointer to it

Delete “24” - One child

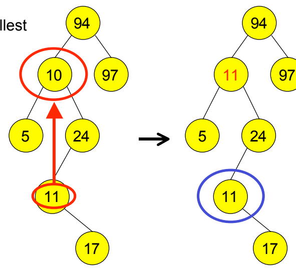
Find 24 node



Then Free the 24 node and replace the pointer to it with a pointer to its child

Delete “10” - two children

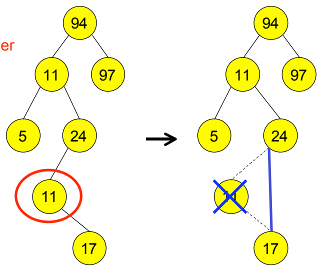
Find 10,
Copy the smallest value in right subtree into the node



Then recursively Delete node with smallest value in right subtree
Note: it does not have two children

Delete “11” - One child

Remember
11 node



Then Free the 11 node and replace the pointer to it with a pointer to its child

Runtimes

- Find? Insert? Delete?
- What is the average height of a BST?
- What is the maximum height?
- What happened when we insert nodes in sorted order?

Balanced BST

Observation

- BST: the shallower the better!
- Simple cases such as insert(1, 2, 3, ..., n) lead to the worst case scenario

Solution: Require a **Balance Condition** that

1. ensures depth is $O(\log n)$ – strong enough!
2. is easy to maintain – not too strong!

Potential Balance Conditions

1. Left and right subtrees of the root have equal number of nodes
2. Left and right subtrees of the root have equal *height*

Potential Balance Conditions

3. Left and right subtrees of *every node* have equal number of nodes
4. Left and right subtrees of *every node* have equal *height*

The AVL Balance Condition

Left and right subtrees of *every node*
have equal *heights* **differing by at most 1**

Define: **balance**(*x*) = height(*x*.left) – height(*x*.right)

AVL property: **–1 ≤ balance(*x*) ≤ 1, for every node *x***

- Ensures small depth
  - Will prove this by showing that an AVL tree of height *h* must have a lot of (i.e. O(2^h)) nodes
- Easy to maintain
  - Using single and double rotations

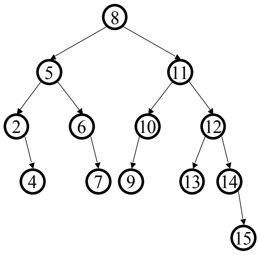
The AVL Tree Data Structure

Structural properties

1. Binary tree property
2. Balance property: balance of every node is between -1 and 1

Result:

Worst case depth is O(log *n*)



Ordering property

- Same as for BST