CSE 326: Data Structures

B Trees

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Splay Tree Summary

- All operations are in amortized O(log *n*) time
- Splaying can be done top-down; this may be better because:
  - only one pass
  - no recursion or parent pointers necessary
  - *we didn't cover top-down in class*
- Splay trees are *very* effective search trees
  - Relatively simple
  - No extra fields required
  - Excellent *locality* properties: frequently accessed keys are cheap to find

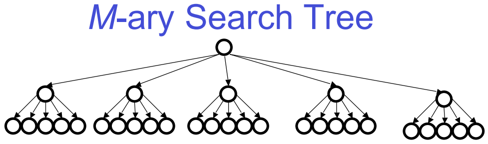
Disk vs. Memory

- Disks many times slower than memory:
  - Processor measured in GH = 10^9 cycles per second
  - Main memory measured in microsec. = 10^6 per second
  - Disk seek measured in miliseconds = 10^3 per second
- i.e. ~ 1 million instructions per disk lookup
- Measuring runtime by pointer lookups meaningless if data can't fit in main memory

Trees on disk

- Each pointer lookup means seeking the disk
- Want as shallow a tree as possible
- Balanced binary tree with N nodes has height ________?
- Balanced M-ary tree with N nodes has height ________?

M-ary Search Tree



- Maximum branching factor of *M*
- Complete tree has height =

disk accesses for *find*:

Runtime of *find*:

Problems with *M*-ary Search Trees

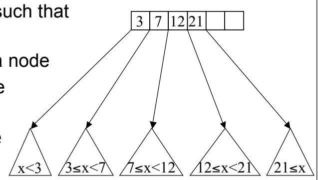
1.

2.

3.

Solution: B-Trees

- B-Trees are specialized *M*-ary search trees
- Each node has many keys (max M-1)
  - › subtree between two keys *x* and *y* contains leaves with *values v* such that $x \le v < y$
  - › binary search within a node to find correct subtree
- Each node takes one full {*page, block*} of memory



B-Trees

What makes them disk-friendly?

1. **Many keys stored in a node**
   - All brought to memory/cache in one access!
2. Internal nodes contain *only* keys;
   Only leaf nodes contain keys and actual data
   - The tree structure can be loaded into memory irrespective of data object size
   - Data actually resides in disk

Example

- 1k byte page
- Key 8 bytes, pointer 4 bytes
- (M-1)8 + 4M = 1024

  12 M = 1032

  $M = \lfloor 1032/12 \rfloor = 86$

B-Trees

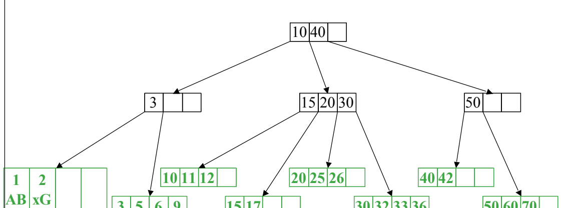
B-Trees are multi-way search trees commonly used in database systems or other applications where data is stored externally on disks and keeping the tree shallow is important.

A B-Tree of order M has the following properties:
1. The root is either a leaf or has between 2 and M children.
2. All nonleaf nodes (except the root) have between $\lceil M/2 \rceil$ and M children.
3. All leaves are at the same depth.

All data records are stored at the leaves.
Leaves store between $\lceil M/2 \rceil$ and M data records.
Internal nodes only used for searching.

B-Tree: Example

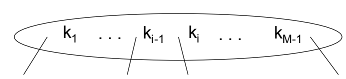
B-Tree with `M = 4` (# pointers in internal node)



Note: All leaves at the same depth!

B-Tree Details

Each (non-leaf) internal node of a B-tree has:
- › Between $\lceil M/2 \rceil$ and M children.
- › up to M-1 keys $k_1 < k_2 < ... < k_{M-1}$

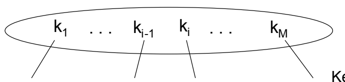


Keys are ordered so that:
$k_1 < k_2 < ... < k_{M-1}$

B-Tree Details

Each leaf node of a B-tree has:

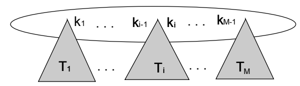
› Between ⌈M/2⌉ and M keys and pointers.



Keys are ordered so that:
$k_1 < k_2 < ... < k_{M-1}$

Keys point to data on other pages.

Properties of B-Trees



Children of each internal node are "between" the items in that node.

Suppose subtree T_i is the i-th child of the node:

all keys in T_i must be between keys k_{i-1} and k_i

i.e. $k_{i-1} \le T_i < k_i$

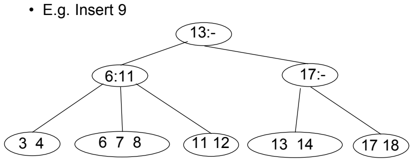
k_{i-1} is the smallest key in T_i

All keys in first subtree $T_1 < k_1$

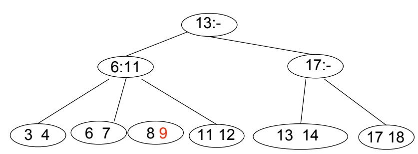
All keys in last subtree $T_M \ge k_{M-1}$

Inserting into B-Trees

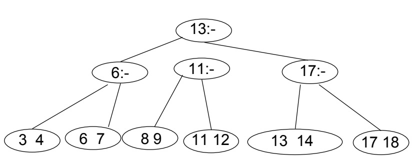
- Insert X: Do a Find on X and find appropriate leaf node
  - › If leaf node is not full, fill in empty slot with X
    - E.g. Insert 5
  - › If leaf node is full, split leaf node and adjust parents up to root node
    - E.g. Insert 9



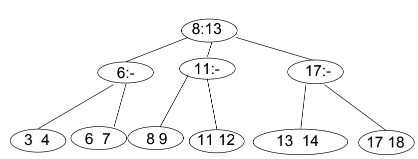
Insert Example



Insert Example

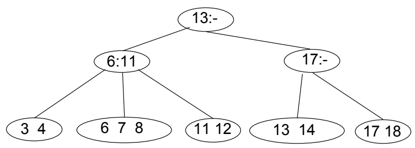


Insert Example

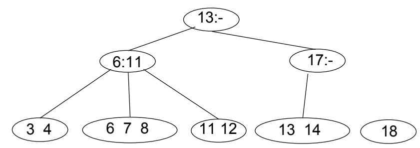


Deleting From B-Trees

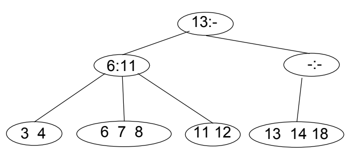
- Delete X : Do a find and remove from leaf
  - › Leaf underflows – borrow from a neighbor
    - E.g. 11
  - › Leaf underflows and can't borrow – merge nodes, delete parent
    - E.g. 17



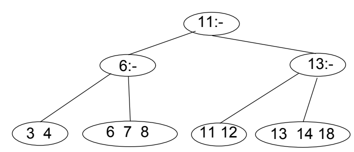
Delete Example



Delete Example



Delete Example



Run Time Analysis of B-Tree Operations

- For a B-Tree of order M
  - › Each internal node has up to M-1 keys to search
  - › Each internal node has between $\lceil M/2 \rceil$ and M children
  - › Depth of B-Tree storing N items is $O(\log_{\lceil M/2 \rceil} N)$
- Example: M = 86
  - › $\log_{43} N = \log_2 N / \log_2 43 = .184 \log_2 N$
  - › $\log_{43} 1{,}000{,}000{,}000 = 5.51$

Summary of Search Trees

- Problem with Search Trees: Must keep tree balanced to allow fast access to stored items
- AVL trees: Insert/Delete operations keep tree balanced
- Splay trees: Repeated Find operations produce balanced trees on average
- Multi-way search trees (e.g. B-Trees): More than two children
  - › per node allows shallow trees; all leaves are at the same depth
  - › keeping tree balanced at all times