CSE 326: Data Structures

Graph Search

Neva Cherniavsky
Summer 2006

Graph Search

- Many problems in computer science correspond to searching for a path in a graph, given a start node and goal criteria
  - › Route planning – Mapquest
  - › Packet-switching
  - › VLSI layout
  - › 6-degrees of Kevin Bacon
  - › Program synthesis
  - › Speech recognition
    - We'll discuss these last two later…

General Graph Search Algorithm

Open – some data structure (e.g., stack, queue, heap)

Criteria – some method for removing an element from Open

```
• Search( Start, Goal_test, Criteria)
•     insert(Start, Open);
•     repeat
•         if (empty(Open)) then return fail;
•         select Node from Open using Criteria;
•         if (Goal_test(Node)) then return Node;
•         for each Child of node do
•             if (Child not already visited) then Insert( Child, Open );
•         Mark Node as visited;
•   end
```

Depth-First Graph Search

Open – Stack

Criteria – Pop

```
• DFS( Start, Goal_test)
•     push(Start, Open);
•     repeat
•         if (empty(Open)) then return fail;
•         Node := pop(Open);
•         if (Goal_test(Node)) then return Node;
•         for each Child of node do
•             if (Child not already visited) then push(Child, Open);
•         Mark Node as visited;
•   end
```

Breadth-First Graph Search

Open – Queue

Criteria – Dequeue (FIFO)

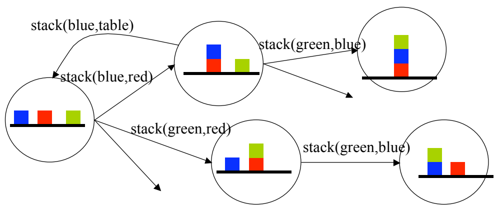
```
• BFS( Start, Goal_test)
•     enqueue(Start, Open);
•     repeat
•         if (empty(Open)) then return fail;
•         Node := dequeue(Open);
•         if (Goal_test(Node)) then return Node;
•         for each Child of node do
•             if (Child not already visited) then enqueue(Child,
  Open);
•         Mark Node as visited;
•   end
```

Two Models

1. Standard Model: Graph given explicitly with n vertices and e edges.
   > Search is O(n + e) time in adjacency list representation
2. AI Model: Graph generated on the fly.
   > Time for search need not visit every vertex.

Planning Example

- A huge graph may be implicitly specified by rules for generating it on-the-fly
- Blocks world:
  - › vertex = relative positions of all blocks
  - › edge = robot arm stacks one block



AI Comparison: DFS versus BFS

- Depth-first search
  - › Does not always find shortest paths
  - › Must be careful to mark visited vertices, or you could go into an infinite loop if there is a cycle
- Breadth-first search
  - › Always finds shortest paths – optimal solutions
  - › Marking visited nodes can improve efficiency, but even without doing so search is guaranteed to terminate

Is BFS always preferable?

DFS Space Requirements

- Assume:
  - › Longest path in graph is length *d*
  - › Highest number of out-edges is *k*
- DFS stack grows at most to size ??
- For *k*=10, *d*=15, size is ??

BFS Space Requirements

- Assume
  - › Distance from start to a goal is *d*
  - › Highest number of out edges is *k* BFS
- Queue could grow to size
  - › For *k*=10, *d*=15, size is ???

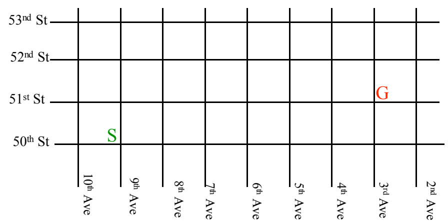
Conclusion

- In the AI Model, DFS is hugely more memory efficient, *if we can limit the maximum path length to some fixed d.*
  - › If we *knew* the distance from the start to the goal in advance, we can just *not add any children to stack after level d*
  - › But what if we don't know *d* in advance?

Problem: Large Graphs

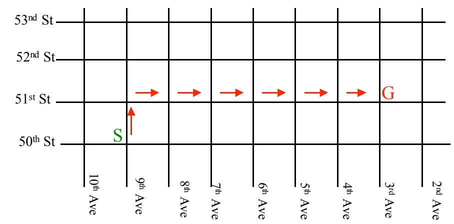
- It is expensive to find optimal paths in large graphs, using BFS or Dijkstra's algorithm (for weighted graphs)
- How can we search large graphs efficiently by using "commonsense" about which direction looks most promising?

Example



Plan a route from 9th & 50th to 3rd & 51st

Example



Plan a route from 9th & 50th to 3rd & 51st

Best-First Search

- The *Manhattan distance* (Δ x+ Δ y) is an estimate of the distance to the goal
  - It is a *search heuristic*
- Best-First Search
  - Order nodes in priority to minimize estimated distance to the goal
- Compare: BFS / Dijkstra
  - Order nodes in priority to minimize distance from the start

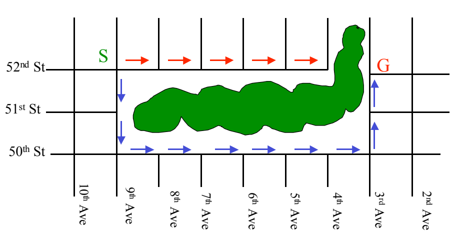
Best-First Search

Open – Heap (priority queue)
Criteria – Smallest key (highest priority)
h(n) – heuristic estimate of distance from n to closest goal

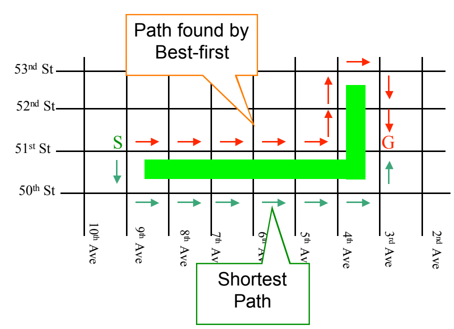
```
• Best_First_Search( Start, Goal_test)
•     insert(Start, h(Start), heap);
•     repeat
•         if (empty(heap)) then return fail;
•         Node := deleteMin(heap);
•         if (Goal_test(Node)) then return Node;
•         for each Child of node do
•             if (Child not already visited) then
•                 insert(Child, h(Child),heap);
•         end
•         Mark Node as visited;
•     end
```

Obstacles

- Best-FS eventually will expand vertex to get back on the right track



Non-Optimality of Best-First



Improving Best-First

- Best-first is often tremendously faster than BFS/Dijkstra, but might stop with a non-optimal solution
- How can it be modified to be (almost) as fast, but guaranteed to find optimal solutions?
- A* - Hart, Nilsson, Raphael 1968
  - › One of the first significant algorithms developed in AI
  - › Widely used in many applications

A*

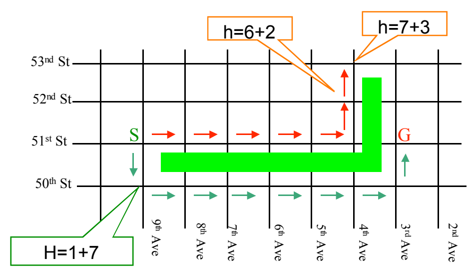
- Exactly like Best-first search, but using a different criteria for the priority queue:
- minimize (distance from start) + (estimated distance to goal)
- priority f(n) = g(n) + h(n)

f(n) = priority of a node
g(n) = true distance from start
h(n) = heuristic distance to goal

Optimality of A*

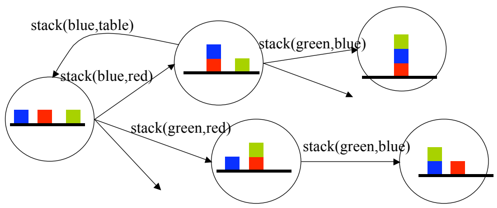
- Suppose the estimated distance is *always* less than or equal to the true distance to the goal
  - › heuristic is a lower bound
- Then: when the goal is removed from the priority queue, we are guaranteed to have found a shortest path!
- Everything else has a higher estimated cost

A* in Action



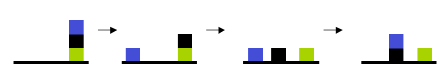
Applications of A*: Planning

- A huge graph may be implicitly specified by rules for generating it on-the-fly
- Blocks world:
  - › vertex = relative positions of all blocks
  - › edge = robot arm stacks one block



Blocks World

- Blocks world:
  - › distance = number of stacks to perform
  - › heuristic lower bound = number of blocks out of place



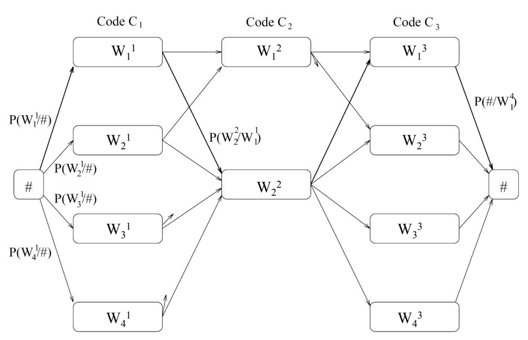
out of place = 2, true distance to goal = 3

Application of A*: Speech Recognition

- (Simplified) Problem:
  - System hears a sequence of 3 words
  - It is unsure about what it heard
    - For each word, it has a set of possible “guesses”
    - E.g.: Word 1 is one of { “hi”, “high”, “I” }
  - What is the most likely sentence it heard?

Speech Recognition as Shortest Path

- Convert to a shortest-path problem:
  - Utterance is a “layered” DAG
  - Begins with a special dummy “start” node
  - Next: A layer of nodes for each word position, one node for each word choice
  - Edges between every node in layer i to every node in layer i+1
    - Cost of an edge is smaller if the pair of words frequently occur together in real speech
      - Technically: - log probability of co-occurrence
  - Finally: a dummy “end” node
  - Find shortest path from start to end node



Summary: Graph Search

- Depth First
  - Little memory required
  - Might find non-optimal path
- Breadth First
  - Much memory required
  - Always finds optimal path
- Dijskstra’s Short Path Algorithm
  - Like BFS for weighted graphs
- Best First
  - Can visit fewer nodes
  - Might find non-optimal path
- A*
  - Can visit fewer nodes than BFS or Dijkstra
  - Optimal if heuristic estimate is a lower-bound