CSE 326: Data Structures
Binomial Queues

Hal Perkins
Spring 2007
Lecture 8

Yet Another Data Structure: Binomial Queues

- Structural property
  - Forest of binomial trees with at most one tree of any height

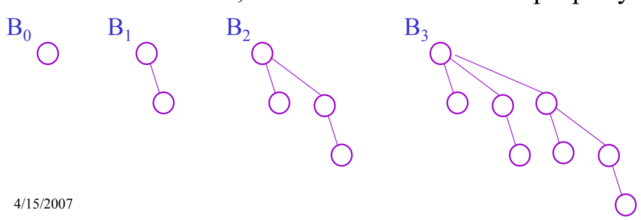
What's a forest?

What's a binomial tree?

- Order property
  - Each binomial tree has the heap-order property

The Binomial Tree, B_h

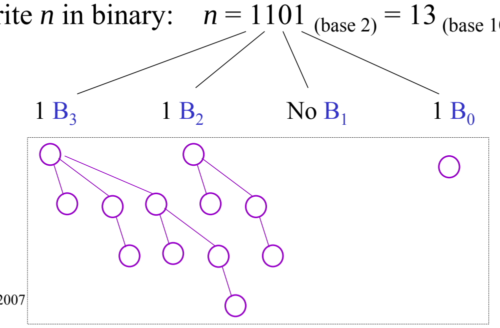
- B_h has height h and exactly 2^h nodes
- B_h is formed by making B_{h-1} a child of another B_{h-1}
- Root has exactly h children
- Number of nodes at depth d is binomial coeff. $\binom{h}{d}$
  - Hence the name; we will *not* use this last property



Binomial Queue with n elements

Binomial Q with n elements has a *unique* structural representation in terms of binomial trees!

Write n in binary: $n = 1101_{\text{(base 2)}} = 13_{\text{(base 10)}}$



Properties of Binomial Queue

- At most one binomial tree of any height
- n nodes $\Rightarrow$ binary representation is of size ?
  $\Rightarrow$ deepest tree has height ?
  $\Rightarrow$ number of trees is ?

Define: height(forest F) = $\max_{\text{tree T in F}}$ { height(T) }

Binomial Q with n nodes has height $\Theta(\log n)$

Operations on Binomial Queue

- Will again define *merge* as the base operation
  - insert, deleteMin, buildBinomialQ will use merge
- Can we do increaseKey efficiently? decreaseKey?
- What about findMin?

Merging Two Binomial Queues

Essentially like adding two binary numbers!

1. Combine the two forests
2. For k from 0 to maxheight {
   a. $m \leftarrow$ total number of B_k's in the two BQs
   b. if m=0: continue;
   c. if m=1: continue;
   d. if m=2: combine the two B_k's to form a B_{k+1}
   e. if m=3: retain one B_k and combine the other two to form a B_{k+1}

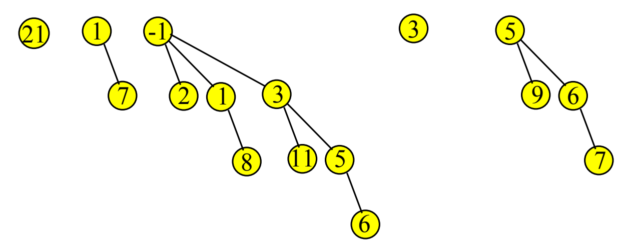
}

# of 1's
0+0 = 0
1+0 = 1
1+1 = 0+c
1+1+c = 1+c

Claim: When this process ends, the forest has at most one tree of any height

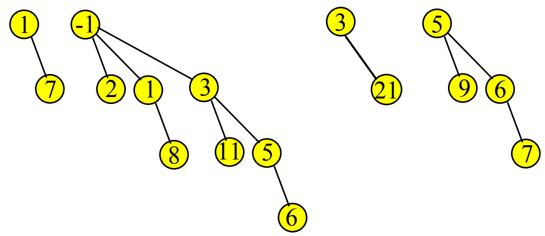
Example: Binomial Queue Merge

H1: H2:



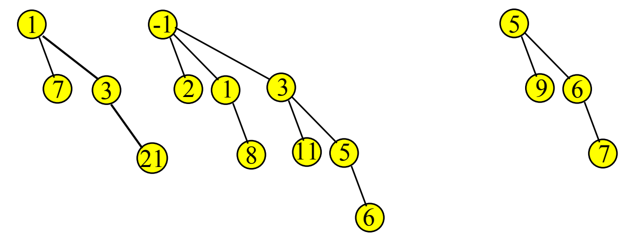
Example: Binomial Queue Merge

H1: H2:



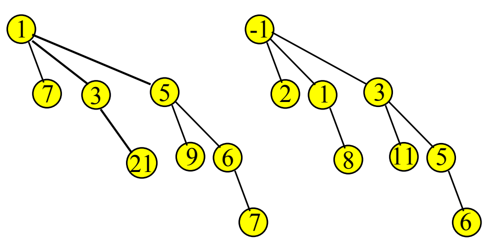
Example: Binomial Queue Merge

H1: H2:



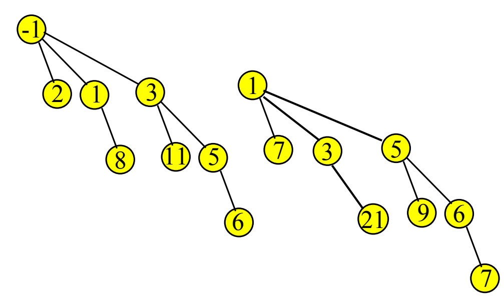
Example: Binomial Queue Merge

H1: H2:



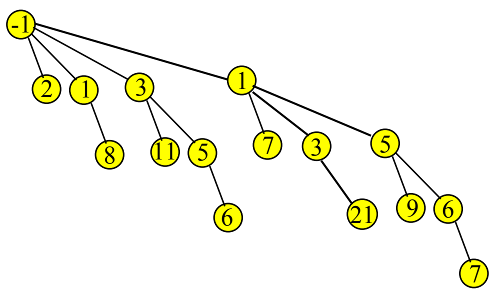
Example: Binomial Queue Merge

H1: H2:



Example: Binomial Queue Merge

H1: H2:



Complexity of Merge

Constant time for each height

Max height is: log n

$\Rightarrow$ worst case running time = $\Theta($ $)$

Insert in a Binomial Queue

Insert(x): Similar to leftist or skew heap

runtime

Worst case complexity: same as merge

O( )

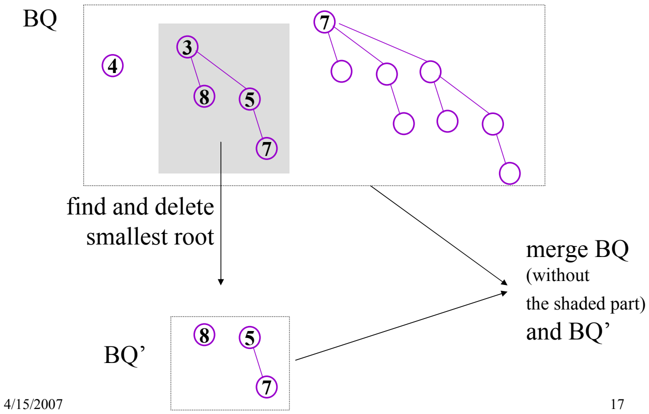
Average case complexity: O(1)

Why?? *Hint: Think of adding 1 to 1101*

deleteMin in Binomial Queue

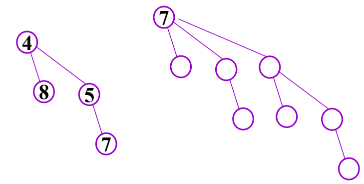
Similar to leftist and skew heaps….

deleteMin: Example



deleteMin: Example

Result:



runtime: