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# CSE 331

## Software Design & Implementation

Topic: Introduction

 **Discussion:** What are you excited for this summer?

# Reminders

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- Read the welcome email
- Check your access to Ed, Gradescope, and Canvas
- Should see email about Gitlab repositories soon

# Upcoming Deadlines

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- Syllabus Quiz      due Thursday (6/23)
- HW1                      due Thursday (6/23)

## Last Time...

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- Welcome email
- Syllabus Overview

## Today's Agenda

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- Upcoming Assignments
- Motivation
- Reasoning

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# Upcoming Assignments

# Syllabus Quiz

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- Due on Thursday night
  - read the syllabus in depth
  - answer a few multiple choice/select questions
  - infinite attempts before deadline
- Why?
  - had a lot of confusion in past quarters
  - make student requests manageable for course staff

# HW1

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- Due on Thursday night
  - practice interview question
  - **write** an algorithm to rearrange array elements as described
  - **argue** in concise, convincing English that it is correct
    - don't just explain *what the code does!*
  - **do not run** your code! (pretend it's on a whiteboard)
    - know that is correct *without* running it (a necessary skill)
- This is expected to be difficult (esp. the "argue" part)
  - graded on effort, not correctness
  - do not spend more than 90 minutes on it
  - want you to see that it is tricky... *without the tools coming next*

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# Motivation

# What are the goals of CSE 331?

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Learn the skills to be able to contribute to a modern software project

- move from CSE 143 problems toward what you'll see in industry and in upper-level courses

Specifically, how to write code of

- higher **quality**
- increased **complexity**

We will discuss *tools* and *techniques* to help with this and the *concepts* and *ideas* behind them

- there are *timeless principles* to both
- widely used across the industry



# What is high quality?

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Code is high quality when it is

1. **Correct**  
Everything else is of secondary importance
2. Easy to **change**  
Most work is making changes to existing systems
3. Easy to **understand**  
Needed for 1 & 2 above

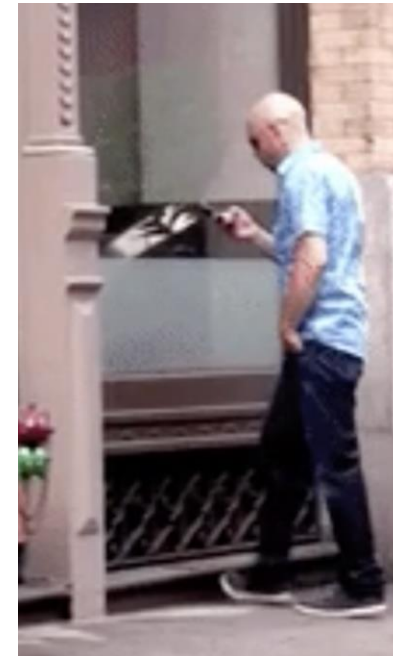
# How do we ensure correctness...

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... when **people** are involved?

People have been known to

- walk into windows
- drive away with a coffee cup on the roof
- drive away still tied to gas pump
- lecture wearing one brown shoe and one black shoe



## Key Insight

1. Can't stop people from making mistakes

# How do we ensure correctness?

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Best practice: use three techniques (we'll study each)

## 1. **Tools**

- type checkers, test runners, etc.

## 2. **Inspection**

- think through your code carefully
- have another person review your code

technical interviews focus on this  
(a.k.a. "reasoning")

## 3. **Testing**

- usually >50% of the work in building software

Together can catch >97% of bugs.

# Scale makes everything harder

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Many studies showing scale makes quality harder to achieve

- Time to write N-line program grows faster than linear
  - Good estimate is  $O(N^{1.05})$  [Boehm, '81]
- Bugs grow like  $\Theta(N \log N)$  [Jones, '12]
  - 10% of errors are between modules [Seaman, '08]
- Communication costs dominate schedules [Brooks, '75]
- Small probability cases become high probability cases
  - Corner cases are more important with more users

**Corollary:** quality must be even higher, per line, in order to achieve overall quality in a *large* program

# How do we cope with scale?

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We tackle increased software scale with **modularity**

- Split code into pieces that can be built independently
- Each must be documented so others can use it
- Also helps understandability and changeability

# What are the goals of CSE 331?

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In summary, we want our code to be:

1. Correct
2. Easy to change
3. Easy to understand
4. Modular

These qualities also allow us to solve more complex problems

- increased complexity = larger scale and sophistication

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# Reasoning

# Our Approach

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- We will learn a set of **formal tools** for proving correctness
  - math can seem daunting – it will connect back!
  - later, this will also allow us to generate the code
- Most professionals can do reasoning like this in their head
  - most do an *informal* version of what we will see
  - with practice, it will be the same for you
- Formal version has key advantages
  - teachable
  - mechanical (no intuition or creativity required)
  - necessary for hard problems
    - we turn to formal tools when problems get too hard



# Formal Reasoning

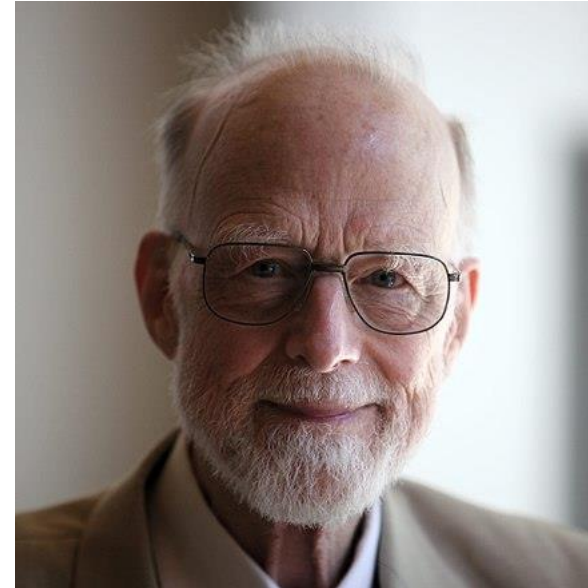
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- Invented by Robert Floyd and Sir Anthony Hoare
  - Floyd won the Turing award in 1978
  - Hoare won the Turing award in 1980



Robert Floyd

picture from [Wikipedia](#)



Tony Hoare

# Terminology of Floyd Logic

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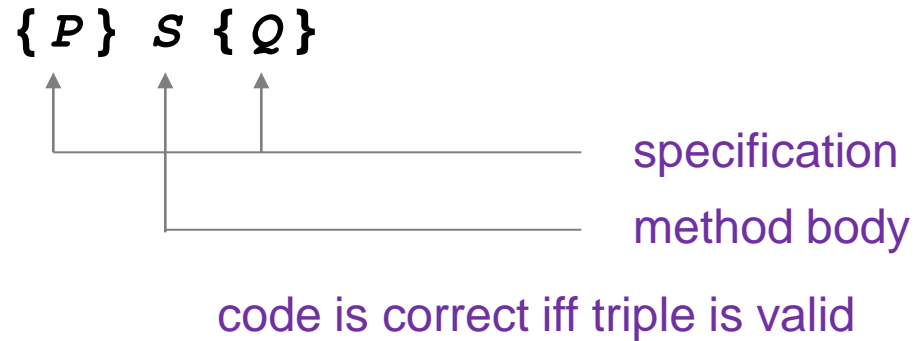
- The *program state* is the values of all the (relevant) variables
- An *assertion* is a true / false claim (proposition) about the state at a given point during execution (e.g., on line 39)
- An assertion *holds* for a program state if the claim is true when the variables have those values
  
- An assertion before the code is a *precondition*
  - these represent assumptions about when that code is used
- An assertion after the code is a *postcondition*
  - these represent what we want the code to accomplish

# Hoare Triples

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- A **Hoare triple** is two assertions and one piece of code:

- $P$  the precondition
- $S$  the code
- $Q$  the postcondition



- A Hoare triple  $\{P\} S \{Q\}$  is called **valid** if:
  - in any state where  $P$  holds, executing  $S$  produces a state where  $Q$  holds
  - i.e., if  $P$  is true before  $S$ , then  $Q$  must be true after it
  - otherwise, the triple is called **invalid**

# Notation

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- Floyd logic writes assertions in  $\{..\}$ 
  - since Java code also has  $\{..\}$ , I will use  $\{\{...\}\}$
  - e.g.,  $\{\{ w \geq 1 \}\} \mathbf{x} = 2 * \mathbf{w}; \{\{ x \geq 2 \}\}$
- Assertions are math / logic not Java
  - you can use the usual math notation
    - (e.g.,  $=$  instead of  $==$  for equals)
  - purpose is communication with other humans (not computers)
  - we will need **and**, **or**, **not** as well
    - can also write use  $\wedge$  (and)  $\vee$  (or) etc.
- The Java language also has assertions (**assert** statements)
  - throws an exception if the condition does not evaluate true
  - we will discuss these more later in the course

# Example 1

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Is the following Hoare triple valid or invalid?

- assume all variables are integers and there is no overflow

$\{\{x \neq 0\}\} y = x * x; \{\{y > 0\}\}$

# Example 1

---

Is the following Hoare triple valid or invalid?

- assume all variables are integers and there is no overflow

$\{\{ \mathbf{x} \neq 0 \}\} \mathbf{y} = \mathbf{x} * \mathbf{x}; \{\{ \mathbf{y} > 0 \}\}$

Valid

- $\mathbf{y}$  could only be zero if  $\mathbf{x}$  were zero (which it isn't)

# Example 2

---

Is the following Hoare triple valid or invalid?

- assume all variables are integers and there is no overflow

$\{\{ z \neq 1 \}\} y = z * z; \{\{ y \neq z \}\}$

# Example 2

---

Is the following Hoare triple valid or invalid?

- assume all variables are integers and there is no overflow

$\{\{z \neq 1\}\} y = z * z; \{\{y \neq z\}\}$

Invalid

- counterexample:  $z = 0$



# Checking Validity

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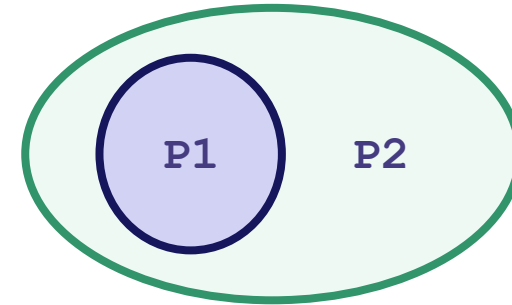
- So far: decided if a Hoare triple is valid by ... **hard** thinking
- Soon: mechanical process for reasoning about
  - assignment statements
  - [next section] conditionals
  - [next lecture] loops
  - (all code can be understood in terms of those 3 elements)
- Next: terminology for comparing different assertions
  - useful, e.g., to compare possible preconditions

# Weaker vs. Stronger Assertions

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If P1 implies P2 (written  $P1 \Rightarrow P2$ ), then:

- P1 is **stronger** than P2
- P2 is **weaker** than P1



Whenever P1 holds, P2 also holds

- So it is more (or at least as) “difficult” to satisfy P1
  - the program states where P1 holds are a subset of the program states where P2 holds
- So P1 puts more constraints on program states
- So it is a stronger set of requirements on the program state
  - P1 gives you more information about the state than P2

# Examples

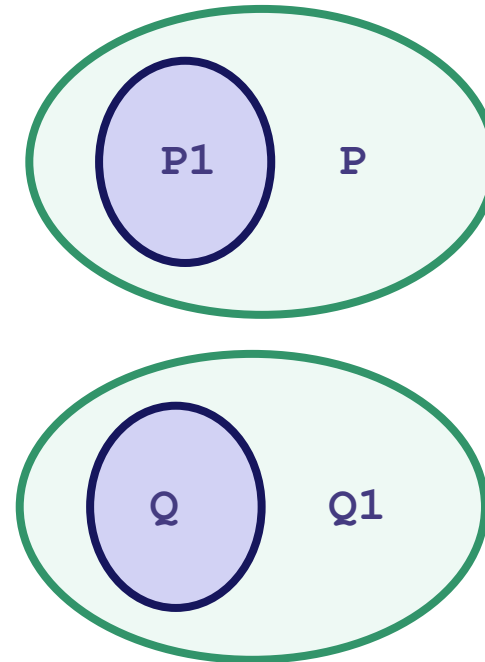
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- $x = 17$  is stronger than  $x > 0$
- $x$  is prime is neither stronger nor weaker than  $x$  is odd
- $x$  is prime and  $x > 2$  is stronger than  $x$  is odd

# Floyd Logic Facts

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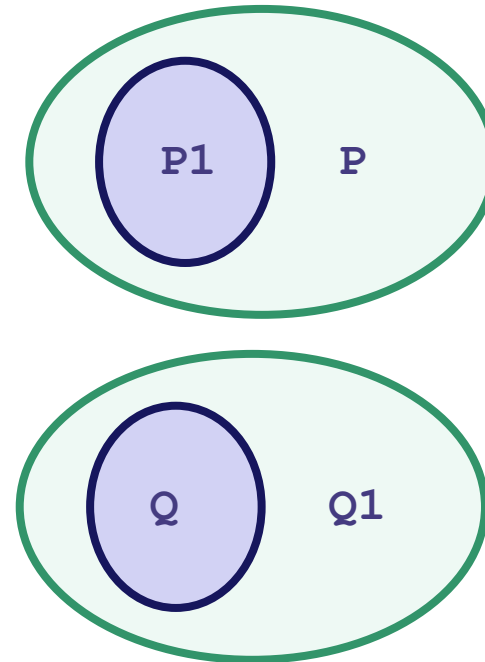
- Suppose  $\{P\} S \{Q\}$  is valid.
- If  $P1$  is stronger than  $P$ , then  $\{P1\} S \{Q\}$  is valid.
- If  $Q1$  is weaker than  $Q$ , then  $\{P\} S \{Q1\}$  is valid.
- Example:
  - Suppose  $P$  is  $x \geq 0$  and  $P1$  is  $x > 0$
  - Suppose  $Q$  is  $y > 0$  and  $Q1$  is  $y \geq 0$
  - Since  $\{\{x \geq 0\}\} y = x+1 \{\{y > 0\}\}$  is valid,  $\{\{x > 0\}\} y = x+1 \{\{y \geq 0\}\}$  is also valid



# Floyd Logic Facts

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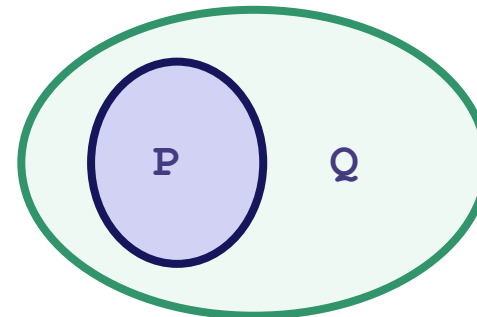
- Suppose  $\{P\} S \{Q\}$  is valid.
- If  $P1$  is stronger than  $P$ , then  $\{P1\} S \{Q\}$  is valid.
- If  $Q1$  is weaker than  $Q$ , then  $\{P\} S \{Q1\}$  is valid.
- **Key points:**
  - always okay to **strengthen** a **precondition**
  - always okay to **weaken** a **postcondition**



# Floyd Logic Facts

---

- When is  $\{P\} ; \{Q\}$  is valid?
  - with no code in between
- Valid if any state satisfying P also satisfies Q
- I.e., if P is **stronger** than Q



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# Forward & Backward Reasoning

# Example of Forward Reasoning

---

Work forward from the precondition

```
{{ w > 0 }}
```

```
  x = 17;
```

```
{{ _____ }}
```

```
  y = 42;
```

```
{{ _____ }}
```

```
  z = w + x + y;
```

```
{{ _____ }}
```



# Example of Forward Reasoning

---

Work forward from the precondition

↓

```
  {{ w > 0 }}  
  x = 17;  
  {{ w > 0 and x = 17 }}  
  y = 42;  
  {{ _____ }}  
  z = w + x + y;  
  {{ _____ }}
```

# Example of Forward Reasoning

---

Work forward from the precondition

$\{\{ w > 0 \}\}$

**$x = 17;$**

$\{\{ w > 0 \text{ and } x = 17 \}\}$

**$y = 42;$**

$\{\{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \}\}$

**$z = w + x + y;$**

$\{\{ \text{_____} \}\}$

# Example of Forward Reasoning

---

Work forward from the precondition

$\{\{ w > 0 \}\}$

$\mathbf{x = 17;}$

$\{\{ w > 0 \text{ and } x = 17 \}\}$

$\mathbf{y = 42;}$

$\{\{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \}\}$

$\mathbf{z = w + x + y;}$

$\{\{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \text{ and } z = w + x + y \}\}$



# Example of Forward Reasoning

---

Work forward from the precondition

$\{\{ w > 0 \}\}$

**$x = 17;$**

$\{\{ w > 0 \text{ and } x = 17 \}\}$

**$y = 42;$**

$\{\{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \}\}$

**$z = w + x + y;$**

$\{\{ w > 0 \text{ and } x = 17 \text{ and } y = 42 \text{ and } z = w + 59 \}\}$

# Forward Reasoning

---

- Start with the **given** precondition
- Fill in the **strongest** postcondition
- For an assignment,  $\mathbf{x} = \mathbf{y}$ ...
  - add the fact “ $x = y$ ” to what is known
  - important subtleties here... (more on those later)
- Later: if statements and loops...

# Example of Backward Reasoning

---

Work backward from the desired postcondition

{{ \_\_\_\_\_ }}

**x = 17;**

{{ \_\_\_\_\_ }}

**y = 42;**

{{ \_\_\_\_\_ }}

**z = w + x + y;**

{{ z < 0 }}

# Example of Backward Reasoning

---

Work backward from the desired postcondition

`{{ _____ }}`

`x = 17;`

`{{ _____ }}`

`y = 42;`

`{{ w + x + y < 0 }}`

`z = w + x + y;`

`{{ z < 0 }}`



# Example of Backward Reasoning

---

Work backward from the desired postcondition

$\{ \underline{\hspace{10em}} \}$

$\mathbf{x = 17;}$

$\{ \{ w + x + 42 < 0 \} \}$

$\mathbf{y = 42;}$

$\{ \{ w + x + y < 0 \} \}$

$\mathbf{z = w + x + y;}$

$\{ \{ z < 0 \} \}$



# Example of Backward Reasoning

---

Work backward from the desired postcondition

$\{\{ w + 17 + 42 < 0 \}\}$

$\mathbf{x} = 17;$

$\{\{ w + x + 42 < 0 \}\}$

$\mathbf{y} = 42;$

$\{\{ w + x + y < 0 \}\}$

$\mathbf{z} = \mathbf{w} + \mathbf{x} + \mathbf{y};$

$\{\{ z < 0 \}\}$



# Backward Reasoning

---

- Start with the **required** postcondition
- Fill in the **weakest** precondition
- For an assignment,  $\mathbf{x} = \mathbf{y}$ :
  - just replace “x” with “y” in the postcondition
  - if the condition using “y” holds beforehand, then the condition with “x” will afterward since  $x = y$  then
- Later: if statements and loops...

# Correctness by Forward Reasoning

---

Use forward reasoning to determine if this code is correct:

`{{ w > 0 }}`

`x = 17;`

`y = 42;`

`z = w + x + y;`

`{{ z > 50 }}`

# Example of Forward Reasoning

---

$\{\{ w > 0 \}\}$

$\mathbf{x = 17;}$

$\{\{ w > 0 \text{ and } x=17 \}\}$

$\mathbf{y = 42;}$

$\{\{ w > 0 \text{ and } x=17 \text{ and } y=42 \}\}$

$\mathbf{z = w + x + y;}$

$\{\{ w > 0 \text{ and } x=17 \text{ and } y=42 \text{ and } z = w + 59 \}\}$

$\{\{ z > 50 \}\}$



Do the facts that are always true  
imply the facts we need?

I.e., is the bottom statement  
**weaker** than the top one?

(Recall that weakening the postcondition is always okay.)

# Correctness by Backward Reasoning

---

Use backward reasoning to determine if this code is correct:

`{{ w < -60 }}`

`x = 17;`

`y = 42;`

`z = w + x + y;`

`{{ z < 0 }}`

# Correctness by Backward Reasoning

---

Use backward reasoning to determine if this code is correct:

$\{\{ w < -60 \}\}$

$\{\{ w + 17 + 42 < 0 \}\}$

**x = 17;**

$\{\{ w + x + 42 < 0 \}\}$

**y = 42;**

$\{\{ w + x + y < 0 \}\}$

**z = w + x + y;**

$\{\{ z < 0 \}\}$

$\Leftrightarrow \{\{ w < -59 \}\}$

Do the facts that are always true imply the facts we need?

I.e., is the top statement **stronger** than the bottom one?

(Recall that strengthening the precondition is always okay.)

# Combining Forward & Backward

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It is okay to use both types of reasoning

- Reason forward from precondition
- Reason backward from postcondition

Will meet in the middle:

`{{ P }}`

`S1`

`S2`

`{{ Q }}`

# Combining Forward & Backward

---

It is okay to use both types of reasoning

- Reason forward from precondition
- Reason backward from postcondition

Will meet in the middle:





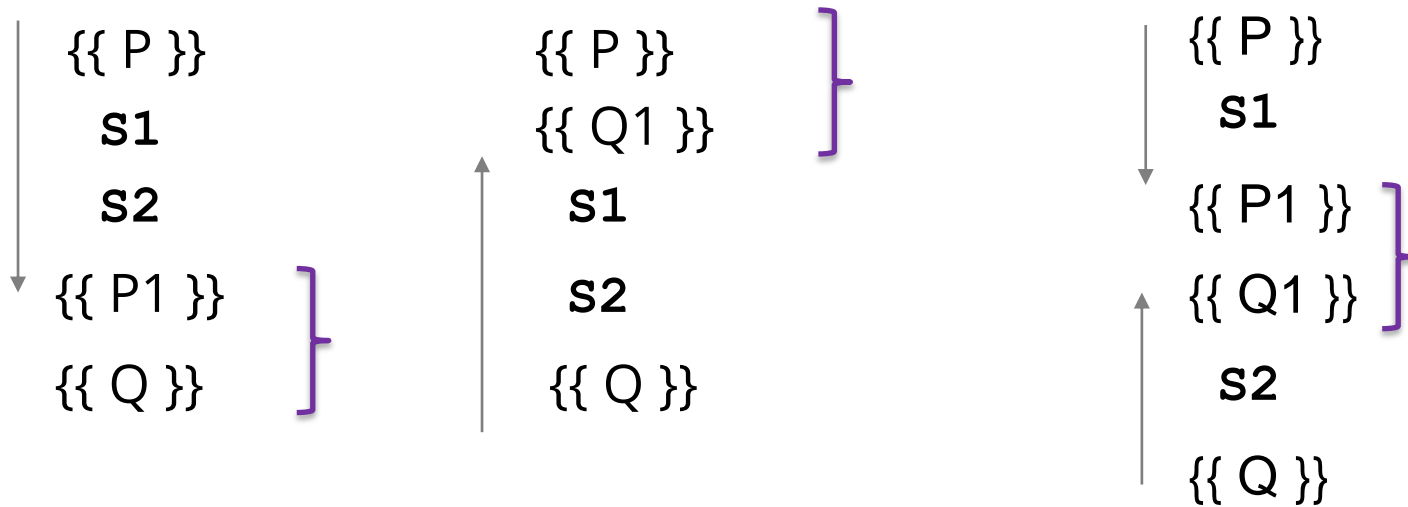
# Combining Forward & Backward

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Reasoning in either direction gives valid assertions

Just need to check adjacent assertions:

- top assertion must imply bottom one



# Subtleties in Forward Reasoning...

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- Forward reasoning can **fail** if applied blindly...

$\{\{ \}$

$w = x + y;$

$\{\{ w = x + y \}$

$x = 4;$

$\{\{ w = x + y \text{ and } x = 4 \}$

$y = 3;$

$\{\{ w = x + y \text{ and } x = 4 \text{ and } y = 3 \}$

This implies that  $w = 7$ , but that is not true!

- $w$  equals whatever  $x + y$  was **before** they were changed

# Fix 1

---

- Use **subscripts** to refer to old values of the variables
- Un-subscripted variables should always mean **current** value

$\{\{ \}$

$w = x + y;$

$\{\{ w = x + y \}$

$x = 4;$

$\{\{ w = x_1 + y \text{ and } x = 4 \}$

$y = 3;$

$\{\{ w = x_1 + y_1 \text{ and } x = 4 \text{ and } y = 3 \}$

# Fix 2 (better)

---

- Express prior values in terms of the current value

$\{\{\}\}$

$w = x + y;$

$\{\{ w = x + y \}\}$

$x = x + 4;$

$\{\{ w = x_1 + y \text{ and } x = x_1 + 4 \}\}$

Now,  $x_1 = x - 4$

so  $w = x_1 + y \Leftrightarrow w = x - 4 + y$

$\Rightarrow \{\{ w = x - 4 + y \}\}$

Note for updating variables, e.g.,  $x = x + 4$ :

- Backward reasoning just substitutes new value (no change)
- Forward reasoning requires you to invert the “+” operation

# Forward vs. Backward

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- Forward reasoning:
  - Find strongest postcondition
  - Intuitive: “simulate” the code in your head
    - BUT you need to change facts to refer to *prior values*
  - Inefficient: Introduces many irrelevant facts
    - usually need to weaken as you go to keep things sane
- Backward reasoning
  - Find weakest precondition
  - Formally simpler, but (initially) unintuitive
  - Efficient

# Before next class...

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1. Familiarize yourself with website:

<http://courses.cs.washington.edu/courses/cse331/22su/>

- read the welcome email
- read the syllabus

2. Try to do [HW1](#) and [syllabus quiz](#) before section tomorrow!

- submit a PDF on Gradescope
- limit this to at most 90 min
- do not use formal reasoning