CSE 332 Autumn 2023
Lecture 2: Algorithm Analysis pt.1

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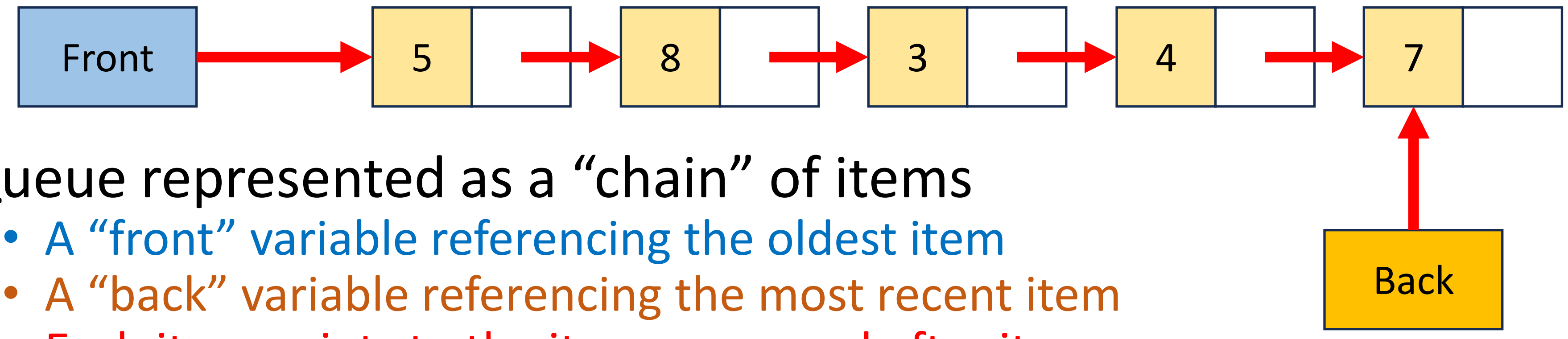
Terminology

- Abstract Data Type (ADT)
  - Mathematical description of a “thing” with set of operations on that “thing”
- Algorithm
  - A high level, language-independent description of a step-by-step process
- Data structure
  - A specific organization of data and family of algorithms for implementing an ADT
- Implementation of a data structure
  - A specific implementation in a specific language

ADT: Queue

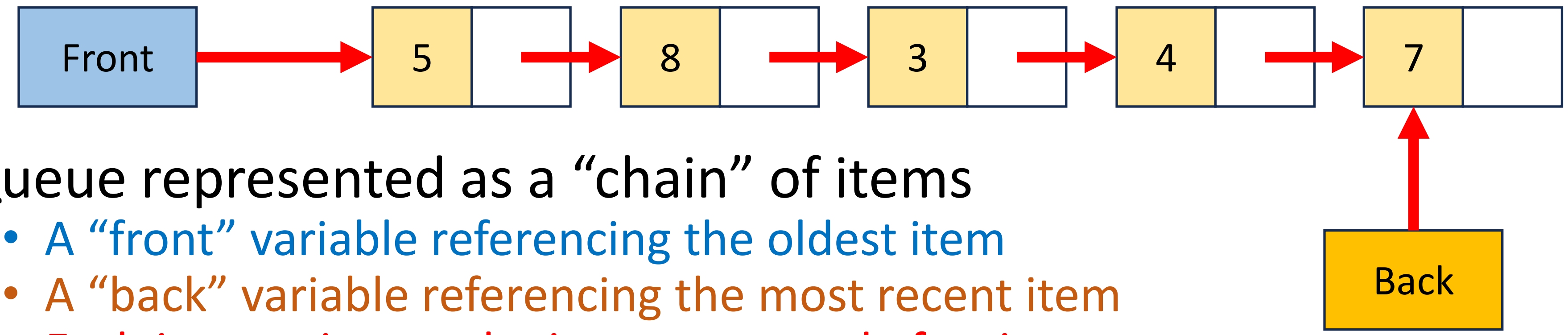
- What is it?
  - A “First In First Out” (FIFO) collection of items
- What Operations do we need?
  - Enqueue
    - Add a new item to the queue
  - Dequeue
    - Remove the “oldest” item from the queue
  - Is_empty
    - Indicate whether or not there are items still on the queue

Linked List – Queue Data Structure



- Queue represented as a "chain" of items
  - A "front" variable referencing the oldest item
  - A "back" variable referencing the most recent item
  - Each item points to the item enqueued after it
- Enqueue Procedure:
- Dequeue Procedure:
- Is_empty Procedure:

Linked List – Queue Data Structure



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  - A "front" variable referencing the oldest item
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  - Each item points to the item enqueued after it
- Enqueue Procedure:

```
enqueue(x){
    last = new Node(x)
    back.next = last
    back = last
}
```

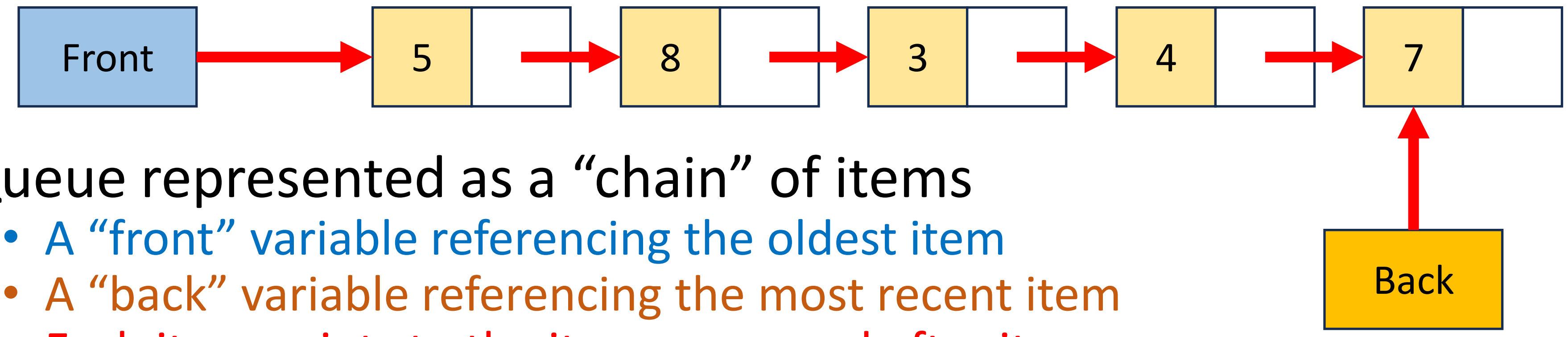
- Dequeue Procedure:

```
dequeue(){
    first = front.item
    front = front.next
    return first
}
```

- Is_empty Procedure:

```
is_empty(){
    return front.equals(Null)
}
```

Circular Array – Queue Data Structure



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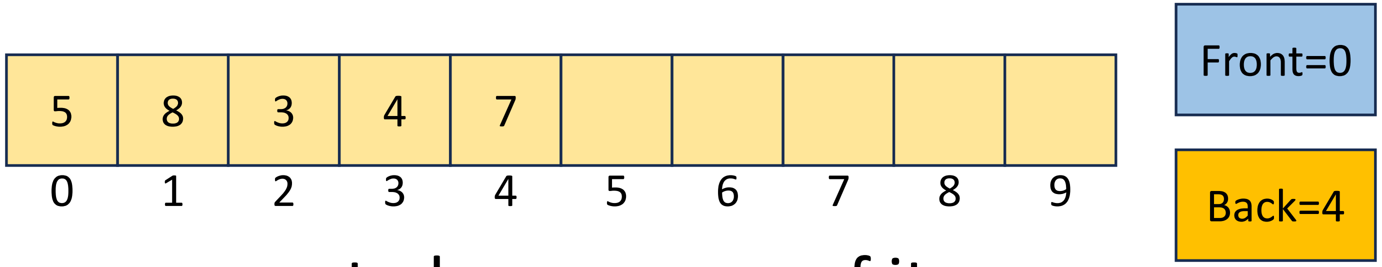
Circular Array – Queue Data Structure



- Queue represented as an array of items
  - A “front” index to indicate the oldest item in the queue
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- Enqueue Procedure:
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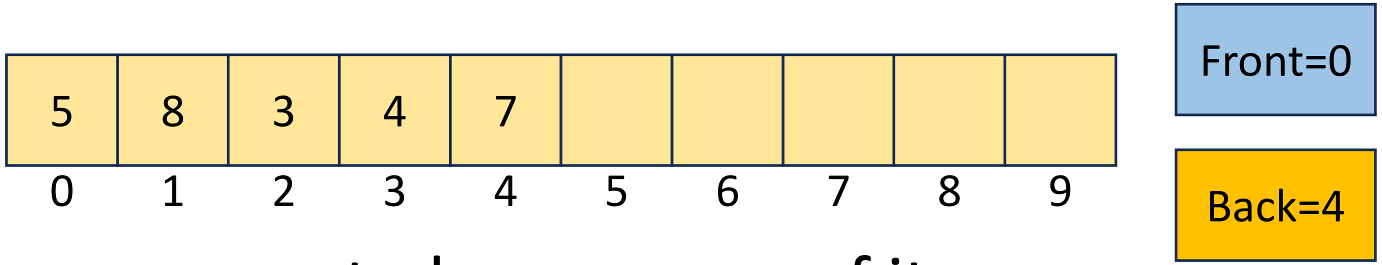
Circular Array – Queue Data Structure



- Queue represented as an array of items
  - A “front” index to indicate the oldest item in the queue
  - A “back” index to indicate the most recent item in the queue
- Enqueue Procedure:

```
enqueue(x){
   queue[back] = x
   back = (back + 1) % queue.length
}
```

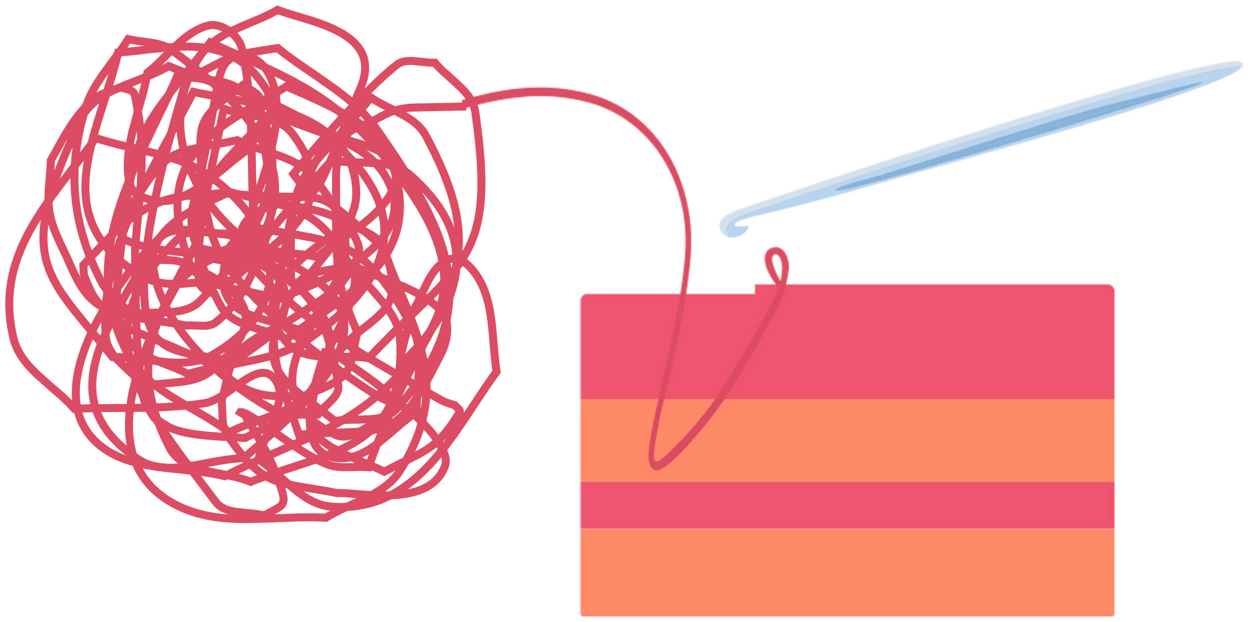
- Dequeue Procedure:

```
dequeue(){
   first = queue[front]
   front = (front + 1) % queue.length
}
```

- Is_empty Procedure:

```
is_empty(){
   return front == back
}
```

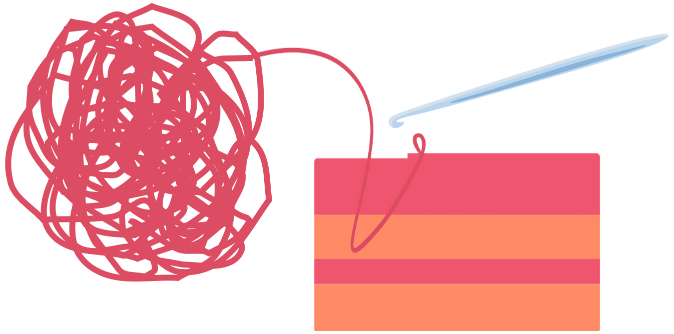
Linked List vs. Circular Array



Warm up:

- I have a pile of string
- I have one end of the string in-hand
- I need to find the other end in the pile
- How can I do this efficiently?

Algorithm Ideas



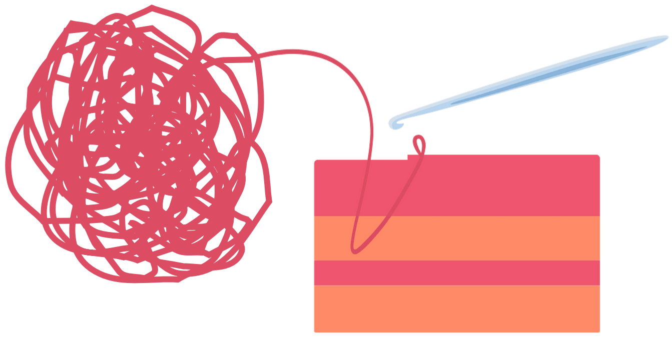
- Ideas:

Algorithm Running Times

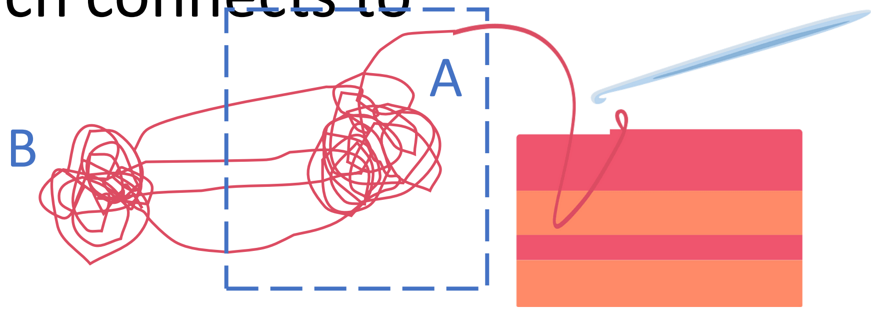
- How do we express running time?
- Units of “time”
- How to express efficiency?

My Approach

End-of-Yarn Finding



1. Set aside the already-obtained "beginning"
2. If you see the end of the yarn, you're done!
3. Separate the pile of yarn into 2 piles, note which connects to the beginning (call it pile A, the other pile B)



Repeat on pile with end

4. Count the number of strands crossing the piles
5. If the count is even, pile A contains the end, else pile B does

Why Do resource Analysis?

- Allows us to compare *algorithms,* not implementations
  - Using observations necessarily couples the algorithm with its implementation
  - If my implementation on my computer takes more time than your implementation on your computer, we cannot conclude your algorithm is better
- We can predict an algorithm’s running time before implementing
- Understand where the bottlenecks are in our algorithm

Goals for Algorithm Analysis

- Identify a *function* which maps the algorithm's input size to a measure of resources used
  - Domain of the function: **sizes** of the input
    - Number of characters in a string, number of items in a list, number of pixels in an image
  - Codomain of the function: **counts** of resources used
    - Number of times the algorithm adds two numbers together, number times the algorithm does a > or < comparison, maximum number of bytes of memory the algorithm uses at any time
- Important note: Make sure you know the "units" of your domain and codomain!

Worst Case Analysis (in general)

- If an algorithm has a worst case resource complexity of $f(n)$
  - Among all possible size-n inputs, the “worst” one will use $f(n)$ “resources”
  - I.e. $f(n)$ gives the maximum count of resources needed from among all inputs of size n

Worst Case Running Time Analysis

- If an algorithm has a worst case running time of $f(n)$
  - Among all possible size-n inputs, the "worst" one will do $f(n)$ "operations"
  - I.e. $f(n)$ gives the maximum operation count from among all inputs of size n

Worst Case Space Analysis

- If an algorithm has a worst case space complexity of $f(n)$
  - Among all possible size-n inputs, the “worst” one will need $f(n)$ “memory units”
  - I.e. $f(n)$ gives the maximum memory unit count from among all inputs of size n

Worst Case Running Time - Example

```
myFunction(List n){
   b = 55 + 5;
   c = b / 3;
   b = c + 100;
   for (i = 0; i < n.size(); i++) {
      b++;
  }
   if (b % 2 == 0) {
      c++;
   }
   else {
      for (i = 0; i < n.size(); i++) {
         c++;
      }
   }
   return c;
}
```

Questions to ask:

- What are the units of the input size?
- What are the operations we're counting?
- For each line:
  - How many times will it run?
  - How long does it take to run?
  - Does this change with the input size?

Worst Case Running Time – Example 2

```
beAnnoying(List n){
    List m = [];
    for (i=0; i < n.size(); i++){
        m.add(n[i]);
        for (j=0; j< n.size(); j++){
            print ("Hi, I'm annoying");
        }
    }
    return;
}
```

Questions to ask:

- What are the units of the input size?
- What are the operations we're counting?
- For each line:
  - How many times will it run?
  - How long does it take to run?
  - Does this change with the input size?

Worst Case Running Time – General Guide

- Add together the time of consecutive statements
- Loops: Sum up the time required through each iteration of the loop
  - If each takes the same time, then [time per loop $\times$ number of iterations]
- Conditionals: Sum together the time to check the condition and time of the slowest branch
- Function Calls: Time of the function’s body
- Recursion: Solve a recurrence relation