## W <br> PAUL G. ALLEN SCHOOL OF COMPUTER SCIENCE \& ENGINEERING

CSE341: Programming Languages
Lecture 14
Thunks, Laziness, Streams, Memoization

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## Delayed evaluation

For each language construct, the semantics specifies when subexpressions get evaluated. In ML, Racket, Java, C:

- Function arguments are eager (call-by-value)
- Evaluated once before calling the function
- Conditional branches are not eager

It matters: calling factorial-bad never terminates:
(define (my-if-bad x y z)
(if x y z))
(define (factorial-bad n)
(my-if-bad (= n 0)
1
(* n (factorial-bad (- n 1)))))

## Thunks delay

We know how to delay evaluation: put expression in a function!

- Thanks to closures, can use all the same variables later

A zero-argument function used to delay evaluation is called a thunk

- As a verb: thunk the expression

This works (but it is silly to wrap if like this):

```
(define (my-if x y z)
    (if x (y) (z)))
(define (fact n)
    (my-if (= n 0)
        (lambda() 1)
        (lambda() (* n (fact (- n 1))))))
```


## The key point

- Evaluate an expression e to get a result:
e
- A function that when called, evaluates $\mathbf{e}$ and returns result
- Zero-argument function for "thunking"


## (lambda () e)

- Evaluate $\mathbf{e}$ to some thunk and then call the thunk


## (e)

- Next: Powerful idioms related to delaying evaluation and/or avoided repeated or unnecessary computations
- Some idioms also use mutation in encapsulated ways


## Avoiding expensive computations

Thunks let you skip expensive computations if they are not needed

Great if take the true-branch:

```
(define (f th)
    (if (...) 0 (... (th) ...)))
```

But worse if you end up using the thunk more than once:

$$
\begin{aligned}
& \text { (define (f th) } \\
& \text { (... (if (...) } 0 \text { (... (th) ...)) } \\
& \text { (if (...) } 0 \text { (... (th) ...)) } \\
& \text { (if (...) } 0 \text { (... (th) ...)))) }
\end{aligned}
$$

In general, might not know many times a result is needed

## Best of both worlds

Assuming some expensive computation has no side effects, ideally we would:

- Not compute it until needed
- Remember the answer so future uses complete immediately Called lazy evaluation

Languages where most constructs, including function arguments, work this way are lazy languages

- Haskell

Racket predefines support for promises, but we can make our own

- Thunks and mutable pairs are enough


## Delay and force

## (define (my-delay th)

(mcons \#f th))
(define (my-force p)
(if (mcar p)
(mcdr p)
(begin (set-mcar! p \#t) (set-mcdr! p ((mcdr p))) (mcdr p))))

An ADT represented by a mutable pair

- \#f in car means cdr is unevaluated thunk
- Really a one-of type: thunk or result-of-thunk
- Ideally hide representation in a module


## Using promises

```
(define (f p)
    (... (if (...) 0 (... (my-force p) ...))
    (if (...) 0 (... (my-force p) ...))
    (if (...) 0 (... (my-force p) ...)))
```

    (f (my-delay (lambda () e)))
    
## Lessons From Example

See code file for example that does multiplication using a very slow addition helper function

- With thunking second argument:
- Great if first argument 0
- Okay if first argument 1
- Worse otherwise
- With precomputing second argument:
- Okay in all cases
- With thunk that uses a promise for second argument:
- Great if first argument 0
- Okay otherwise


## Streams

- A stream is an infinite sequence of values
- So cannot make a stream by making all the values
- Key idea: Use a thunk to delay creating most of the sequence
- Just a programming idiom

A powerful concept for division of labor:

- Stream producer knows how to create any number of values
- Stream consumer decides how many values to ask for

Some examples of streams you might (not) be familiar with:

- User actions (mouse clicks, etc.)
- UNIX pipes: cmd1 | cmd2 has cmd2 "pull" data from cmd1
- Output values from a sequential feedback circuit


## Using streams

We will represent streams using pairs and thunks

Let a stream be a thunk that when called returns a pair:
'(next-answer . next-thunk)

So given a stream s, the client can get any number of elements

- First: (car (s))
- Second: (car ((cdr (s))))
- Third: (car ((cdr ((cdr (s))))))
(Usually bind (cdr (s)) to a variable or pass to a recursive function)


## Example using streams

This function returns how many stream elements it takes to find one for which tester does not return \#f

- Happens to be written with a tail-recursive helper function
(define (number-until stream tester)
(letrec ([f (lambda (stream ans)
(let ([pr (stream)])
(if (tester (car pr)) ans
(f (cdr pr) (+ ans 1)))))])
(f stream 1)))
- (stream) generates the pair
- So recursively pass (cdr pr), the thunk for the rest of the infinite sequence


## Streams

Coding up a stream in your program is easy

- We will do functional streams using pairs and thunks

Let a stream be a thunk that when called returns a pair: '(next-answer . next-thunk)

Saw how to use them, now how to make them...

- Admittedly mind-bending, but uses what we know


## Making streams

- How can one thunk create the right next thunk? Recursion!
- Make a thunk that produces a pair where cdr is next thunk
- A recursive function can return a thunk where recursive call does not happen until thunk is called
(define ones (lambda () (cons 1 ones)))
(define nats
(letrec ([f (lambda (x)
(cons x (lambda () (f (+ x 1)))))])
(lambda () (f 1))))
(define powers-of-two
(letrec ([f (lambda (x)
(cons x (lambda () (f (* x 2)))))])
(lambda () (f 2))))


## Getting it wrong

- This uses a variable before it is defined
(define ones-really-bad (cons 1 ones-really-bad))
- This goes into an infinite loop making an infinite-length list (define ones-bad (lambda () cons 1 (ones-bad))) (define (ones-bad) (cons 1 (ones-bad)))
- This is a stream: thunk that returns a pair with cdr a thunk (define ones (lambda () (cons 1 ones))) (define (ones) (cons 1 ones))


## Memoization

- If a function has no side effects and does not read mutable memory, no point in computing it twice for the same arguments
- Can keep a cache of previous results
- Net win if (1) maintaining cache is cheaper than recomputing and (2) cached results are reused
- Similar to promises, but if the function takes arguments, then there are multiple "previous results"
- For recursive functions, this memoization can lead to exponentially faster programs
- Related to algorithmic technique of dynamic programming


## How to do memoization: see example

- Need a (mutable) cache that all calls using the cache share
- So must be defined outside the function(s) using it
- See code for an example with Fibonacci numbers
- Good demonstration of the idea because it is short, but, as shown in the code, there are also easier less-general ways to make fibonacci efficient
- (An association list (list of pairs) is a simple but sub-optimal data structure for a cache; okay for our example)


## aSSOC

- Example uses assoc, which is just a library function you could look up in the Racket reference manual:
(assoc v lst) takes a list of pairs and locates the first element of lst whose car is equal to $\mathbf{v}$ according to isequal?. If such an element exists, the pair (i.e., an element of lst) is returned. Otherwise, the result is \#f.
- Returns \#f for not found to distinguish from finding a pair with \#f in cdr

