Overview

- Last lecture
  - Sequential Logic Examples
- Today
  - State encoding
    - One-hot encoding
    - Output encoding

State encoding

- Assume n state bits and m states
  - $2^n! / (2^n – m)!$ possible encodings [$m \geq n \geq \log_2(m)$]
    - From binomial expansion
    - Example: 3 state bits, 4 states, 1680 possible state assignments
- Hard problem, with no known algorithmic solution
  - Can try heuristic approaches
  - Can try to optimize some metric
    - FSM size (amount of logic and number of FFs)
    - FSM speed (depth of logic and fanout)
    - FSM dependencies (decomposition)
- Need to consider startup
  - Self-starting FSM or explicit reset input

State-encoding strategies

- No guarantee of optimality
  - An intractable problem
- Most common strategies
  - Binary (sequential) – number states as in the state table
  - Random – computer tries random encodings
  - Heuristic – rules of thumb that seem to work well
    - e.g. Gray-code – try to give adjacent states (states with an arc between them) codes that differ in only one bit position
  - One-hot – use as many state bits as there are states
  - Output – use outputs to help encode states

One-hot encoding

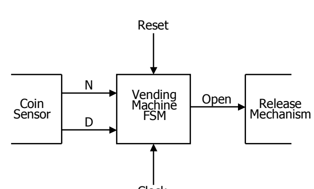
- One-hot: Encode n states using n flip-flops
  - Assign a single "1" for each state
    - Example: 0001, 0010, 0100, 1000
  - Propagate a single "1" from one flip-flop to the next
    - All other flip-flop outputs are "0"
- The inverse: One-cold encoding
  - Assign a single "0" for each state
    - Example: 1110, 1101, 1011, 0111
  - Propagate a single "0" from one flip-flop to the next
    - All other flip-flop outputs are "1"
- "almost one-hot" encoding
  - Use no-hot (000…0) for the initial (reset state)
  - Assumes you never revisit the reset state

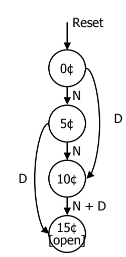
One-hot encoding (con't)

- Often the best approach for FPGAs
  - FPGAs have many flip-flops
  - One-hot machines use the least next-state logic
- Draw FSM directly from the state diagram
  - One product term per incoming arc
  - But complex state diagram $\Rightarrow$ complex design
- One-hot designs have many possible failure modes
  - All states that aren't one-hot
  - Can create logic to reset the FSM if it enters illegal state
- Large machines require many flip-flops
  - Decompose design into smaller one-hot encoded sub-designs
    - n+m states for two machines versus n*m states for one

Vending machine again...

- Release item after receiving 15 cents
  - Single coin slot for dimes and nickels
    - Sensor specifies coin type
  - Machine does not give change



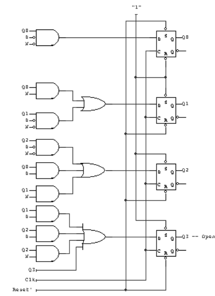


One-hot encoded transition table

present state $Q_3Q_2Q_1Q_0$	inputs D N	next state $D_3D_2D_1D_0$	output open
0 0 0 1	0 0	0 0 0 1	0
	0 1	0 0 1 0	0
	1 0	0 1 0 0	0
	1 1	– – – –	–
0 0 1 0	0 0	0 0 1 0	0
	0 1	0 1 0 0	0
	1 0	1 0 0 0	0
	1 1	– – – –	–
0 1 0 0	0 0	0 1 0 0	0
	0 1	1 0 0 0	0
	1 0	1 0 0 0	0
	1 1	– – – –	–
1 0 0 0	– –	1 0 0 0	1

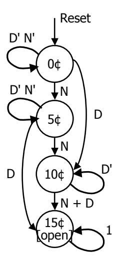
$D_0 = Q_0D'N'$
$D_1 = Q_0N + Q_1D'N'$
$D_2 = Q_0D + Q_1N + Q_2D'N'$
$D_3 = Q_1D + Q_2D + Q_2N + Q_3$
OPEN = Q_3

One-hot encoded vending machine



$D_0 = Q_0D'N'$
$D_1 = Q_0N + Q_1D'N'$
$D_2 = Q_0D + Q_1N + Q_2D'N'$
$D_3 = Q_1D + Q_2D + Q_2N + Q_3$
OPEN = Q_3

Designing from the state diagram



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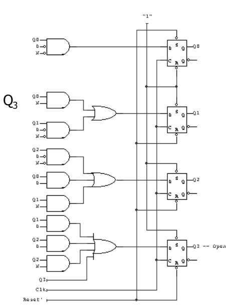
OPEN = Q_3

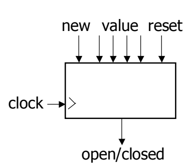


Output encoding

- Reuse outputs as state bits
  - Why create new functions when you can use outputs?
  - Bits from state assignments are the outputs for that state
    - Take outputs directly from the flip-flops
- ad hoc - no tools
  - Yields small circuits for most FSMs
  - Fits nicely with synchronous Mealy machines

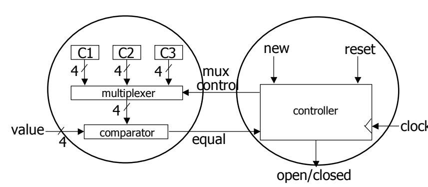
Digital combination lock again...

- An output-encoded FSM
  - Punch in 3 values in sequence and the door opens
  - If there is an error the lock must be reset
  - After the door opens the lock must be reset
  - Inputs: sequence of number values, reset
  - Outputs: door open/close

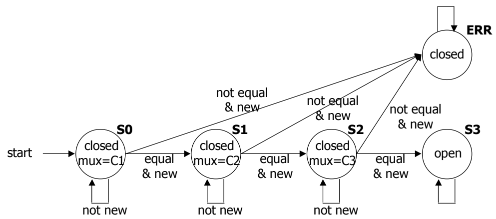


Separate data path and control

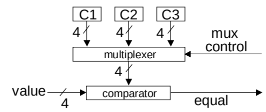
- Design datapath first
  - After the state diagram
  - Before the state encoding
- Control has 2 outputs
  - Mux control to datapath
  - Lock open/closed

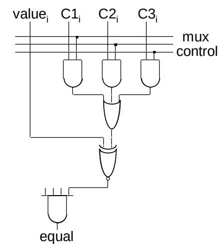


Draw the state diagram



Design the datapath





- Choose simple control
  - 3-wire mux for datapath
    - Control is 001, 010, 100
  - Open/closed bit for lock state
    - Control is 0/1

Output encode the FSM

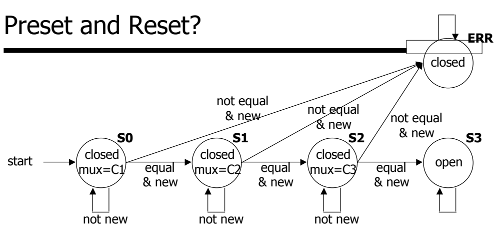
- FSM outputs
  - Mux control is 100, 010, 001
  - Lock control is 0/1
- State are: S0, S1, S2, S3, or ERR
  - Can use 3, 4, or 5 bits to encode
  - Have 4 outputs, so choose 4 bits
    - Encode mux control and lock control in state bits
    - Lock control is first bit, mux control is last 3 bits

      S0 = 0001 (lock closed, mux first code)
      S1 = 0010 (lock closed, mux second code)
      S2 = 0100 (lock closed, mux third code)
      S3 = 1000 (lock open)
      ERR = 0000 (error, lock closed)

FSM has 4 state bits and 2 inputs...

- Output encoded!
  - Outputs and state bits are the same
- How do we minimize the logic?
  - FSM has 4 state bits and 2 inputs (equal, new)
  - 6-variable kmap?
- Notice the state assignment is close to one-hot
  - ERR state (0000) is only deviation
  - Is there a clever design we can use?

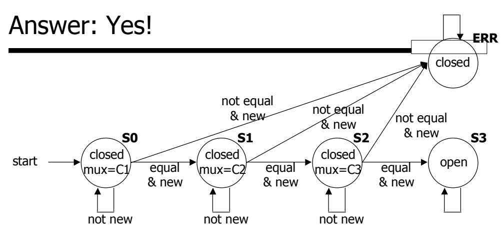
Preset and Reset?



Assume flip-flops have preset & reset inputs

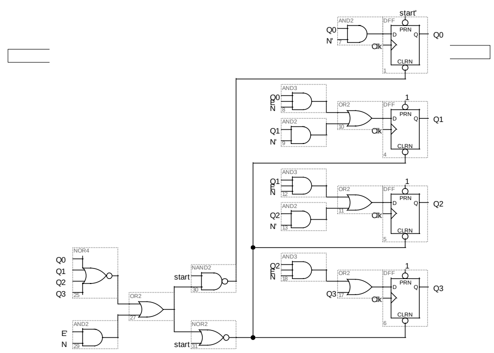
Can we encode the ERR state as reset?

Answer: Yes!



$D_0 = Q_0N'$
$D_1 = Q_0EN + Q_1N'$
$D_2 = Q_1EN + Q_2N'$
$D_3 = Q_2EN + Q_3$

$Preset_0 = start$
$Preset_{1,2,3} = 0$
$Reset_0 = start'(E'N + (Q_0+Q_1+Q_2+Q_3)')$
$Reset_{1,2,3} = start + (E'N + (Q_0+Q_1+Q_2+Q_3)')$



FSM design: A 5-step process

1. Understand the problem
   - State diagram and state-transition table
2. Determine the machine's states
   - Consider missing transitions: Will the machine start?
   - Minimize the state diagram: Reuse states where possible
3. Encode the states
   - Encode states, outputs with a reasonable encoding choice
   - Consider the implementation target
4. Design the next-state logic
   - Minimize the combinational logic
   - Choices made in steps 2 & 3 affect the logic complexity
5. Implement the FSM