Sequential logic examples

- Basic design approach: a 4-step design process
- Hardware description languages and finite state machines
- Implementation examples and case studies
  - finite-string pattern recognizer
  - complex counter
  - traffic light controller
  - door combination lock

General FSM design procedure

- (1) Determine inputs and outputs
- (2) Determine possible states of machine
  - state minimization
- (3) Encode states and outputs into a binary code
  - state assignment or state encoding
  - output encoding
  - possibly input encoding (if under our control)
- (4) Realize logic to implement functions for states and outputs
  - combinational logic implementation and optimization
  - choices in steps 2 and 3 can have large effect on resulting logic

Finite string pattern recognizer (step 1)

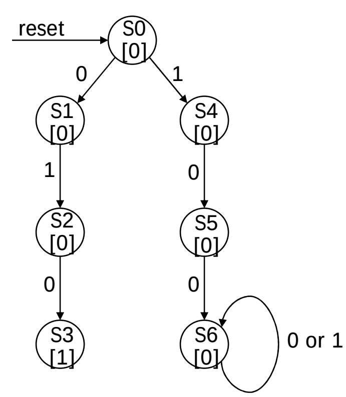
- Finite string pattern recognizer
  - one input (X) and one output (Z)
  - output is asserted whenever the input sequence …010… has been
    observed, as long as the sequence …100… has never been seen
- Step 1: understanding the problem statement
  - sample input/output behavior:

X: 0 0 1 0 1 0 1 0 0 1 0 …
Z: 0 0 0 1 0 1 0 1 0 0 0 …

X: 1 1 0 1 1 0 1 0 0 1 0 …
Z: 0 0 0 0 0 0 0 1 0 0 0 …

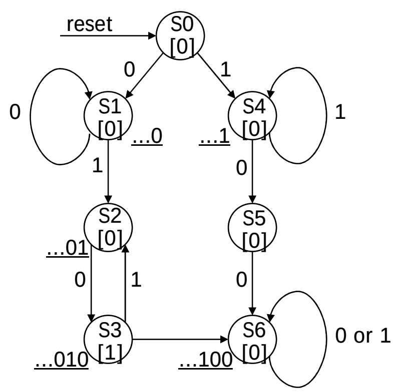
Finite string pattern recognizer (step 2)

- Step 2: draw state diagram
  - for the strings that must be recognized, i.e., 010 and 100
  - a Moore implementation



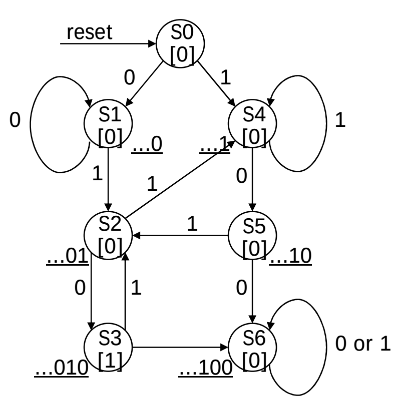
Finite string pattern recognizer (step 2, cont'd)

- Exit conditions from state S3: have recognized …010
  - if next input is 0 then have …0100 = ...100 (state S6)
  - if next input is 1 then have …0101 = …01 (state S2)
- Exit conditions from S1: recognizes strings of form …0 (no 1 seen)
  - loop back to S1 if input is 0
- Exit conditions from S4: recognizes strings of form …1 (no 0 seen)
  - loop back to S4 if input is 1



Finite string pattern recognizer (step 2, cont'd)

- S2 and S5 still have incomplete transitions
  - S2 = …01; If next input is 1,
    then string could be prefix of (01)1(00)
    S4 handles just this case
  - S5 = …10; If next input is 1,
    then string could be prefix of (10)1(0)
    S2 handles just this case
- Reuse states as much as possible
  - look for same meaning
  - state minimization leads to smaller number of bits to represent states
- Once all states have a complete set of transitions we have a final state diagram



Finite string pattern recognizer (step 3)

- Verilog description including state assignment (or state encoding)

```
module string (clk, X, rst, Q0, Q1, Q2, Z);
input clk, X, rst;
output Q0, Q1, Q2, Z;

parameter S0 = [0,0,0]; //reset state
parameter S1 = [0,0,1]; //strings ending in   ...0
parameter S2 = [0,1,0]; //strings ending in  ...01
parameter S3 = [0,1,1]; //strings ending in ...010
parameter S4 = [1,0,0]; //strings ending in   ...1
parameter S5 = [1,0,1]; //strings ending in  ...10
parameter S6 = [1,1,0]; //strings ending in ...100

reg state[0:2];

assign Q0 = state[0];
assign Q1 = state[1];
assign Q2 = state[2];
assign Z = (state == S3);
```

```
always @(posedge clk) begin
  if (rst) state = S0;
  else
    case (state)
      S0: if (X) state = S4 else state = S1;
      S1: if (X) state = S2 else state = S1;
      S2: if (X) state = S4 else state = S3;
      S3: if (X) state = S2 else state = S6;
      S4: if (X) state = S4 else state = S5;
      S5: if (X) state = S2 else state = S6;
      S6: state = S6;
      default: begin
        $display ("invalid state reached");
        state = 3'bxxx;
      end
    endcase
end

endmodule
```


Finite string pattern recognizer

- Review of process
  - understanding problem
    - write down sample inputs and outputs to understand specification
  - derive a state diagram
    - write down sequences of states and transitions for sequences to be recognized
  - minimize number of states
    - add missing transitions; reuse states as much as possible
  - state assignment or encoding
    - encode states with unique patterns
  - simulate realization
    - verify I/O behavior of your state diagram to ensure it matches specification

Complex counter

- A synchronous 3-bit counter has a mode control M
  - when M = 0, the counter counts up in the binary sequence
  - when M = 1, the counter advances through the Gray code sequence

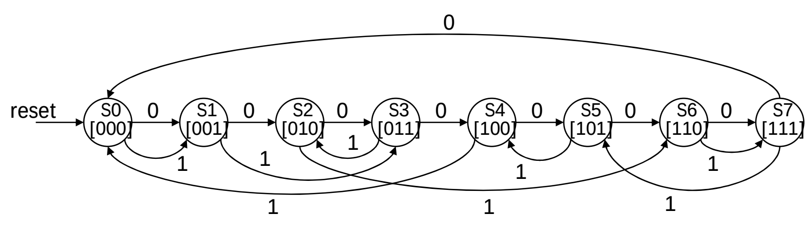
binary: 000, 001, 010, 011, 100, 101, 110, 111
Gray: 000, 001, 011, 010, 110, 111, 101, 100

- Valid I/O behavior (partial)

Mode Input M	Current State	Next State
0	000	001
0	001	010
1	010	110
1	110	111
1	111	101
0	101	110
0	110	111

Complex counter (state diagram)

- Deriving state diagram
  - one state for each output combination
  - add appropriate arcs for the mode control



Complex counter (state encoding)

- Verilog description including state encoding

```
module string (clk, M, rst, Z0, Z1, Z2);
input clk, X, rst;
output Z0, Z1, Z2;

parameter S0 = [0,0,0];
parameter S1 = [0,0,1];
parameter S2 = [0,1,0];
parameter S3 = [0,1,1];
parameter S4 = [1,0,0];
parameter S5 = [1,0,1];
parameter S6 = [1,1,0];
parameter S7 = [1,1,1];

reg state[0:2];

assign Z0 = state[0];
assign Z1 = state[1];
assign Z2 = state[2];
```

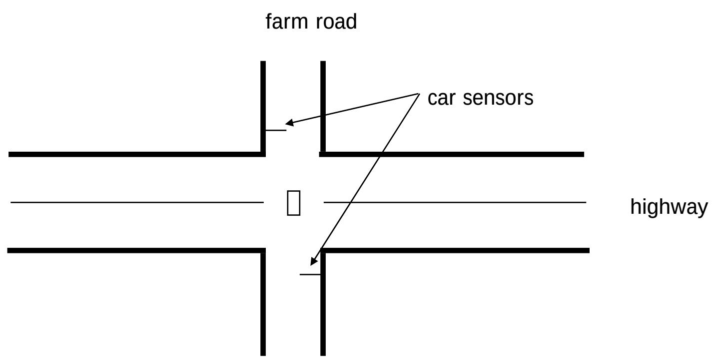
```
always @(posedge clk) begin
  if rst state = S0;
  else
    case (state)
      S0: state = S1;
      S1: if (M) state = S3 else state = S2;
      S2: if (M) state = S6 else state = S3;
      S3: if (M) state = S2 else state = S4;
      S4: if (M) state = S0 else state = S5;
      S5: if (M) state = S4 else state = S6;
      S6: if (M) state = S7 else state = S7;
      S7: if (M) state = S5 else state = S0;
 endcase

end

endmodule
```


Example: traffic light controller

- A busy highway is intersected by a little used farmroad
- Detectors C sense the presence of cars waiting on the farmroad
  - with no car on farmroad, light remain green in highway direction
  - if vehicle on farmroad, highway lights go from Green to Yellow to Red, allowing the farmroad lights to become green
  - these stay green only as long as a farmroad car is detected but never longer than a set interval
  - when these are met, farm lights transition from Green to Yellow to Red, allowing highway to return to green
  - even if farmroad vehicles are waiting, highway gets at least a set interval as green
- Assume you have an interval timer that generates:
  - a short time pulse (TS) and
  - a long time pulse (TL),
  - in response to a set (ST) signal.
  - TS is to be used for timing yellow lights and TL for green lights

Example: traffic light controller (cont')

- Highway/farm road intersection



Example: traffic light controller (cont')

- Tabulation of inputs and outputs

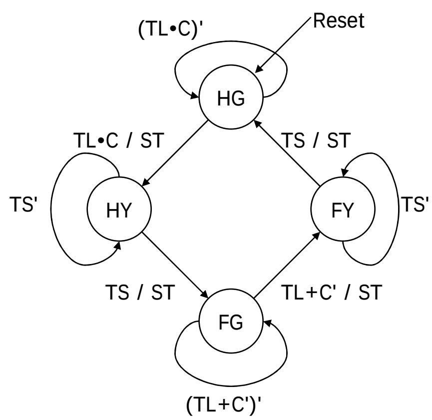
inputs	description	outputs	description
reset	place FSM in initial state	HG, HY, HR	assert green/yellow/red highway lights
C	detect vehicle on the farm road	FG, FY, FR	assert green/yellow/red highway lights
TS	short time interval expired	ST	start timing a short or long interval
TL	long time interval expired		

- Tabulation of unique states – some light configurations imply others

state	description
HG	highway green (farm road red)
HY	highway yellow (farm road red)
FG	farm road green (highway red)
FY	farm road yellow (highway red)

Example: traffic light controller (cont')

- State diagram



Example: traffic light controller (cont')

- Generate state table with symbolic states
- Consider state assignments

output encoding – similar problem to state assignment (Green = 00, Yellow = 01, Red = 10)

Inputs C	TL	TS	Present State	Next State	Outputs ST	H	F
0	–	–	HG	HG	0	Green	Red
–	0	–	HG	HG	0	Green	Red
1	1	–	HG	HY	1	Green	Red
–	–	0	HY	HY	0	Yellow	Red
–	–	1	HY	FG	1	Yellow	Red
1	0	–	FG	FG	0	Red	Green
0	–	–	FG	FY	1	Red	Green
–	1	–	FG	FY	1	Red	Green
–	–	0	FY	FY	0	Red	Yellow
–	–	1	FY	HG	1	Red	Yellow

SA1:	HG = 00	HY = 01	FG = 11	FY = 10	
SA2:	HG = 00	HY = 10	FG = 01	FY = 11	
SA3:	HG = 0001	HY = 0010	FG = 0100	FY = 1000	(one-hot)

Logic for different state assignments

- SA1

  NS1 = C•TL'•PS1•PS0 + TS•PS1'•PS0 + TS•PS1•PS0' + C'•PS1•PS0 + TL•PS1•PS0
  NS0 = C•TL•PS1'•PS0' + C•TL'•PS1•PS0 + PS1'•PS0

  ST = C•TL•PS1'•PS0' + TS•PS1'•PS0 + TS•PS1•PS0' + C'•PS1•PS0 + TL•PS1•PS0
  H1 = PS1 H0 = PS1'•PS0
  F1 = PS1' F0 = PS1•PS0'

- SA2

  NS1 = C•TL•PS1' + TS'•PS1 + C'•PS1'•PS0
  NS0 = TS•PS1•PS0' + PS1'•PS0 + TS'•PS1•PS0

  ST = C•TL•PS1' + C'•PS1'•PS0 + TS•PS1
  H1 = PS0 H0 = PS1•PS0'
  F1 = PS0' F0 = PS1•PS0

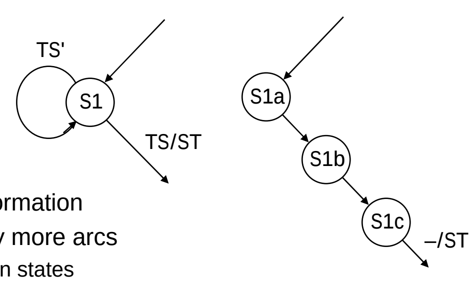
- SA3

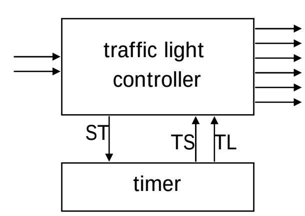
  NS3 = C'•PS2 + TL•PS2 + TS'•PS3 NS2 = TS•PS1 + C•TL'•PS2
  NS1 = C•TL•PS0 + TS'•PS1 NS0 = C'•PS0 + TL'•PS0 + TS•PS3

  ST = C•TL•PS0 + TS•PS1 + C'•PS2 + TL•PS2 + TS•PS3
  H1 = PS3 + PS2 H0 = PS1
  F1 = PS1 + PS0 F0 = PS3

Traffic light controller as two communicating FSMs

- Without separate timer
  - S0 would require 7 states
  - S1 would require 3 states
  - S2 would require 7 states
  - S3 would require 3 states
  - S1 and S3 have simple transformation
  - S0 and S2 would require many more arcs
    - C could change in any of seven states
- By factoring out timer
  - greatly reduce number of states
    - 4 instead of 20
  - counter only requires seven or eight states
    - 12 total instead of 20





Traffic light controller FSM

- Specification of inputs, outputs, and state elements

```
module FSM(HR, HY, HG, FR, FY, FG, ST, TS, TL, C, reset, Clk);
  output     HR;
  output     HY;
  output     HG;
  output     FR;
  output     FY;
  output     FG;
  output     ST;
  input      TS;
  input      TL;
  input      C;
  input      reset;
  input      Clk;

  reg [6:1]  state;
  reg        ST;
```

```
parameter highwaygreen   = 6'b001100;
parameter highwayyellow  = 6'b010100;
parameter farmroadgreen  = 6'b100001;
parameter farmroadyellow = 6'b100010;

assign HR = state[6];
assign HY = state[5];
assign HG = state[4];
assign FR = state[3];
assign FY = state[2];
assign FG = state[1];
```

specify state bits and codes for each state as well as connections to outputs

Traffic light controller FSM (cont'd)

```
  initial begin state = highwaygreen; ST = 0; end

  always @(posedge Clk)
    begin
      if (reset)
        begin state = highwaygreen; ST = 1; end
      else
        begin
          ST = 0;
          case (state)
            highwaygreen:
              if (TL & C) begin state = highwayyellow; ST = 1; end
            highwayyellow:
              if (TS) begin state = farmroadgreen; ST = 1; end
            farmroadgreen:
              if (TL | !C) begin state = farmroadyellow; ST = 1; end
            farmroadyellow:
              if (TS) begin state = highwaygreen; ST = 1; end
          endcase
        end
    end
endmodule
```

case statement triggerred by clock edge

Timer for traffic light controller

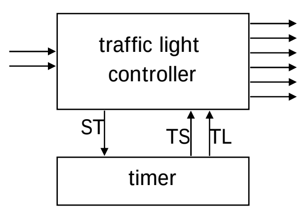
- Another FSM

```
module Timer(TS, TL, ST, Clk);
  output TS;
  output TL;
  input    ST;
  input    Clk;
  integer   value;

  assign TS = (value >=  4); //  5 cycles after reset
  assign TL = (value >= 14); // 15 cycles after reset

  always @(posedge ST) value = 0; // async reset

  always @(posedge Clk) value = value + 1;

endmodule
```


Complete traffic light controller

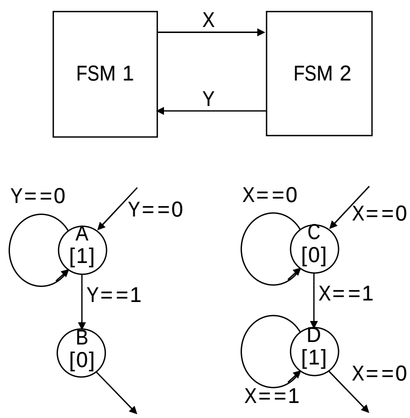
- Tying it all together (FSM + timer)
  - structural Verilog (same as a schematic drawing)

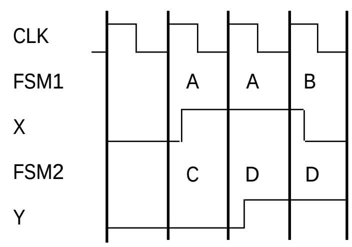
```
module main(HR, HY, HG, FR, FY, FG, reset, C, Clk);
 output HR, HY, HG, FR, FY, FG;
 input  reset, C, Clk;

  Timer part1(TS, TL, ST, Clk);
  FSM   part2(HR, HY, HG, FR, FY, FG, ST, TS, TL, C, reset, Clk);
endmodule
```



Communicating finite state machines

- One machine's output is another machine's input

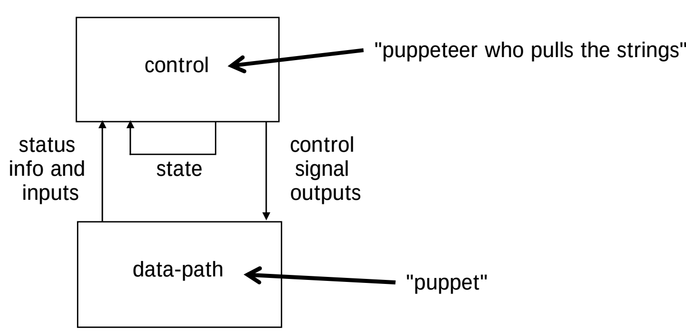




machines advance in lock step
initial inputs/outputs: X = 0, Y = 0

Data-path and control

- Digital hardware systems = data-path + control
  - datapath: registers, counters, combinational functional units (e.g., ALU), communication (e.g., busses)
  - control: FSM generating sequences of control signals that instructs datapath what to do next



Digital combinational lock

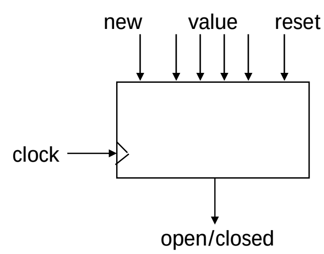
- Door combination lock:
  - punch in 3 values in sequence and the door opens; if there is an error the lock must be reset; once the door opens the lock must be reset
  - inputs: sequence of input values, reset
  - outputs: door open/close
  - memory: must remember combination or always have it available
  - open questions: how do you set the internal combination?
    - stored in registers (how loaded?)
    - hardwired via switches set by user

Implementation in software

```
integer combination_lock ( ) {
    integer v1, v2, v3;
    integer error = 0;
    static integer c[3] = 3, 4, 2;

    while (!new_value( ));
    v1 = read_value( );
    if (v1 != c[1]) then error = 1;

    while (!new_value( ));
    v2 = read_value( );
    if (v2 != c[2]) then error = 1;

    while (!new_value( ));
    v3 = read_value( );
    if (v2 != c[3]) then error = 1;

    if (error == 1) then return(0); else return (1);
}
```

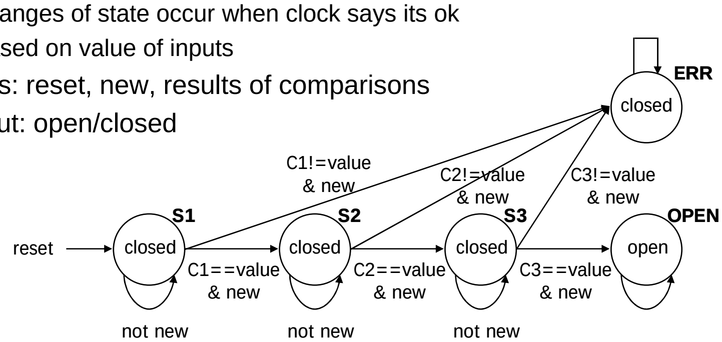
Determining details of the specification

- How many bits per input value?
- How many values in sequence?
- How do we know a new input value is entered?
- What are the states and state transitions of the system?



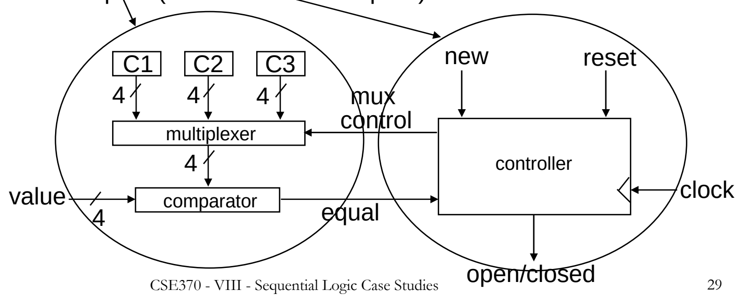
Digital combination lock state diagram

- States: 5 states
  - represent point in execution of machine
  - each state has outputs
- Transitions: 6 from state to state, 5 self transitions, 1 global
  - changes of state occur when clock says its ok
  - based on value of inputs
- Inputs: reset, new, results of comparisons
- Output: open/closed



Data-path and control structure

- Data-path
  - storage registers for combination values
  - multiplexer
  - comparator
- Control
  - finite-state machine controller
  - control for data-path (which value to compare)



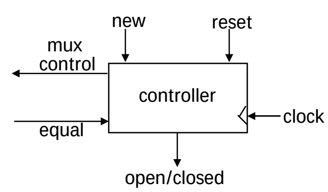
State table for combination lock

- Finite-state machine
  - refine state diagram to take internal structure into account
  - state table ready for encoding

reset	new	equal	state	next state	mux	open/closed
1	–	–	–	S1	C1	closed
0	0	–	S1	S1	C1	closed
0	1	0	S1	ERR	–	closed
0	1	1	S1	S2	C2	closed
...						
0	1	1	S3	OPEN	–	open
...						

Encodings for combination lock

- Encode state table
  - state can be: S1, S2, S3, OPEN, or ERR
    - needs at least 3 bits to encode: 000, 001, 010, 011, 100
    - and as many as 5: 00001, 00010, 00100, 01000, 10000
    - choose 4 bits: 0001, 0010, 0100, 1000, 0000
  - output mux can be: C1, C2, or C3
    - needs 2 to 3 bits to encode
    - choose 3 bits: 001, 010, 100
  - output open/closed can be: open or closed
    - needs 1 or 2 bits to encode
    - choose 1 bit: 1, 0

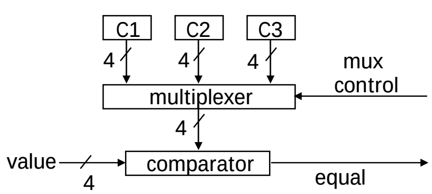


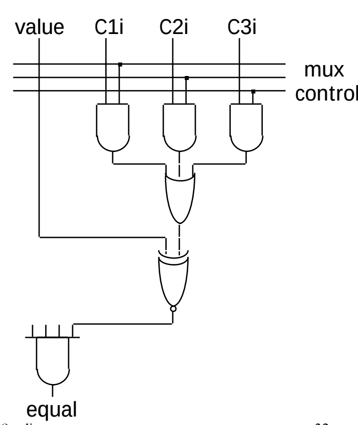
reset	new	equal	state	next state	mux	open/closed
1	–	–	–	0001	001	0
0	0	–	0001	0001	001	0
0	1	0	0001	0000	–	0
0	1	1	0001	0010	010	0
...						
0	1	1	0100	1000	–	1
...						

mux is identical to last 3 bits of state
open/closed is identical to first bit of state
therefore, we do not even need to implement
FFs to hold state, just use outputs

Data-path implementation for combination lock

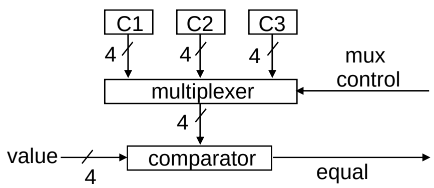
- Multiplexer
  - easy to implement as combinational logic when few inputs
  - logic can easily get too big for most PLDs

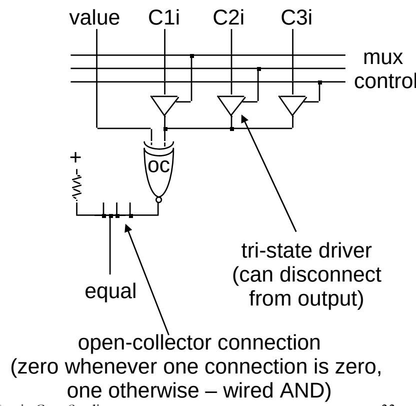




Data-path implementation (cont'd)

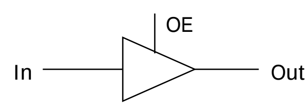
- Tri-state logic
  - utilize a third output state: "no connection" or "float"
  - connect outputs together as long as only one is "enabled"
  - open-collector gates can only output 0, not 1
    - can be used to implement logical AND with only wires





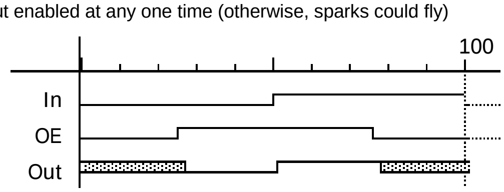
Tri-state gates

- The third value
  - logic values: "0", "1"
  - don't care: "X" (must be 0 or 1 in real circuit!)
  - third value or state: "Z" — high impedance, infinite R, no connection
- Tri-state gates
  - additional input – output enable (OE)
  - output values are 0, 1, and Z
  - when OE is high, the gate functions normally
  - when OE is low, the gate is disconnected from wire at output
  - allows more than one gate to be connected to the same output wire
    - as long as only one has its output enabled at any one time (otherwise, sparks could fly)



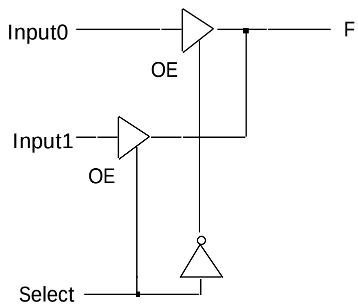
non-inverting tri-state buffer

In	OE	Out
X	0	Z
0	1	0
1	1	1



Tri-state and multiplexing

- When using tri-state logic
  - (1) make sure never more than one "driver" for a wire at any one time (pulling high and low at the same time can severely damage circuits)
  - (2) make sure to only use value on wire when its being driven (using a floating value may cause failures)
- Using tri-state gates to implement an economical multiplexer



when Select is high
Input1 is connected to F

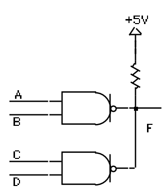
when Select is low
Input0 is connected to F

this is essentially a 2:1 mux

Open-collector gates and wired-AND

- Open collector: another way to connect gate outputs to the same wire
  - gate only has the ability to pull its output low
  - it cannot actively drive the wire high (default – pulled high through resistor)
- Wired-AND can be implemented with open collector logic
  - if A and B are "1", output is actively pulled low
  - if C and D are "1", output is actively pulled low
  - if one gate output is low and the other high, then low wins
  - if both gate outputs are "1", the wire value "floats", pulled high by resistor
    - low to high transition usually slower than it would have been with a gate pulling high
  - hence, the two NAND functions are ANDed together

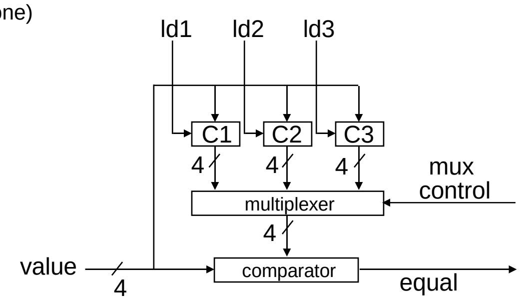
open-collector
NAND gates



with ouputs wired together
using "wired-AND"
to form (AB)'(CD)'

Digital combination lock (new data-path)

- Decrease number of inputs
- Remove 3 code digits as inputs
  - use code registers
  - make them loadable from value
  - need 3 load signal inputs (net gain in input (4*3)–3=9)
    - could be done with 2 signals and decoder (ld1, ld2, ld3, load none)



Sequential logic case studies summary

- FSM design
  - understanding the problem
  - generating state diagram
  - communicating state machines
  - implementation using PLDs
- Four case studies
  - understand I/O behavior
  - draw diagrams
  - enumerate states for the "goal"
  - expand with error conditions
  - reuse states whenever possible