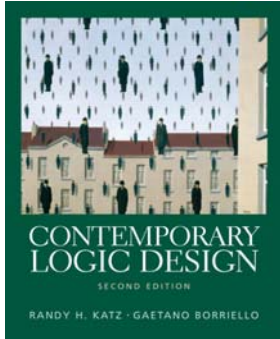


# CSE 370 Spring 2006

## Introduction to Digital Design

### Lecture 6: Karnaugh Maps



#### Last Lecture

- Canonical Forms
- Sum of Products
- Product of Sums
- Boolean Cubes

#### Today

- Karnaugh Maps

## Administrivia

- Turn in Homework #2.
- Homework #3 available this afternoon on website.
- Office Hours: Firat Kiyak, Th 10-12am, in CSE 003
- Lab 3 available on website.
- Reading: Reading: pp. 93-114, 139-145, Verilog Reference (on website, see master calendar)

## QUIZ #1

## Karnaugh Maps

- Flat map of Boolean cube
  - wrap-around at edges
  - hard to draw and visualize for more than 4 dimensions
  - virtually impossible for more than 6 dimensions
- Alternative to truth-tables to help visualize adjacencies
  - guide to applying the uniting theorem
  - on-set elements with only one variable changing value are adjacent unlike the situation in a linear truth-table

	A	0	1	
B				
0		1	2	1
1		0	3	0

A	B	F
0	0	1
0	1	0
1	0	1
1	1	0

# Karnaugh Maps Continued

- Numbering scheme based on Gray-code
  - e.g., 00, 01, 11, 10
  - only a single bit changes in code for adjacent map cells

	AB	00	01	11	10
C	0	0	2	6	4
	1	1	3	7	5

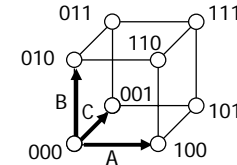
	A	0	4	12	8
	1	5	13	9	
	3	7	15	11	
C	2	6	14	10	

13 = 1101 = ABC'D

# Adjacencies in Karnaugh Maps

- Wrap from first to last column
- Wrap top row to bottom row

	A	000	010	110	100
C	001	011	111	101	



# Karnaugh Map Examples

- $F =$   $AB + ACin + BCin$
  - $Cout =$   $AC + B'C$
  - $f(A,B,C) = \sum m(0,4,5,7)$   $AC + B'C + \cancel{AB}$
- obtain the complement of the function by covering 0s with subcubes

# More Karnaugh Map Examples

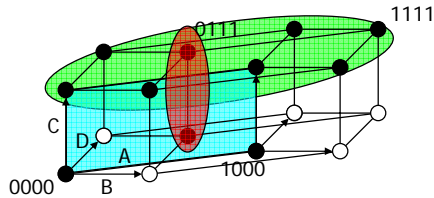
- $G(A,B,C) = A$
  - $F(A,B,C) = \sum m(0,4,5,7) = AC + B'C$
  - $F'(A,B,C) = \sum m(1,2,3,6) = BC' + A'C$
- $F'$  simply replace 1's with 0's and vice versa

# A Four Variable Example

■  $F(A,B,C,D) = \sum m(0,2,3,5,6,7,8,10,11,14,15)$

$F = C + A'BD + B'D'$

	A			
	1	0	0	1
	0	1	0	0
	1	1	1	1
C	1	1	1	1
	1	1	1	1
	B			



find the smallest number of the largest possible subcubes to cover the ON-set (fewer terms with fewer inputs per term)

# Karnaugh Map Don't Cares

■  $f(A,B,C,D) = \sum m(1,3,5,7,9) + d(6,12,13)$

■ without don't cares

■  $f = A'D + B'C'D$

	A			
	0	0	X	0
	1	1	X	1
	1	1	0	0
C	0	X	0	0
	B			

# Karnaugh Map Don't Cares

■  $f(A,B,C,D) = \sum m(1,3,5,7,9) + d(6,12,13)$

■  $f = A'D + B'C'D$

without don't cares

■  $f = A'D + C'D$

with don't cares

	A			
	0	0	X	0
	1	1	X	1
	1	1	0	0
C	0	X	0	0
	B			

by using don't care as a "1" a 2-cube can be formed rather than a 1-cube to cover this node

don't cares can be treated as 1s or 0s depending on which is more advantageous

# Exercise

■ Minimize the function  $F = \sum m(0, 2, 7, 8, 14, 15) + d(3, 6, 9, 12, 13)$