

Lecture 7: Minimization with Karnaugh Maps

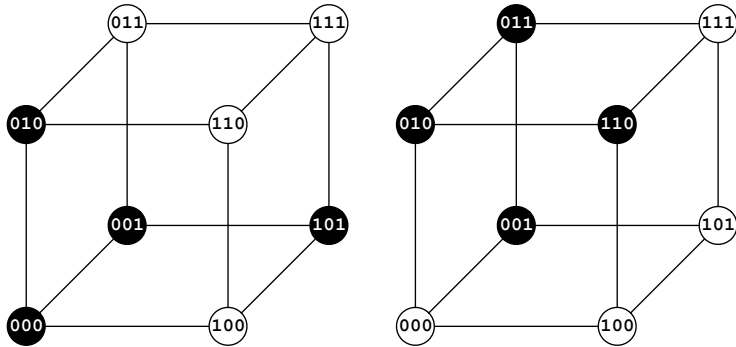
CSE 370, Autumn 2007
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Daily Quiz

- Draw Boolean cubes for these two functions. I recommend putting the input vector $X=0, Y=0, Z=0$ in the lower left corner, making the X axis left-right, the Y axis up-down and the Z axis depth.

$$F(X, Y, Z) = \sum m(0, 1, 2, 5) \quad G(X, Y, Z) = \prod M(0, 4, 5, 7)$$

Daily Quiz Solution



Last Lecture's Daily Quiz

- In product-of-sums form, an input 1 leads to inversion and an input 0 leads to no inversion
- Don't optimize when the exercise tells you not to optimize
- Some students drew circuits that were not sum-of-products or product-of-sums form
- It's a lot faster to draw 2-level circuits without the input inverters

Where We Are

- Last lecture: Boolean cubes and K-maps
- This lecture: Minimization with K-maps
- Next lecture: Combinational Verilog
- Homework 2 due today
- Lab 2 ongoing

Two-Level Simplification

- Key tool: The uniting theorem
 - $A(B + \neg B) = A$
 - $A + (B\neg B) = A$
- We will start with lots of “big” terms, then shrink and eliminate them with the uniting theorem

Implicants

- Any valid rectangle

		A B			
		00	01	11	10
C D	00	1	1	1	1
	01	1	1	0	1
	11	1	0	0	0
	10	1	1	0	1

Prime Implicants

- Implicants that are not "contained" within a larger implicant

		A B			
		00	01	11	10
C D	00	1	1	1	1
	01	1	1	0	1
	11	1	0	0	0
	10	1	1	0	1

Essential Prime Implicants

- Prime implicants that cover individual squares not covered by any other implicant

		A		B	
		C	D	00	01
00	0	1	0	0	
01	0	1	1	1	Not essential
11	1	1	1	0	
10	0	0	1	0	

Interesting Example

- No essential prime implicants
- You choose a “cover”
 - Set of implicants
 - Together include all 1's
 - Usually prime
 - Usually non-redundant

		A		B	
		C	D	00	01
00	0	0	1	1	
01	0	1	1	0	
11	1	1	0	0	
10	1	0	0	1	

Translate Each Implicant

- $A\bar{C}\bar{D}$
- $B\bar{C}D$
- $\bar{A}CD$
- $\bar{B}C\bar{D}$

		A			
		B			
C	D	00	01	11	10
00	0	0	0	1	1
01	0	0	1	1	0
11	1	1	1	0	0
10	1	1	0	0	1

• $F(A,B,C,D) = A\bar{C}\bar{D} + B\bar{C}D + \bar{A}CD + \bar{B}C\bar{D}$

Example with Don't Cares

- BCD increment

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

		A			
		B			
C	D	00	01	11	10
00					
01					
11					
10					

Top Bit

- BCD increment

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

		A		B	
		C	D	00	01
C	00	0	0	X	1
	01	0	0	X	0
	11	0	1	X	X
	10	0	0	X	X

One Way to Cover the 0's

- BCD increment

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

		A		B	
		C	D	00	01
C	00	0	0	X	1
	01	0	0	X	0
	11	0	1	X	X
	10	0	0	X	X

And the Resulting Expression

- BCD increment

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

		A		B			
		C	D	00	01	11	10
C	D	00	0	0	X	1	
		01	0	0	X	0	
	11	0	1	X	X		
	10	0	0	X	X		

$$E = (A+C)(B+\neg D)(\neg C+D)$$

Now the Second Bit

- BCD increment

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

		A		B			
		C	D	00	01	11	10
C	D	00					
		01					
	11						
	10						

Now the Second Bit

- BCD increment

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

		A			
		B			
C	D	00	01	11	10
	00	0	1	X	0
01	0	1	X	0	
11	1	0	X	X	
10	0	1	X	X	

Now the Second Bit

- BCD increment

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

		A			
		B			
C	D	00	01	11	10
	00	0	1	X	0
01	0	1	X	0	
11	1	0	X	X	
10	0	1	X	X	

And the Resulting Expression

- BCD increment

A	B	C	D	E	F	G	H
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	X	X	X	X	X
1	1	X	X	X	X	X	X

		A/B			
		00	01	11	10
C/D	00	0	1	X	0
	01	0	1	X	0
11	11	1	0	X	X
	10	0	1	X	X

$$F = (B+C)(B+D)(\neg B+\neg C+\neg D)$$

5 Variable K-maps

		B/C			
		00	01	11	10
A=0	D/E	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

		B/C			
		00	01	11	10
A=1	D/E	16	20	28	24
	01	17	21	29	25
	11	19	23	31	27
	10	18	22	30	26

6 Variable K-maps

		$\begin{matrix} C \\ \diagdown \\ D \end{matrix}$						$\begin{matrix} C \\ \diagdown \\ D \end{matrix}$			
		$\begin{matrix} E \\ \diagdown \\ F \end{matrix}$		00	01			11	10	$\begin{matrix} E \\ \diagdown \\ F \end{matrix}$	
AB=00	00	0	4	12	8	AB=01	00	16	20	28	24
	01	1	5	13	9		01	17	21	29	25
	11	3	7	15	11		11	19	23	31	27
	10	2	6	14	10		10	18	22	30	26

		$\begin{matrix} C \\ \diagdown \\ D \end{matrix}$						$\begin{matrix} C \\ \diagdown \\ D \end{matrix}$			
		$\begin{matrix} E \\ \diagdown \\ F \end{matrix}$		00	01			11	10	$\begin{matrix} E \\ \diagdown \\ F \end{matrix}$	
AB=10	00	32	36	44	40	AB=11	00	48	52	60	56
	01	33	37	45	41		01	49	53	61	57
	11	35	39	47	43		11	51	55	63	59
	10	34	38	46	42		10	50	54	62	58

K-map Minimization Summary

- Fill out the table with 1's 0's and x's
- Find all the prime implicants
 - Try to "grow" non-prime implicants in each direction
- Select cover
 - All essential prime implicants
 - However many additional implicants are needed

Thank You for Your Attention

- Read lab 2
- Continue homework 2
- Continue reading the book