CSE 370 Lecture 3

Wrapping up 2's Complement. Starting Boolean Algebra.

Lecture 2 Recap:

• Hexadecimal has 16 symbols, Decimal 10, Octal 8, Binary has 2
•Learnt the kung-fu required to switch between bases
•Binary in digital systems:
  •Finite and Fixed word length
  •How do we decide on a word length for our system?
  •Log_2 x = No. of bits we will need
•What about Negative Numbers?
•Sign Magnitude
•One's complement
•2's complement

Some Observations about 2's complement

- Range: - 2^{N-1} to 2^{N-1} -1
- The weird number: most negative number
- Trick: Fast way to do 2's complement

Major topic: Combinational logic

- Axioms and theorems of Boolean algebra
- Logic functions and truth tables
  - AND, OR, Buffer, NAND, NOR, NOT, XOR, XNOR
- Gate logic
  - Networks of Boolean functions
- Canonical forms
  - Sum of products and product of sums
- Simplification
  - Boolean cubes and Karnaugh maps
  - Two-level simplification

Boolean Logic/Algebra

- Notation for writing down precise logical statements (in propositional logic)
- Primitives: true, false, variables
- Connectives: NOT, AND, OR, IMPLIES, …
- (Almost) all memoryless digital circuits can be seen as Boolean algebra expressions
- Understanding Boolean logic helps us design "simpler" circuits, both by hand and automatically
- ((A AND B) OR (NOT A AND B)) AND A
- Equivalent to: A AND B

Boolean algebra

- A Boolean algebra comprises...
  - A set of elements B
  - Binary operators {+ , •}
  - A unary operation { ' }
- ...and the following axioms
  - 1. The set B contains at least two elements {a b} with a ≠ b
  - 2. Closure: a+b is in B   a•b is in B
  - 3. Commutative: a+b = b+a   a•b = b•a
  - 4. Associative: a+(b+c) = (a+b)+c   a•(b•c) = (a•b)•c
  - 5. Identity: a+0 = a   a•1 = a
  - 6. Distributive: a+(b•c)=(a+b)•(a+c)   a•(b+c)=(a•b)+(a•c)
  - 7. Complementarity: a+a' = 1   a•a' = 0

Digital (binary) logic is a Boolean algebra

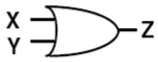
- Substitute
  - {0, 1} for B
  - AND for •   Boolean Product
  - OR for +   Boolean Sum
  - NOT for '
- All the axioms hold for binary logic
- Definitions
  - Boolean function
    - Maps inputs from the set {0,1} to the set {0,1}
  - Boolean expression
    - An algebraic statement of Boolean variables and operators

AND, OR, Not

- AND X•Y XY

X	Y	Z
0	0	
0	1	
1	0	
1	1	

- OR X+Y



X	Y	Z
0	0	
0	1	
1	0	
1	1	

- NOT $\overline{X}$ X'

X	Y
0	
1	

Logic functions and Boolean algebra

- *Any* logic function that is expressible as a truth table can be written in Boolean algebra using +, •, and '

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

Z=

X	Y	X'	Z
0	0	1	0
0	1	1	1
1	0	0	0
1	1	0	0

Z=

X	Y	X'	Y'	X • Y	X' •Y'	Z
0	0	1	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	1	0	1

Z=

Two key concepts

- Duality (a meta-theorem— *a theorem about theorems*)
  - All Boolean expressions have logical duals
  - Any theorem that can be proved is also proved for its dual
  - Replace: • with +, + with •, 0 with 1, and 1 with 0
  - Leave the variables unchanged
- de Morgan's Theorem
  - Procedure for complementing Boolean functions
  - Replace: • with +, + with •, 0 with 1, and 1 with 0
  - Replace all variables with their complements

Useful laws and theorems

Identity:	X + 0 =	Dual: X • 1 =
Null:	X + 1 =	Dual: X • 0 =
Idempotent:	X + X =	Dual: X • X =
Involution:	(X')' =	
Complementarity:	X + X' =	Dual: X • X' =
Commutative:	X + Y =	Dual: X • Y =
Associative:	(X+Y)+Z=	Dual: (X•Y)•Z=
Distributive:	X•(Y+Z)=	Dual: X+(Y•Z)=
Uniting:	X•Y+X•Y'=X	Dual: (X+Y)•(X+Y')=X

Useful laws and theorems (con't)

Absorption: $X+X\bullet Y=X$ Dual: $X\bullet(X+Y)=X$

Absorption (#2): $(X+Y')\bullet Y=X\bullet Y$ Dual: $(X\bullet Y')+Y=X+Y$

de Morgan's: $(X+Y+\ldots)'=X'\bullet Y'\bullet\ldots$ Dual: $(X\bullet Y\bullet\ldots)'=X'+Y'+\ldots$

Duality: $(X+Y+\ldots)^D=X\bullet Y\bullet\ldots$ Dual: $(X\bullet Y\bullet\ldots)^D=X+Y+\ldots$

Multiplying & factoring: $(X+Y)\bullet(X'+Z)=X\bullet Z+X'\bullet Y$
Dual: $X\bullet Y+X'\bullet Z=(X+Z)\bullet(X'+Y)$

Consensus: $(X\bullet Y)+(Y\bullet Z)+(X'\bullet Z)= X\bullet Y+X'\bullet Z$
Dual: $(X+Y)\bullet(Y+Z)\bullet(X'+Z)=(X+Y)\bullet(X'+Z)$

Proving theorems

- Example 1: Prove the uniting theorem-- $X\bullet Y+X\bullet Y'=X$

- Example 2: Prove the absorption theorem-- $X+X\bullet Y=X$

Logic simplification

- Use the axioms to simplify logical expressions
  - Why? To use less hardware
- Example: A two-level logic expression

$$Z = A'BC + AB'C' + AB'C + ABC' + ABC$$