

## Lecture 9

---

### ◆ Logistics

- HW2 due now
- HW3 due Monday
- Lab --- stay on track
- Tuesday review 6pm(ish) place TBD
- Nikhil's office hour location moved: CSE 218 same time (M1:30)

### ◆ Last lecture

- "Switching-network" logic blocks
  - ▣ Multiplexers and Demultiplexers

### ◆ Today

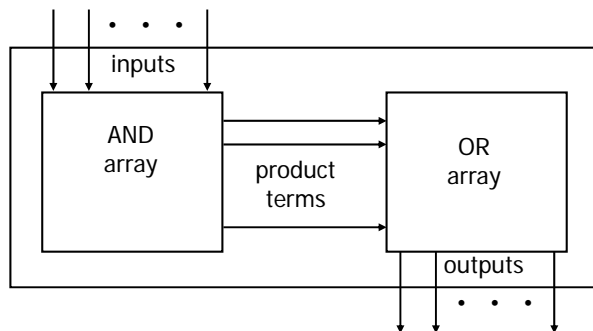
- PLDs
  - ▣ PLAs
  - ▣ PALS
- ROMs

## Programmable logic (PLAs & PALs )

---

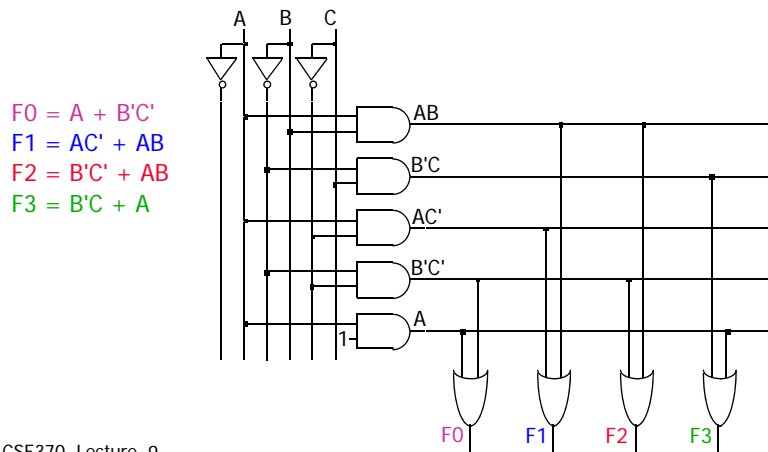
### ◆ Concept: Large array of uncommitted AND/OR gates

- Actually NAND/NOR gates
- You program the array by making or breaking connections
  - ▣ Programmable block for sum-of-products logic



## Programming the wire connections

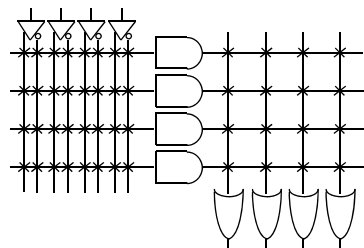
- Fuse: Comes connected; break unwanted connections
- Anti-fuse: Comes disconnected; make wanted connections



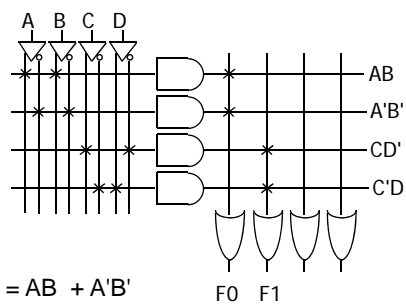
## Short-hand notation

- Draw multiple wires as a single wire or bus
- × signifies a connection

Before Programming



After Programming

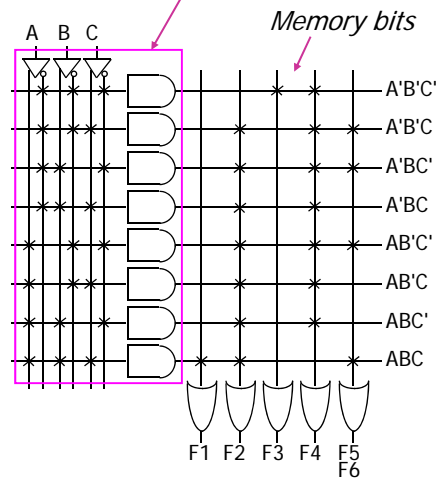


## PLA example

$$\begin{aligned}
 F1 &= ABC \\
 F2 &= A + B + C \\
 F3 &= A' B' C' \\
 F4 &= A' + B' + C' \\
 F5 &= A \text{ xor } B \text{ xor } C \\
 F6 &= A \text{ xnor } B \text{ xnor } C
 \end{aligned}$$

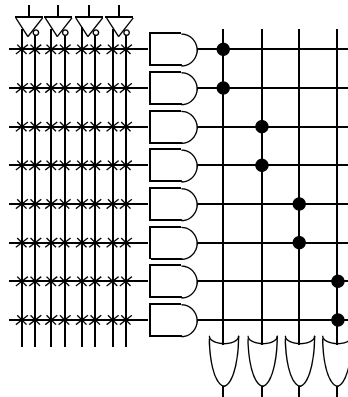
A	B	C	F1	F2	F3	F4	F5	F6
0	0	0	0	0	1	1	0	0
0	0	1	0	1	0	1	1	1
0	1	0	0	1	0	1	1	1
0	1	1	0	1	0	1	0	0
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	0
1	1	0	0	1	0	1	0	0
1	1	1	1	1	0	0	1	1

Think of as a memory-address decoder



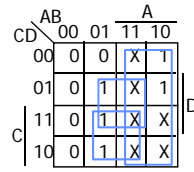
## PLAs versus PALs

- ◆ We've been looking at PLAs
  - Fully programmable AND / OR arrays
- ◆ Programmable array logic (PAL)
  - Programmable AND array
  - OR array is prewired
    - ↳ Cheaper and faster than PLAs

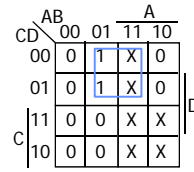


## Example: BCD to Gray code converter

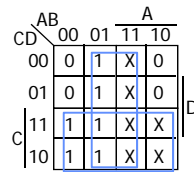
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	1	1	1	0
0	1	1	0	1	0	1	0
0	1	1	1	1	0	1	1
1	0	0	0	1	0	0	1
1	0	0	1	1	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X



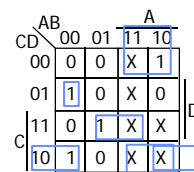
K-map for W



K-map for X



K-map for Y

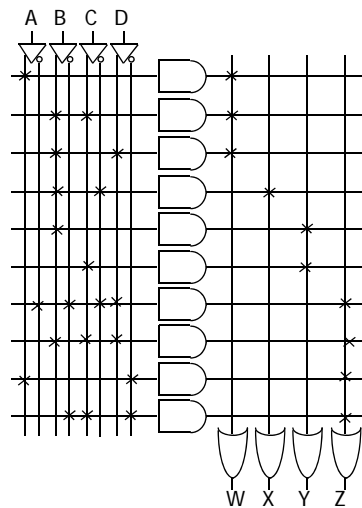


K-map for Z

## Example: BCD to Gray --- Wire a PLA

Minimized functions:

$$\begin{aligned}
 W &= A + BC + BD \\
 X &= BC' \\
 Y &= B + C \\
 Z &= A'B'C'D + BCD \\
 &\quad + AD' + B'CD'
 \end{aligned}$$



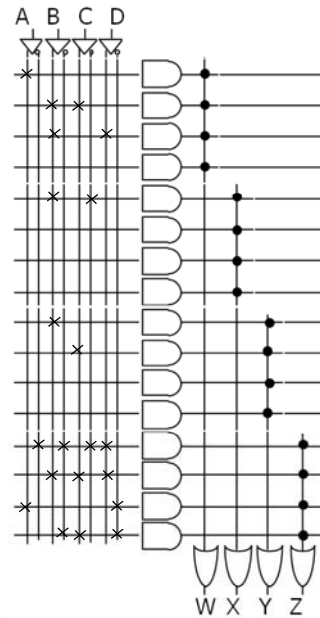
## Example: Wire a PAL

Minimized functions:

$$\begin{aligned}W &= A + BC + BD \\X &= BC' \\Y &= B + C \\Z &= A'B'C'D + BCD \\&\quad + AD' + B'CD'\end{aligned}$$

Fine example for the use of PAL  
(because no shared AND terms)

Many AND gates wasted, but  
still faster and cheaper than PLA

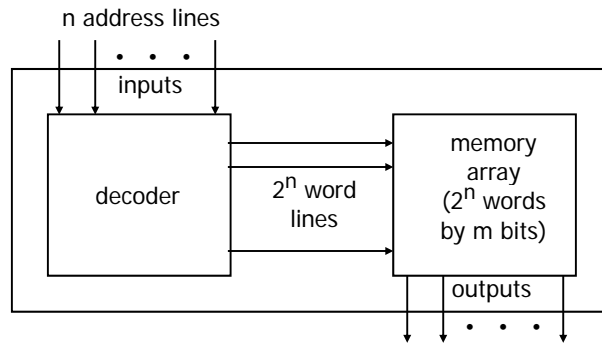


## Compare implementations for this example

- ◆ PLA:
  - No shared logic terms in this example
  - 10 decoded functions (10 AND gates)
- ◆ PAL:
  - Z requires 4 product terms
    - ↳ 16 decoded functions (16 AND gates)
    - ↳ 6 unused AND gates
- ◆ This decoder is a best candidate for PLAs/PALs
  - 10 of 16 possible inputs are decoded
  - No sharing among AND terms
- ◆ Another option?
  - Yes — a ROM

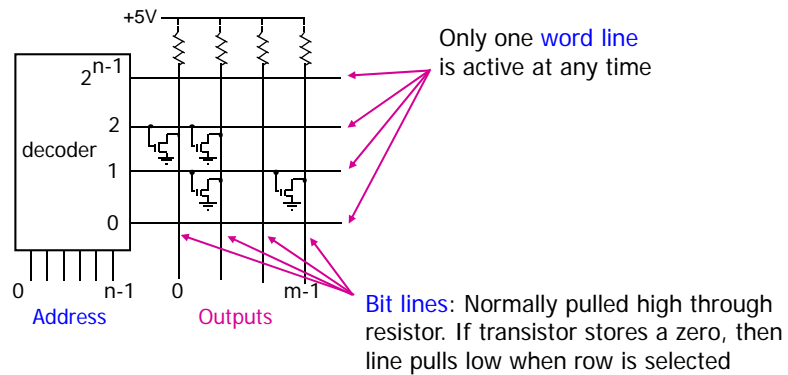
## Read-only memories (ROMs)

- ◆ Two dimensional array of stored 1s and 0s
  - Input is an address  $\Rightarrow$  ROM decodes all possible input addresses
  - Stored row entry is called a "word"
  - ROM output is the decoded word



## ROM details

- Similar to a PLA but with a fully decoded AND array
- Completely flexible OR array (unlike a PAL)
- Extremely dense: One transistor per stored bit



## Two-level combinational logic using a ROM

- ◆ Use a ROM to directly store a truth table

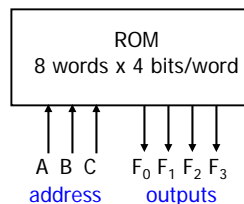
- No need to minimize logic
  - Example:
 
$$F_0 = A'B'C + AB'C' + AB'C$$

$$F_1 = A'B'C + A'BC' + ABC$$

$$F_2 = A'B'C' + A'B'C + AB'C'$$

$$F_3 = A'BC + AB'C' + ABC'$$

A	B	C	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
0	0	0	0	0	1	0
0	0	1	1	1	1	0
0	1	0	0	1	0	0
0	1	1	0	0	0	1
1	0	0	1	0	1	1
1	0	1	1	0	0	0
1	1	0	0	0	0	1
1	1	1	0	1	0	0



You specify whether to store 1 or 0 in each location in the ROM

## ROMs versus PLAs/PALs

- ◆ ROMs

- Benefits
    - ☛ Quick to design, simple, dense
  - Limitations
    - ☛ Size doubles for each additional input
    - ☛ Can't exploit don't cares

- ◆ PLAs/PALs

- Benefits
    - ☛ Logic minimization reduces size
    - ☛ PALs faster/cheaper than PLAs
  - Limitations
    - ☛ PAL OR-plane has hard-wired fan-in

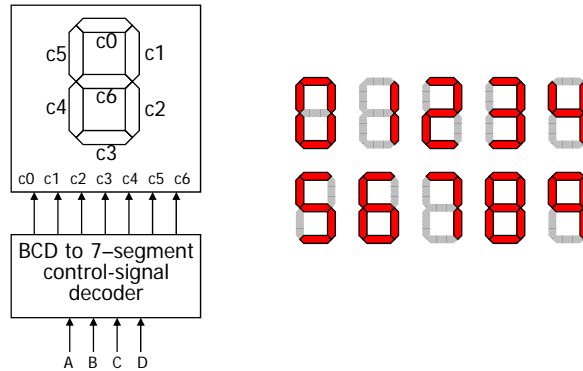
- ◆ Another alternative: Field programmable gate arrays

- Learn a bit more later in this class

## Example: BCD to 7-segment display controller

### ◆ The problem

- Input is a 4-bit BCD digit (A, B, C, D)
- Need signals to drive a display (7 outputs C0 – C6)



## Formalize the problem

### ◆ Truth table

- Many don't cares

### ◆ Choose implementation target

- If ROM, we are done
- Don't cares imply PAL/PLA may be good choice

### ◆ Implement design

- Minimize the logic
- Map into PAL/PLA

A	B	C	D	C0	C1	C2	C3	C4	C5	C6
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	0	0	1	1
1	0	1	X	X	X	X	X	X	X	X
1	1	X	X	X	X	X	X	X	X	X



## Sum-of-products implementation

- ◆ 15 unique product terms if we minimize individually

$C0 = A + B D + C + B' D'$   
 $C1 = C' D' + C D + B'$   
 $C2 = B + C' + D$   
 $C3 = B' D' + C D' + B C' D + B' C$   
 $C4 = B' D' + C D'$   
 $C5 = A + C' D' + B D' + B C'$   
 $C6 = A + C D' + B C' + B' C$

4 input, 7 output

CSE370, Lecture 9

PLA: 15 AND gates

PAL: 4 product terms per output (28 AND gates)

17

## If choosing PLA: better SOP implementation

- ◆ Can do better than 15 product terms
  - Share terms among outputs  $\Rightarrow$  only 9 unique product terms
  - ▣ Each term not necessarily minimized

$C0 = A + B D + C + B' D'$   
 $C1 = C' D' + C D + B'$   
 $C2 = B + C' + D$   
 $C3 = B' D' + C D' + B C' D + B' C$   
 $C4 = B' D' + C D'$   
 $C5 = A + C' D' + B D' + B C'$   
 $C6 = A + C D' + B C' + B' C$

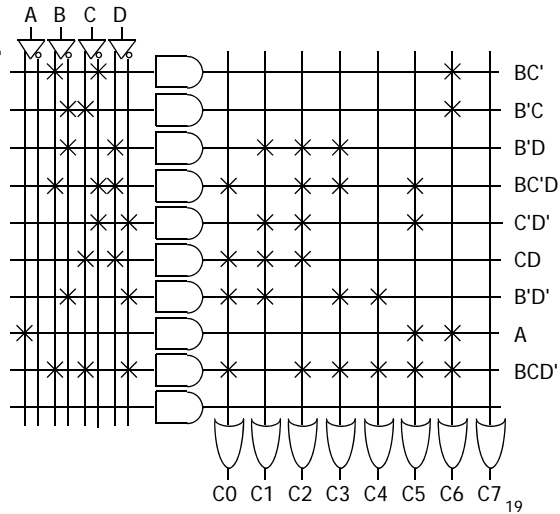
$C0 = B C' D + C D + B' D' + B C D' + A$   
 $C1 = B' D + C' D' + C D + B' D'$   
 $C2 = B' D + B C' D + C' D' + C D + B C D'$   
 $C3 = B C' D + B' D + B' D' + B C D'$   
 $C4 = B' D' + B C D'$   
 $C5 = B C' D + C' D' + A + B C D'$   
 $C6 = B' C + B C' + B C D' + A$

CSE370, Lecture 9

18

## PLA implementation

$$\begin{aligned}
 C0 &= BC'D + CD + B'D' + BCD' + A \\
 C1 &= B'D + C'D' + CD + B'D' \\
 C2 &= B'D + BC'D + C'D' + CD + BCD' \\
 C3 &= BC'D + B'D + B'D' + BCD' \\
 C4 &= B'D' + BCD' \\
 C5 &= BC'D + C'D' + A + BCD' \\
 C6 &= B'C + BC' + BCD' + A
 \end{aligned}$$



CSE370, Lecture 9

## Example: Logical function unit

- ◆ Multipurpose functional block
  - 3 control inputs (**C**) specify function
  - 2 data inputs (operands) **A** and **B**
  - 1 output (same bit-width as input operands)

C0	C1	C2	Function	Comments
0	0	0	1	always 1
0	0	1	$A + B$	logical OR
0	1	0	$(A \cdot B)'$	logical NAND
0	1	1	$A \text{ xor } B$	logical xor
1	0	0	$A \text{ xnor } B$	logical xnor
1	0	1	$A \cdot B$	logical AND
1	1	0	$(A + B)'$	logical NOR
1	1	1	0	always 0

3 control inputs: C0, C1, C2  
 2 data inputs: A, B  
 1 output: F

CSE370, Lecture 9

20

## Formalize the problem and solve

C0	C1	C2	A	B	F
0	0	0	0	0	1
0	0	0	0	1	1
0	0	0	1	0	1
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	0	1	1
0	0	1	1	0	1
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	0	1	1
0	1	0	1	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	0	1	1
0	1	1	1	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	0	1	1	1
1	0	1	0	0	0
1	0	1	0	1	0
1	0	1	1	0	0
1	0	1	1	1	1
1	1	0	0	0	1
1	1	0	0	1	0
1	1	0	1	0	0
1	1	0	1	1	0
1	1	1	0	0	0
1	1	1	0	1	0
1	1	1	1	0	0
1	1	1	1	1	0

Implementation choice:  
multiplexer with discrete gates

