CSE 373 Lecture 18: The Fastest Sorting Algorithm

- Today's topic: Quicksort – fastest known sorting algorithm in practice
  - Algorithm description
  - Example
  - Partitioning in place during Quicksort
  - Performance analysis

- Covered in Chapter 7 of the textbook

Quicksort Description

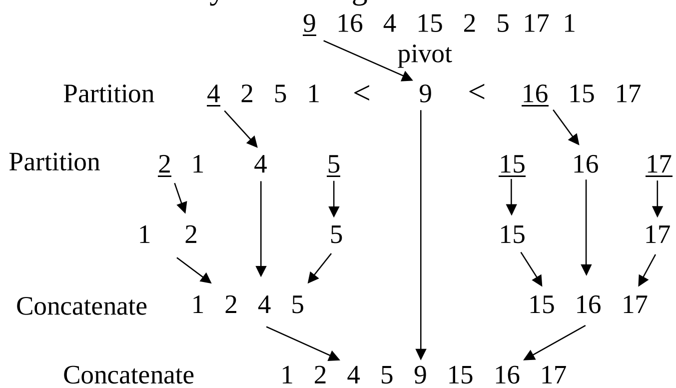
- Quicksort Algorithm:
  1. Partition array into left and right sub-arrays such that:
     - Elements in left sub-array < elements in right sub-array
  2. Recursively sort left and right sub-arrays
  3. Concatenate left and right sub-arrays with pivot in middle

- How to Partition the Array:
  1. Choose an element from the array as the pivot
  2. Move all elements < pivot into left sub-array and all elements > pivot into right sub-array

- Pivot? One choice → use first element in array

Quicksort Example

- Sort the array containing:



Partitioning In Place

- Hmmm…seems like we need an *extra array* for partitioning and concatenating left/right sub-arrays
  - No!
- Algorithm for In Place Partitioning:
  1. Swap pivot with last element → swap pivot and A[N-1]
  2. Set pointers i and j to beginning and end of array
  3. Increment i until you hit an element A[i] > pivot
  4. Decrement j until you hit an element A[j] < pivot
  5. Swap A[i] and A[j]
  6. Repeat until i and j cross (i exceeds or equals j)
  7. Swap pivot (= A[N-1]) with A[i]
- On-Board Example: Partition in place:
  - 9 16 4 15 2 5 17 1 (pivot = A[0] = 9)

Choosing the Pivot

- First Idea: Pick the *first* element in (sub-)array as pivot
  - What if it is the smallest or largest?
  - What if the array is sorted? How many recursive calls does quicksort make?
- 2nd Idea: Pick a *random* element
  - Gets rid of asymmetry in left/right sizes
  - But…requires calls to pseudo-random number generator – expensive/error-prone
- Third idea: Pick *median* (N/2th largest element)
  - Hard to compute without sorting!
  - Compromise: Pick median of three elements

$\underline{9}$ 16 4 15 2

$\underline{2}$ 16 4 15 9

$\underline{2}$ 4 9 15 16

Median-of-Three Pivot

- Find the median of the first, middle and last element

2 4 9 15 16 → 9

5 4 2 15 16 → 5

- Takes only O(1) time and not error-prone like the pseudo-random pivot choice
- Less chance of poor performance as compared to looking at only 1 element
- For sorted inputs, splits array nicely in half each recursion
  - Good performance

Quicksort Performance Analysis

- Best Case Performance: Algorithm always chooses best pivot and keeps splitting sub-arrays in half at each recursion
  - T(0) = T(1) = O(1) (constant time if 0 or 1 element)
  - For N > 1, 2 recursive calls plus linear time for partitioning
  - T(N) = 2T(N/2) + O(N) (Same recurrence relation as Mergesort)
  - T(N) = ?

Quicksort Performance Analysis

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  - T(N) = O(N log N)
- Worst Case Performance: What is the worst case?

Quicksort Performance Analysis

- <u>Best Case Performance</u>: Algorithm always chooses best pivot and keeps splitting sub-arrays in half at each recursion
  - $T(0) = T(1) = O(1)$ and $T(N) = 2T(N/2) + O(N)$
  - $T(N) = \underline{O(N \log N)}$
- <u>Worst Case Performance</u>: Algorithm keeps picking the worst pivot – one sub-array empty at each recursion
  - $T(0) = T(1) = O(1)$
  - $T(N) = T(N-1) + O(N)$
  - $T(N) = ?$

Quicksort Performance Analysis

- <u>Best Case Performance</u>: Algorithm always chooses best pivot and keeps splitting sub-arrays in half at each recursion
  - $T(0) = T(1) = O(1)$ and $T(N) = 2T(N/2) + O(N)$
  - $T(N) = \underline{O(N \log N)}$
- <u>Worst Case Performance</u>: Algorithm keeps picking the worst pivot – one sub-array empty at each recursion
  - $T(0) = T(1) = O(1)$
  - $T(N) = T(N-1) + O(N)$
  - $\quad = T(N-2) + O(N-1) + O(N) = \ldots = T(0) + O(1) + \ldots + O(N)$
  - $T(N) = \underline{O(N^2)}$
- Fortunately, *average case performance* is <u>O(N log N)</u> (see text for proof)

Can We Sort Any Faster?

- Heapsort, Mergesort, and Quicksort all run in O(N log N) best case running time
- Can we do any better?
- Can Joe Smartypants from Softwareville, USA come up with an O(N log log N) sorting algorithm?

Answer in next class…

To do:

Finish reading chapter 7

Start reading chapter 8