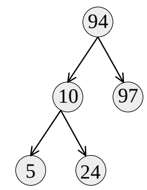
CSE 373 Lecture 7: More on Search Trees

- Today's Topics:
  - Array Implementation of Trees
  - Lazy Deletion
  - Run Time Analysis of Binary Search Tree Operations
  - AVL Trees
  - Splay Trees
- Covered in Chapter 4 of the text

Array Implementation of Trees

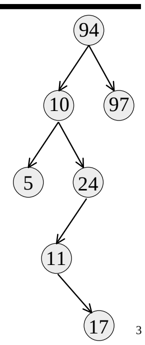
- Used mostly for complete binary trees
  - A complete tree has no gaps when you scan the nodes left-to-right, top-to-bottom
- Idea: Use left-to-right scan to impose a linear order on the tree nodes
- Implementation:
  - Children of A[i] = A[2i+1], A[2i+2]
  - Use a default value to indicate empty node
  - Exercise: Draw array for the tree shown
- Why is this implementation inefficient for non-complete trees?



Pointer Implementation: Delete Operation

- Problem: When you delete a node, what do you replace it by?
- Solution:
  1. If it has no children, by NULL
  2. If it has 1 child, by that child
  3. If it has 2 children, by the node with the smallest value in its right subtree, (or largest value in left subtree)

  Recursively delete node being used in 2 and 3
- Worst case: Recursion propagates all the way to a leaf node – time is O(depth of tree)



Lazy Deletion

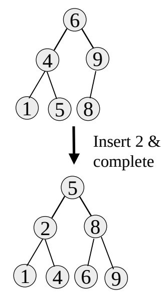
- A "lazy" operation is one that puts off work as much as possible in the hope that a future operation will make the current operation unnecessary
- Idea: Mark node as deleted; no need to reorganize tree
  - Skip marked nodes during Find or Insert
  - Reorganize tree only when number of marked nodes exceeds a percentage of real nodes (e.g. 50%)
  - Constant time penalty due to marked nodes – depth increases only by a constant amount if 50% are marked undeleted nodes
- Modify Insert to make use of marked nodes whenever possible e.g. when deleted value is re-inserted
- Can also use lazy deletion for Lists

Run Time Analysis of Binary Search Trees

- All BST operations (except MakeEmpty) are O(d), where d is tree depth
  - MakeEmpty takes O(N) for a tree with N nodes – frees all nodes
- From last time, we know: $\log N \le d \le N-1$ for a binary tree with N nodes
  - What is the best case tree? What is the worst case tree?
- So, best case running time of BST operations is O(log N)
  - In fact, average case is also O(log N) – see text
- Worst case running time is O(N)
  - E.g. What happens when you Insert elements in ascending order?
    - Insert: 2, 4, 6, 8, 10, 12 into an empty BST
  - Problem: Lack of "balance": compare depths of left and right subtree

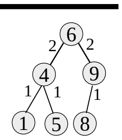
Balancing Trees

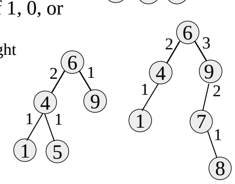
- Many algorithms exist for keeping trees balanced
  - Adelson-Velskii and Landis (AVL) trees (1962)
  - Splay trees and other self-adjusting trees (1978)
  - B-trees and other multiway search trees (1972)
- First try at balancing trees: Perfect balance
  - Want a complete tree after every operation
  - Too expensive E.g. Insert 2
  - Need a looser constraint…



AVL Trees

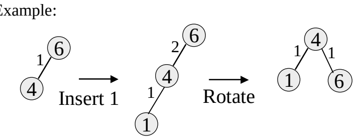
- AVL trees are height-balanced binary search trees
- Balance factor of a node = height(left subtree) - height(right subtree)
- An AVL tree has balance factor of 1, 0, or –1 at every node
  - For every node, heights of left and right subtree differ by no more than 1
  - Store current heights in each node
- Can prove: Height is O(log N)
  All operations (e.g. Find) are O(log N) except Insert (assume lazy deletion)



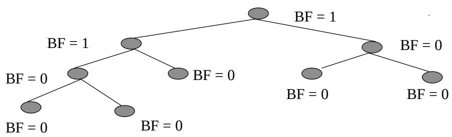


Insert and Rotation in AVL Trees

- Insert operation may cause balance factor to become 2 or –2 for some node on the path from insertion point to root node
  - After Insert, back up to root updating heights
  - If difference = 2 or –2, adjust tree by rotation around deepest such node
  - Example:

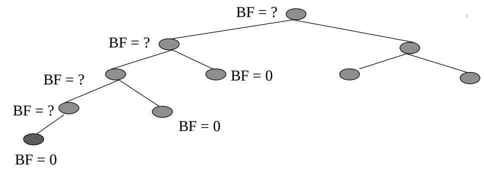


Insertion: Another Example



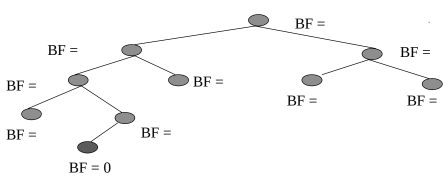
Tree before insertion (BF = Balance Factor)

Insertion: Example 1 (Outside case)



Tree after insertion

Insertion: Example 2 (Inside case)



Tree after insertion

Insertions in AVL Trees

Let the node that needs rebalancing be α.

There are 4 cases:

Outside Cases (require single rotation) :

1. Insertion into left subtree of left child of α.
2. Insertion into right subtree of right child of α.

Inside Cases (require double rotation) :

3. Insertion into right subtree of left child of α.
4. Insertion into left subtree of right child of α.

The rebalancing is performed through four separate rotation algorithms – on board examples. See text for details.