Hashing

CSE 373 - Data Structures

April 22, 2002

Readings and References

- Reading
  - › Chapter 5, *Data Structures and Algorithm Analysis in C*, Weiss
- Other References
  - › Hashing, *Introduction to Algorithms*, Cormen, Leiserson and Rivest

The need for speed

- Data structures we have looked at so far
  - › Use comparison operations to find items
  - › Need O(N) or O(log N) time for Find and Insert
- In real world applications, N is typically between 100 and 100,000 (or more)
  - › log N is between 6.6 and 16.6
- Hash tables are an abstract data type designed for O(1) Find and Inserts

Fewer functions faster

- compare lists and stacks
  - by reducing the flexibility of what we are allowed to do, we can increase the performance of the remaining operations
  - insert(L,X) into a list versus push(S,X) onto a stack
- compare trees and hash tables
  - trees provide for known ordering of all elements
  - hash tables just let you (quickly) find an element

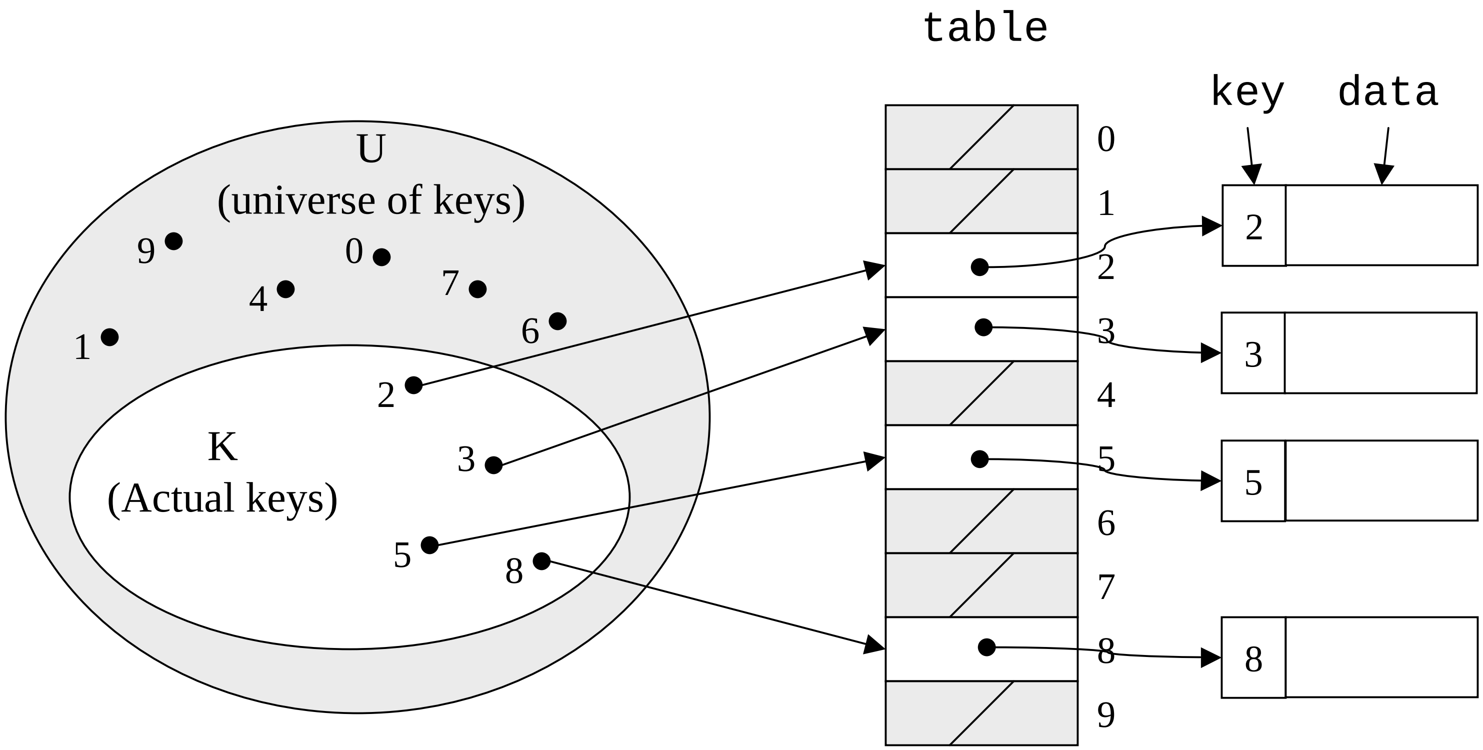
Limited Set of Hash Operations

- For many applications, a limited set of operations is all that is needed
  - Insert, Find, and Delete
  - Note that no ordering of elements is implied
- For example, a compiler needs to maintain information about the symbols in a program
  - user defined
  - language keywords

Direct Address Tables

- Direct addressing using an array is very fast
- Assume
  - › keys are integers in the set U={0,1,…*m*-1}
  - › *m* is small
  - › no two elements have the same key
- Then just store each element at the array location array[key]
  - › search, insert, and delete are trivial

Direct Access Table



[Cormen, et al]

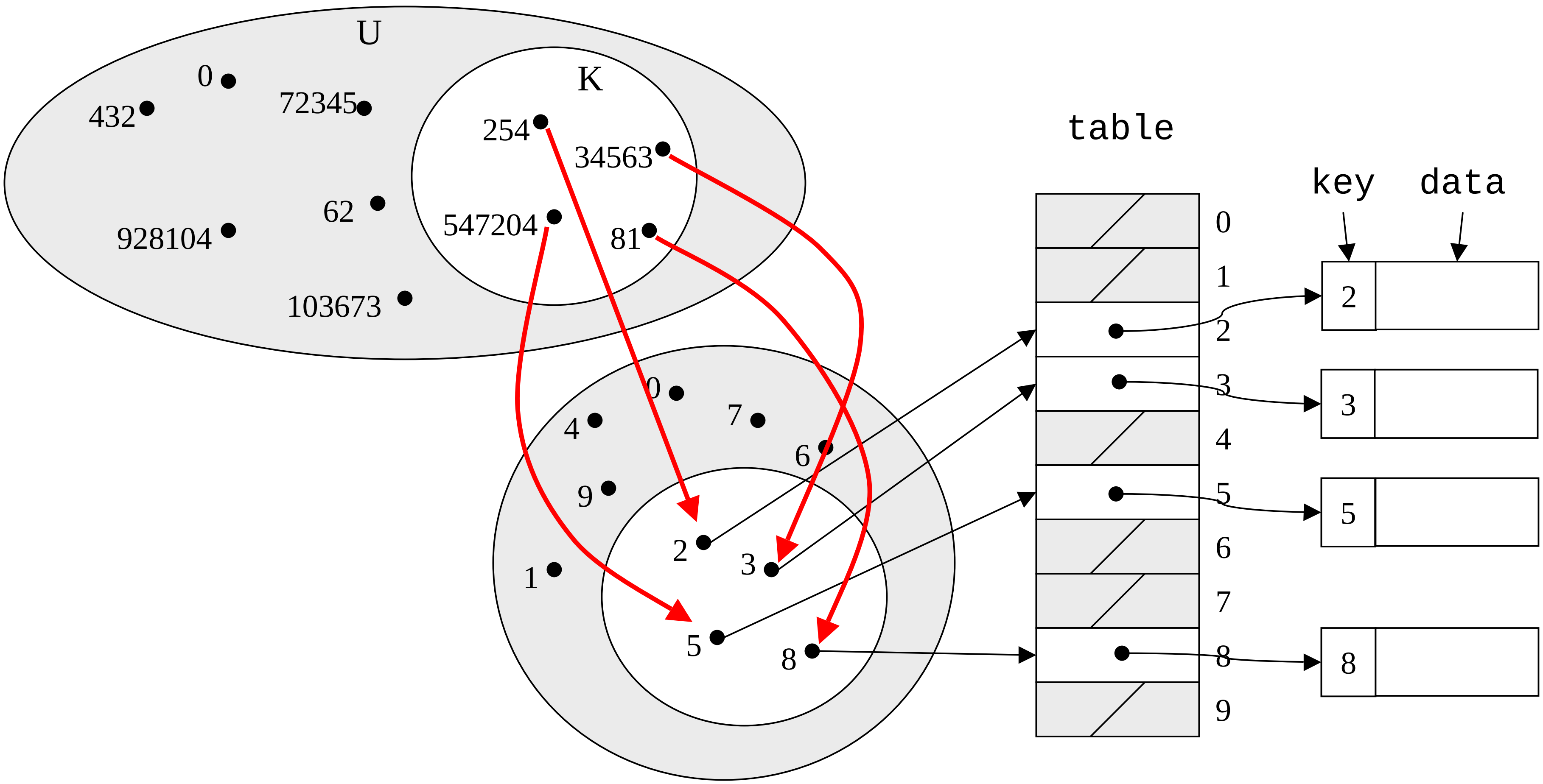
Direct Address Implementation

```
Delete(Table t, ElementType x)
  T[key[x]] = NULL

Insert(Table t, ElementType x)
  T[key[x]] = x

Find(Table t, Key k)
  return T[k]
```

An Issue

- The largest possible key in U may be much larger than the number of elements actually stored (|U| much greater than |K|)
  - › the table is very sparse and wastes space
  - › in worst case, table too large to have in memory
- If most keys in U are used
  - › direct addressing can work very well
- If most keys in U are not used
  - › need to map U to a smaller set closer in size to K

Mapping the Keys



Hashing schemes

- We want to store N items in a table of size M, at a location computed from the key K
- Hash function
  - Method for computing table index from key
- Collision resolution strategy
  - How to handle two keys that hash to the same index

Looking for an element

- Data records can be stored in arrays.
  - › A[0] = {“CHEM 110”, Size 89}
  - › A[3] = {“CSE 142”, Size 251}
  - › A[17] = {“CSE 373”, Size 85}
- Class size for CSE 373?
  - › Linear search the array – O(N) worst case time
  - › Binary search - O(log N) worst case

Go directly to the element

- What if we could directly index into the array using the key?
  - › A[“CSE 373”] = {Size 85}
- Main idea behind hash tables
  - › Use a key based on some aspect of the data element to index directly into an array
  - › O(1) time to access records

Indexing into hash table

- Need a fast *hash function* to convert the element key (string or number) to an integer (the *hash value*) (ie, map from U to index)
  - › Then use this value to index into an array
  - › Hash(“CSE 373”) = 157, Hash(“CSE 143”) = 101
- Output of the hash function
  - › must always be less than size of array
  - › must be as evenly distributed as possible

Choosing the hash function

- What properties do we want from a hash function?
  - › Want universe of hash values to be distributed randomly to minimize collisions
  - › Don’t want systematic nonrandom pattern in selection of keys to lead to systematic collisions
  - › Want hash value to depend on all values in entire key and their positions

The key values are important

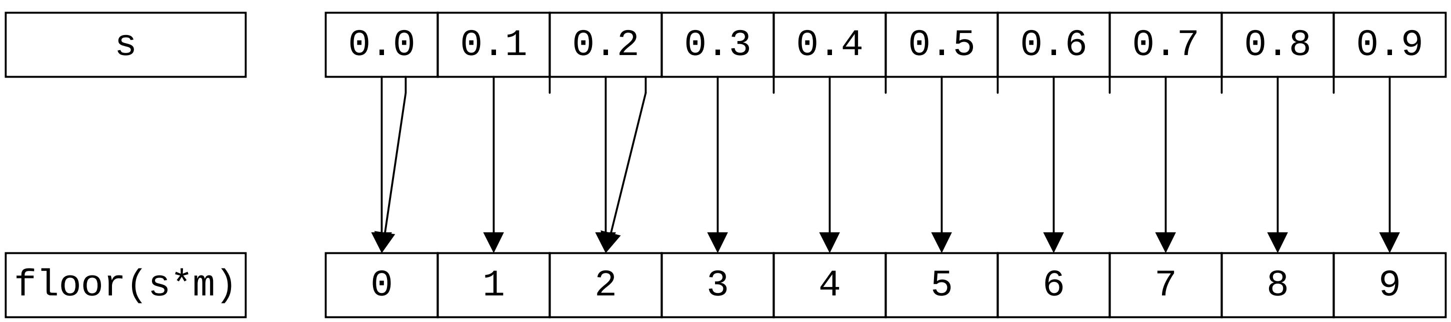
- Notice that one key issue with all the hash functions is that the actual content of the key set matters
- The elements in K (the keys that are used) are quite possibly a restricted subset of U, not just a random collection
  - › variable names, words in the English language, reserved keywords, telephone numbers, etc, etc

Simple hashes

- It's possible to have very simple hash functions if you are certain of your keys
- For example,
  - › suppose we know that the keys *s* will be real numbers uniformly distributed over $0 \le s < 1$
  - › Then a very fast, very good hash function is
    - hash(s) = floor($s \cdot m$)
    - where *m* is the size of the table

very simple mapping

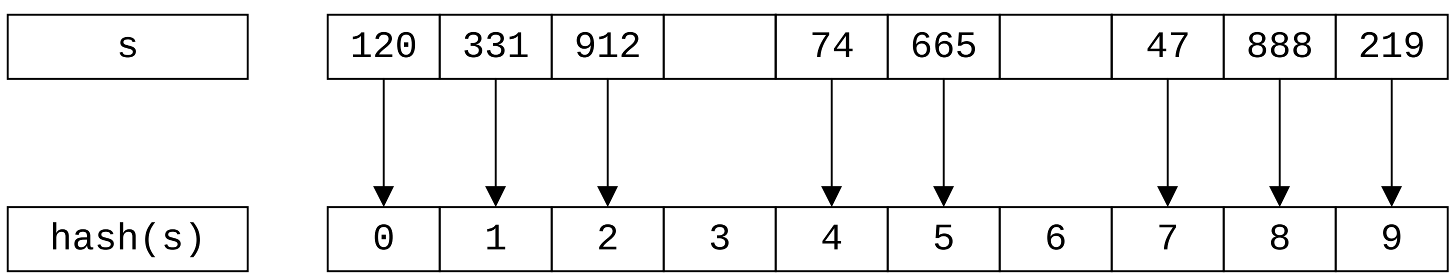
- hash(s) = floor($s \cdot m$) maps from $0 \le s < 1$ to 0..m-1
  - › m = 10



Note the even distribution. There are collisions, but we will deal with them later.

Perfect hashing

- In some cases it's possible to map a known set of keys uniquely to a set of index values
- You must know every single key beforehand and be able to derive a function that works *one-to-one* (not necessarily *onto*)

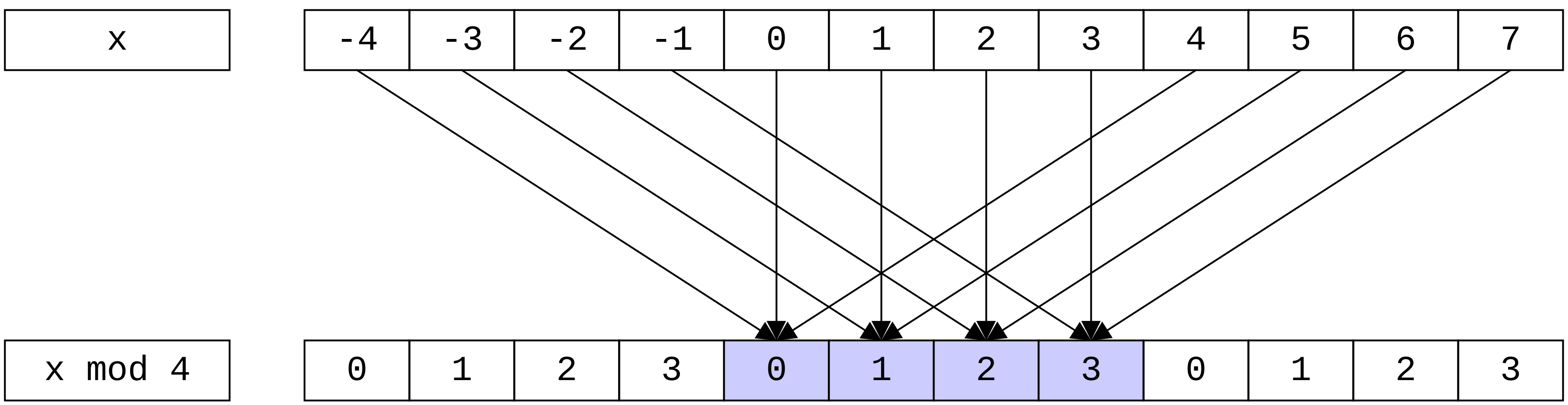


integer key *modulo* table size

- One solution for a less constrained key set
  - › modular arithmetic
- `a` **`mod`** `size`
  - › remainder when "a" is divided by "size"
  - › in C this is written as **`r = a % size;`**
  - › If TableSize = 251
    - 408 mod 251 = 157
    - 352 mod 251 = 101

modulo mapping

- *a* mod *m* maps from integers to 0..m-1
  - one to one? no
  - onto? yes



Hash function : mod

- If keys are integers, we can use the hash function:
  - › Hash(*key*) = *key* mod *TableSize*
- Problem 1: What if *TableSize* is 11 and all keys are 2 repeated digits? (eg, 22, 33, …)
  - › all keys map to the same index
  - › Need to pick *TableSize* carefully: often, a prime number

Keys as Natural Numbers

- Most hash functions assume that the universe of keys is the natural numbers **N**={0,1,…}
- Need to find a function to convert the actual key to a natural number quickly and effectively before or during the hash calculation
- Generally work with the ASCII character codes when converting strings to numbers

Hash Function : add chars

- If keys are strings can get an integer by adding up ASCII values of characters in *key*

```
hashValue = 0;
while (*key != '\0')
  hashValue += *key++;
```

character	C	S	E		3	7	3	<0>
ASCII value	67	83	69	32	51	55	51	0

- We are converting a very large number ($c_0c_1c_2c_3c_4$) to a relatively small number ($c_0+c_1+c_2+c_3+c_4$)

Hash must cover the whole table

- Problem 2: What if *TableSize* is 10,000 and all keys are 8 or less characters long?
  - › chars have values between 0 and 127
  - › Keys will hash only to positions 0 through 8*127 = 1016
- Need to distribute keys over the entire table or the extra space is wasted

Issues with hash add char

- Problems with adding up character values for string keys
  - › If string keys are short, will not hash evenly to all of the hash table
  - › Different character combinations hash to same value
    - "abc", "bca", and "cab" all add up to the same value

Hash function : chars as digits

- Suppose keys can use any of 26 characters plus blank (27 characters numbered 0 to 26)
  - › these are digits in a base 27 representation of a number
  - › can use 32 instead of 27 and shift left by 5 bits for fast multiplication, ie, consider the number to be a base 32 value
- A key conversion function for short strings
  - › “abc” = $1*32^2 + 2*32^1 + 3$ =1091
  - › “bca” = $2*32^2 + 3*32^1 + 1$ =2243
  - › “cab” = $3*32^2 + 1*32^1 + 2$ =6342

Collisions

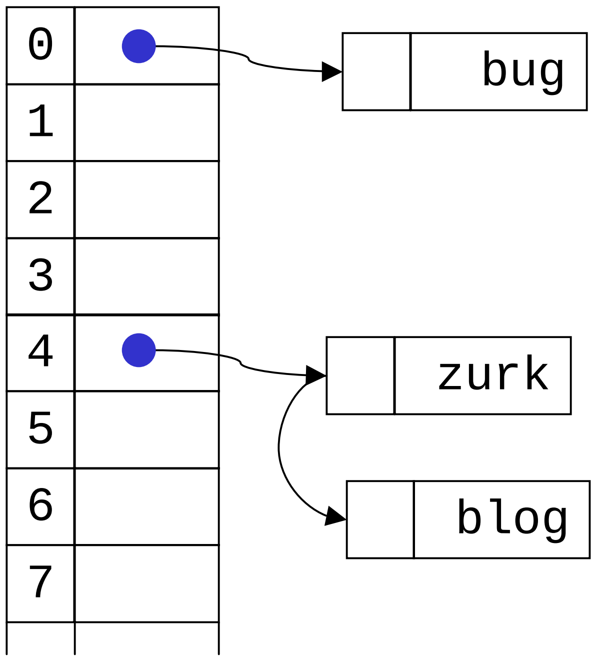
- A collision occurs when two different keys hash to the same value
  - E.g. For *TableSize* = 17, the keys 18 and 35 hash to the same value
  - 18 mod 17 = 1 and 35 mod 17 = 1
- Cannot store both data records in the same slot in array!

Collision Resolution

- Separate Chaining
  - › Use data structure (such as a linked list) to store multiple items that hash to the same slot
- Open addressing (or probing)
  - › search for empty slots using a second function and store item in first empty slot that is found

Resolution by Separate Chaining

- Each hash table cell holds pointer to linked list of records with same hash value (i, j, k in figure)
- Collision: Insert item into linked list
- To Find an item: compute hash value, then do Find on linked list
- Note that there are potentially as many as *TableSize* lists



Why lists?

- Can use List ADT for Find/Insert/Delete in linked list
  - › O(N) runtime where N is the number of elements in the particular chain
- Can also use Binary Search Trees
  - › O(log N) time instead of O(N)
  - › But the number of elements to search through should be small
  - › generally not worth the overhead of BSTs

Load Factor of a Hash Table

- Let N = number of items to be stored
- **Load factor λ = N/*TableSize***
  - › *TableSize* = 101 and N =505, then $\lambda = 5$
  - › *TableSize* = 101 and N = 10, then $\lambda = 0.1$
- Average length of chained list = λ and so average time for accessing an item = $O(1) + O(\lambda)$
  - › Want λ to be close to 1 (i.e. *TableSize* $\approx$ N)
  - › But chaining continues to work for $\lambda > 1$

Resolution by Open addressing

- No links, all keys are in the table
  - › reduced overhead saves space
- When searching for **`X`**, check locations $h_1(X),\ h_2(X),\ h_3(X),\ \ldots$ until either
  - › **`X`** is found; or
  - › we find an empty location (**`X`** not present)
- Various flavors of open addressing differ in which probe sequence they use

Cell Full? Keep looking.

- `h`$_i$`(X)=(Hash(X)+F(i)) mod TableSize`
  - Define F(0) = 0
- F is the collision resolution function. Some possibilities:
  - Linear: F(i) = i
  - Quadratic: F(i) = i^2
  - Double Hashing: F(i) = i·$Hash_2(X)$

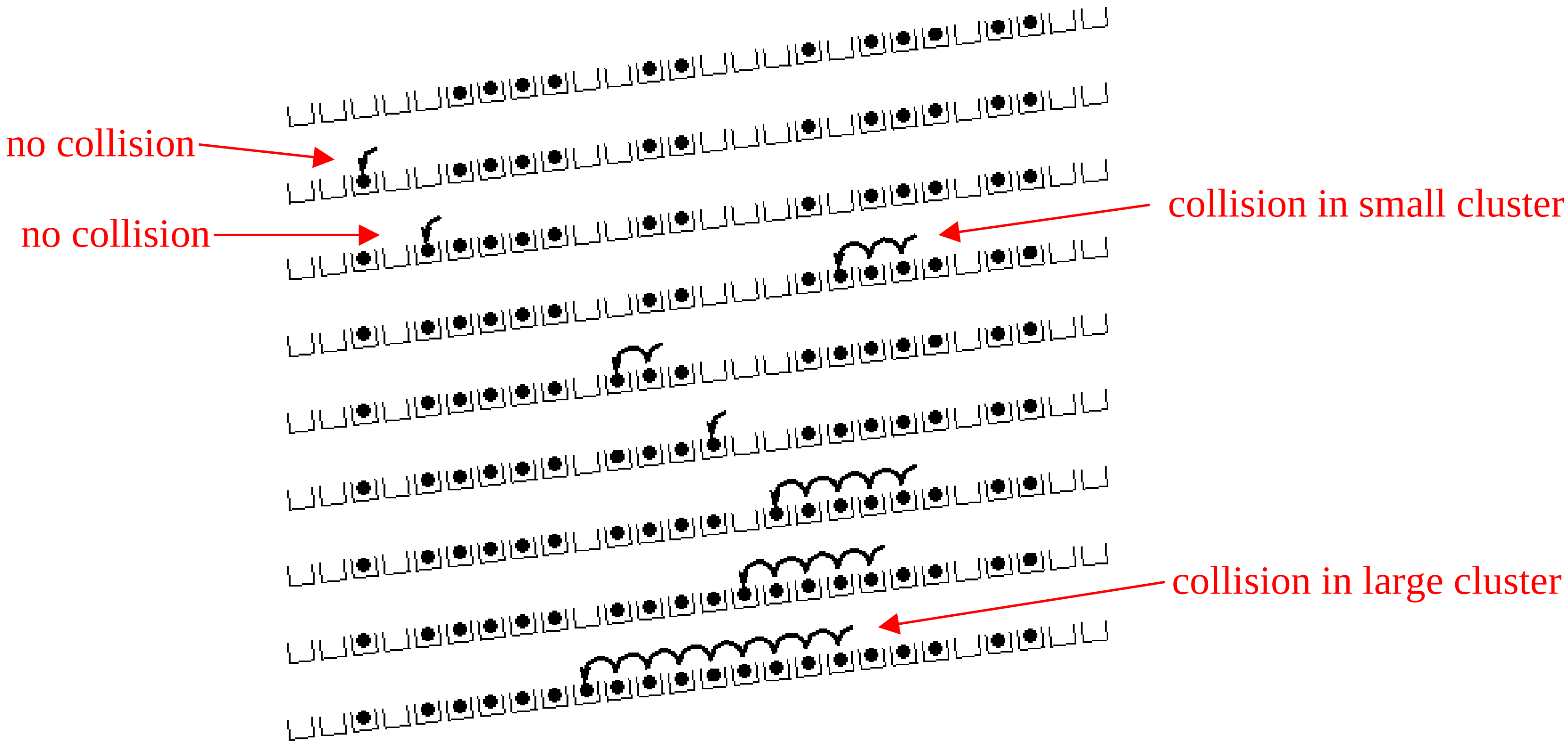
Linear probing

- When searching for **`K`**, check locations **`h(K), h(K)+1, h(K)+2, …`** until either
  - › **`K`** is found; or
  - › we find an empty location (**`K`** not present)
- If table is very sparse, almost like separate chaining.
- When table starts filling, we get clustering but still constant average search time.
- Full table $\Rightarrow$ infinite loop.

Primary clustering phenomenon

- Once a block of a few contiguous occupied positions emerges in table, it becomes a “target” for subsequent collisions
- As clusters grow, they also merge to form larger clusters.
- Primary clustering: elements that hash to different cells probe same alternative cells

Linear probing -- clustering



[R. Sedgewick]

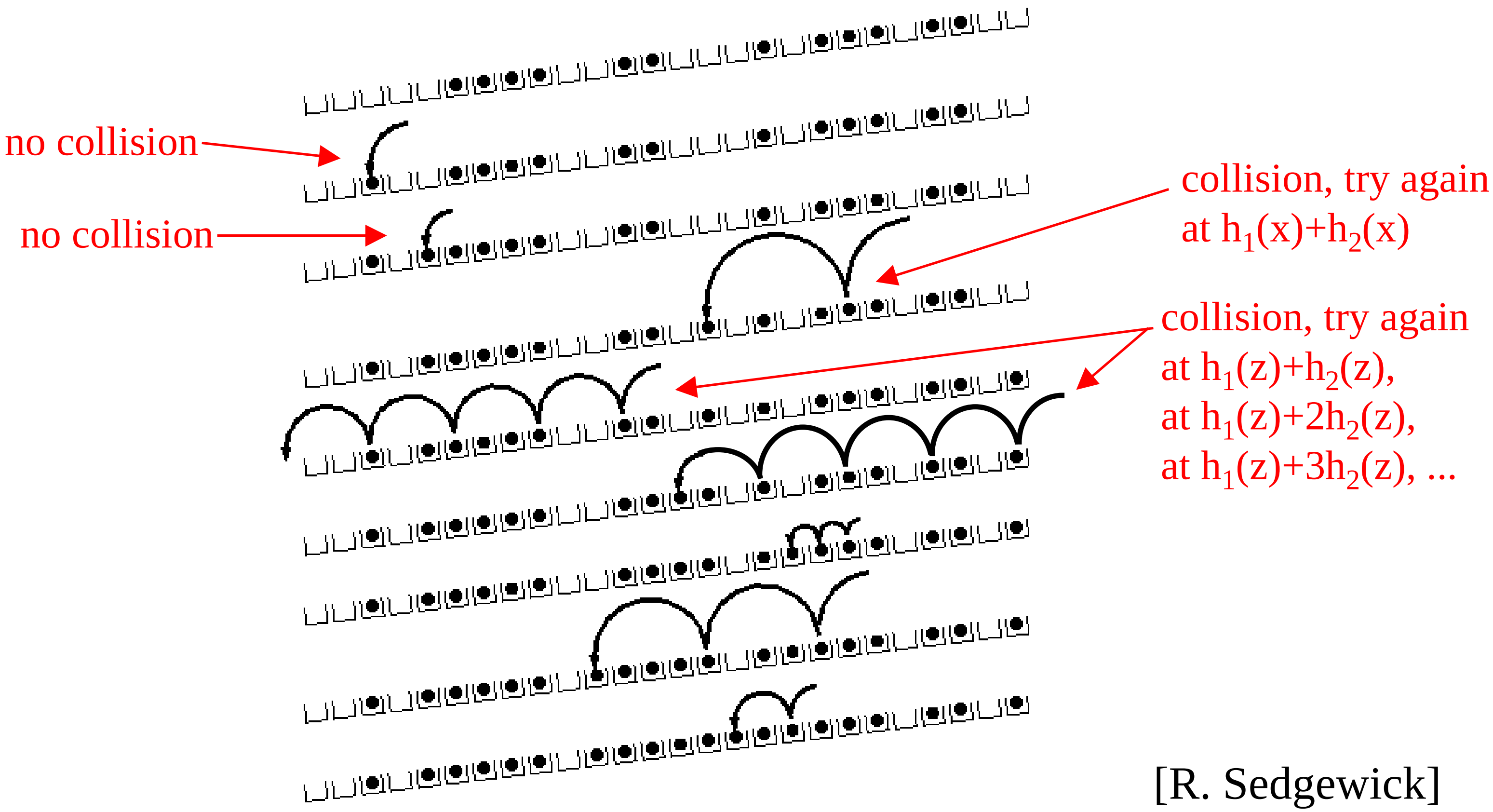
Quadratic Probing

- When searching for **`X`**, check locations $h_1(X)$, $h_1(X)+ i^2$, $h_1(X)+i^3$,… until either
  - **`X`** is found; or
  - we find an empty location (**`X`** not present)
- No primary clustering but secondary clustering possible

Double hashing

- When searching for **`X`**, check locations $h_1(X)$, $h_1(X)+ h_2(X)$, $h_1(X)+2*h_2(X)$, … until either
  - **`X`** is found; or
  - we find an empty location (**`X`** not present)
- Must be careful about $h_2(X)$
  - Not 0 and not a divisor of **`M`**
  - eg, $h_1(k) = k \bmod m_1$, $h_2(k)=1+(k \bmod m_2)$
  - where m_2 is slightly less than m_1

Double hashing



[R. Sedgewick]

Rules of thumb

- Separate chaining is simple but wastes space…
- Linear probing uses space better, is fast when tables are sparse, interacts well with paging
- Double hashing is space efficient, fast (get initial hash and increment at the same time), needs careful implementation
- For average cost t
  - › Max load for Linear Probe is $1-\frac{1}{\sqrt{t}}$
  - › Max load for Double Hashing is $1-\frac{1}{t}$

Rehashing - rebuild the table

- Need to use *lazy deletion* if we use probing (why?)
  - › Need to mark array slots as deleted after Delete
  - › consequently, deleting doesn't make the table any less full than it was before the delete
- If table gets too full ($\lambda \approx 1$) or if many deletions have occurred, running time gets too long and Inserts may fail

Rehashing

- Build a bigger hash table (of size 2**TableSize*) when λ exceeds a particular value
  - › Go through old hash table, ignoring items marked deleted
  - › Recompute hash value for each non-deleted key and put the item in new position in new table
  - › Cannot just copy data from old table because the bigger table has a new hash function
- Running time is O(N) but happens very infrequently

Caveats

- Hash functions are very often the cause of performance bugs.
- Hash functions often make the code not portable.
- Sometime a poor HF distribution-wise is faster overall.
- Always check where the time goes

Appendix

Positional Notation

- Each column in a number represents an additional power of the base number
- in base ten
  - › $1=1*10^0$, $30=3*10^1$, $200=2*10^2$
- in base sixteen
  - › $1=1*16^0$, $30=3*16^1$, $200=2*16^2$
  - › we use A,B,C,D,E,F to represent the numbers between 9_{16} and 10_{16}

Binary, Hex, and Decimal

$2^8=256_{10}$	$2^7=128_{10}$	$2^6=64_{10}$	$2^5=32_{10}$	$2^4=16_{10}$	$2^3=8_{10}$	$2^2=4_{10}$	$2^1=2_{10}$	$2^0=1_{10}$	Hex_{16}	$Decimal_{10}$
							1	1	3	3
					1	0	0	1	9	9
					1	0	1	0	A	10
					1	1	1	1	F	15
				1	0	0	0	0	10	16
				1	1	1	1	1	1F	31
		1	1	1	1	1	1	1	7F	127
	1	1	1	1	1	1	1	1	FF	255

Binary, Hex, and Decimal

$Binary_2$	$16^4=65536_{10}$	$16^3=4096_{10}$	$16^2=256_{10}$	$16^1=16_{10}$	$16^0=1_{10}$	$Decimal_{10}$
11					3	3
1001					9	9
1010					A	10
1111					F	15
1 0000				1	0	16
1 1111				1	F	31
111 1111				7	F	127
1111 1111				F	F	255

Binary, Hex, and Decimal

$Binary_2$	Hex_{16}	$10^3=1000_{10}$	$10^2=100_{10}$	$10^1=10_{10}$	$10^0=1_{10}$
11	3				3
1001	9				9
1010	A			1	0
1111	F			1	5
1 0000	10			1	6
1 1111	1F			3	1
111 1111	7F		1	2	7
1111 1111	FF		2	5	5