Heap Sort

CSE 373 - Data Structures

May 10, 2002

Readings and References

- Reading
  - › Sections 7.5, *Data Structures and Algorithm Analysis in C,* Weiss
- Other References

Binary Search Trees for Sorting?

- Shell sort with Hibbard's increments got us to $O(N^{1.5})$
- Can we beat O(**N^1.5**) using a BST to sort N elements?
  - › Insert each element into an initially empty BST
  - › Do an In-Order traversal to get sorted output
- Running time:
  - › N Inserts at O(log N) apiece = O(N log N)
  - › plus O(N) for In-Order traversal
  - › **O(N log N)** total which is $o(N^{1.5})$

Binary Search Tree sort issue

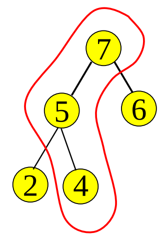
- Extra Space
  - › Need to allocate space for tree nodes and pointers
  - › O(N) extra space, not *in place* sorting
- What if the tree is complete, and we use an array representation – can we sort in place?
  - › Recall your favorite data structure with the initials B. H.

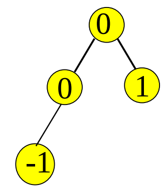
Binary Heaps

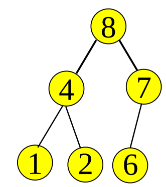
- A binary heap is a binary tree that is:
  - › Complete: the tree is completely filled except possibly the bottom level, which is filled from left to right
  - › Satisfies the heap order property
    - every node is less than or equal to its children
    - or every node is greater than or equal to its children
- The root node is always the smallest node
  - › or the largest, depending on the heap order

Heap order property

- A heap provides limited ordering information
- Each *path* is sorted, but the subtrees are not sorted relative to each other
  - › A binary heap is NOT a binary search tree







These are all valid binary heaps (maximum)

Structure property

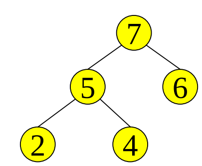
- A binary heap is a complete tree
  - › All nodes are in use except for possibly the right end of the bottom row
- Array implementation is compact because we know how many children there are and we know that they are all present
  - › no pointers are needed, we can directly calculate subscript offsets to the nodes of the tree

Heap Sort using an array

- Root node = A[0]
- Children of A[i] = A[2i+1], A[2i+2]
- Keep track of current size N (number of nodes)

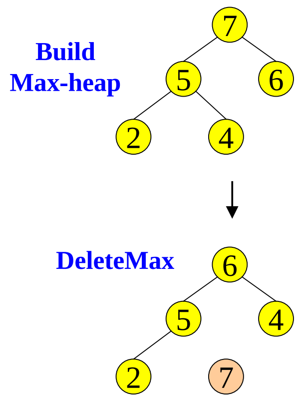
value	7	5	6	2	4			
index	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**

N = 5



Using Binary Heaps for Sorting

- Build a max-heap
- Do N DeleteMax operations and store each Max element as it comes out of the heap
- Data comes out in largest to smallest order
- Where can we put the elements as they are removed from the heap?

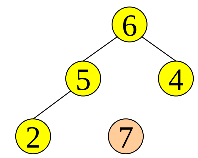


1 Removal = 1 Addition

- Every time we do a DeleteMin, the heap gets smaller by one node, and we have one more node to store
  - › Store the data at the end of the heap array
  - › Not "in the heap" but it is in the heap array

value	6	5	4	2	7			
index	0	1	2	3	4	5	6	7

N = 4

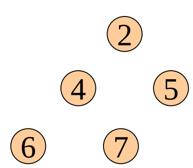


Heap Sort is In-place

- After all the DeleteMins, the heap is gone but the array is full and is in sorted order
- Note that this heap implementation uses index 0 for data and has no sentinel value

value	2	4	5	6	7			
index	0	1	2	3	4	5	6	7

N = 0



Heapsort: Analysis

- Running time
  - › time to build max-heap is O(N)
  - › time for N DeleteMax operations is N O(log N)
  - › total time is **O(N log N)**
- Can also show that running time is Ω(N log N) for some inputs,
  - › so *worst case* is **Θ(N log N)**
  - › *Average case* running time is also O(N log N)