Quick Sort

CSE 373 - Data Structures

May 15, 2002

Readings and References

- Reading
  - › Section 7.7, *Data Structures and Algorithm Analysis in C*, Weiss
- Other References
  - › C LR

Sorting Ideas - swap adjacent

- Swap adjacent elements
  - › Bubble sort
    - it works, but it's always slow
  - › Insertion sort
    - works well on already sorted or partially sorted input
    - low overhead so it works well on small inputs or as the basic sorter for a larger algorithm

Sorting Ideas - swap non-adjacent

- Swap non-adjacent elements
  - › Shell sort
    - resolves multiple inversions with a single swap
    - does an insertion sort of variable sized sub-arrays
    - choice of increments critical
  - › Heap sort
    - resolves multiple inversions with a single swap
    - does insertion sort of paths through a binary heap

Sorting Ideas - recursion and merge

- Merging two sorted arrays is *fast*
  - › Partition the array and sort each part separately, then merge the results
  - › The merge can resolve many inversions with each element merged
- Merge sort
  - › Fast
  - › requires extra O(N) temporary array

Sorting Ideas - recursion and join

- Joining two sorted arrays can be *very fast*
  - › Partition the array into a set of little elements and a set of big elements, sort each partition, and join them
  - › The partitioning operation can move elements a long way towards the final location in one move
- Quick Sort
  - › Fast
  - › in-place sort (no extra space required)

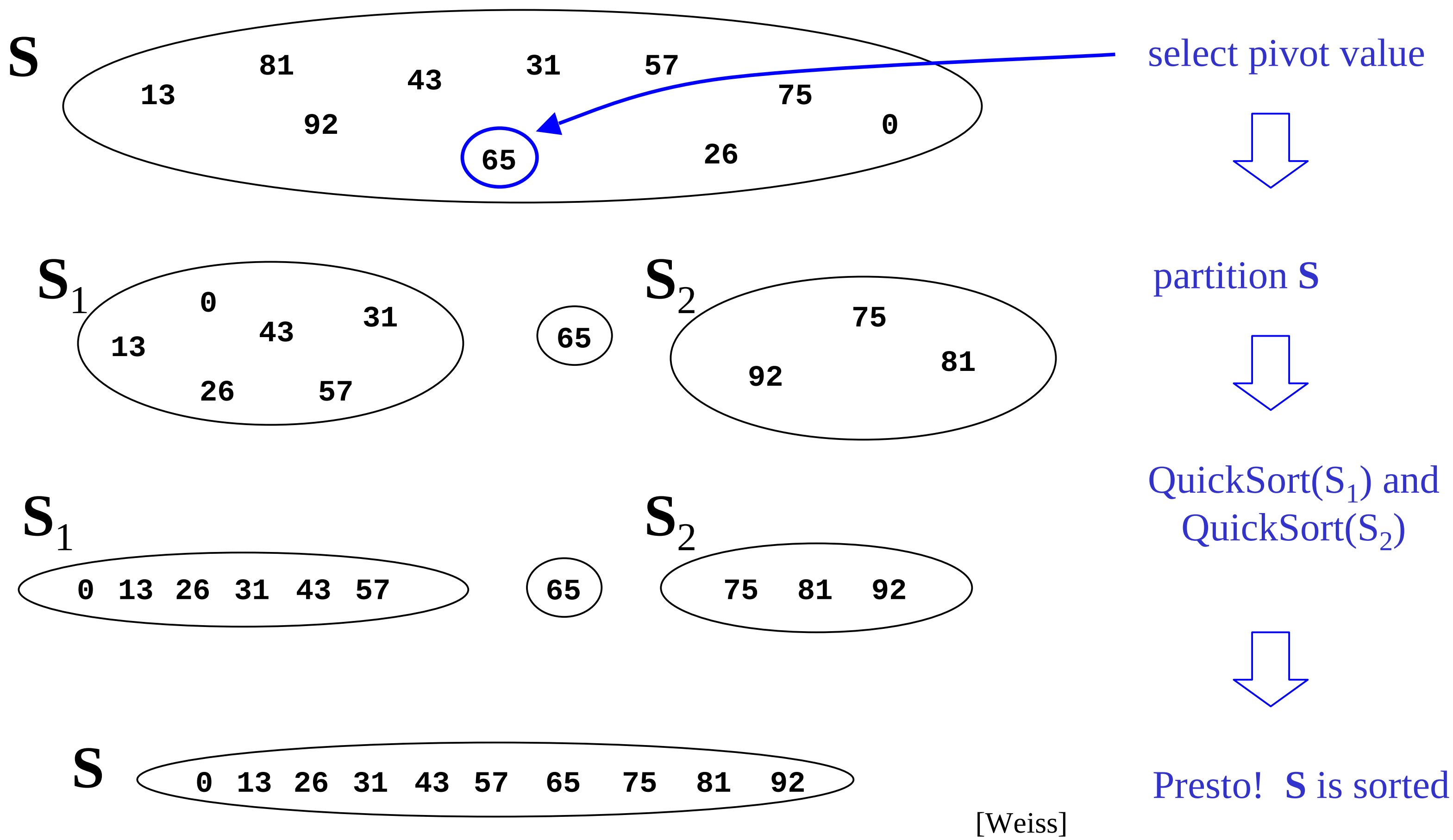
Quicksort

- Quicksort uses a divide and conquer strategy, but does not require the O(N) extra space that MergeSort does
  - › Partition array into left and right sub-arrays
    - the elements in left sub-array are all less than pivot
    - elements in right sub-array are all greater than pivot
  - › Recursively sort left and right sub-arrays
  - › Concatenate left and right sub-arrays in O(1) time

"Four easy steps"

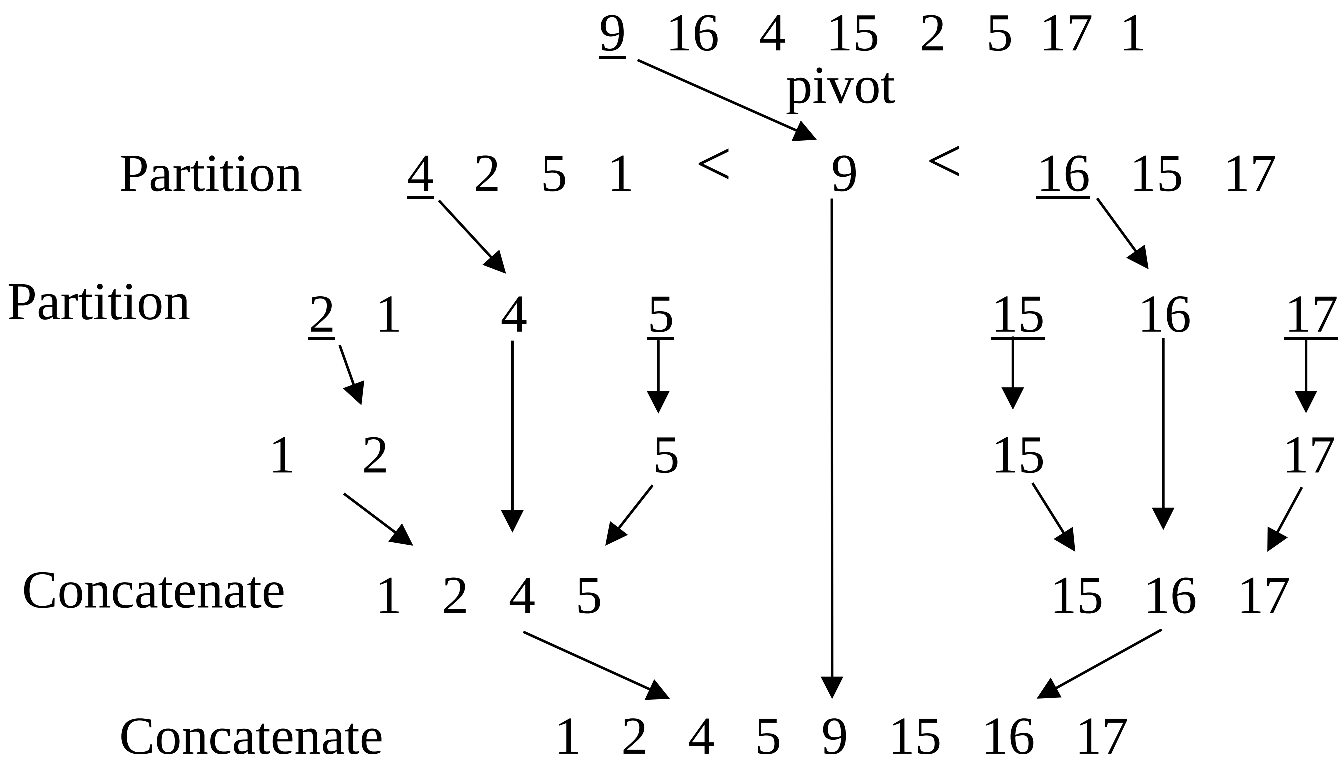
- To sort an array **S**
  - If the number of elements in **S** is 0 or 1, then return. The array is sorted.
  - Pick an element *v* in **S**. This is the *pivot* value.
  - Partition **S**-{*v*} into two disjoint subsets, $\mathbf{S}_1$ = {all values $x \leq v$}, and $\mathbf{S}_2$ = {all values $x \geq v$}.
  - Return QuickSort($\mathbf{S}_1$), *v*, QuickSort($\mathbf{S}_2$)

The steps of QuickSort



Quicksort Example

- Sort the array containing:



Details, details

- "The algorithm so far lacks quite a few of the details"
- Implementing the actual partitioning
- Picking the pivot
  - › want a value that will cause $|S_1|$ and $|S_2|$ to be non-zero, and close to equal in size if possible
- Dealing with cases where the element equals the pivot

Quicksort Partitioning

- Need to partition the array into left and right sub-arrays
  - › the elements in left sub-array are ≤ pivot
  - › elements in right sub-array are ≥ pivot
- How do the elements get to the correct partition?
  - › Choose an element from the array as the pivot
  - › Make one pass through the rest of the array and swap as needed to put elements in partitions

Partitioning is done In-Place

- One implementation (there are others)
  - › median3 finds pivot and sorts left, center, right
  - › Swap pivot with next to last element
  - › Set pointers i and j to start and end of array
  - › Increment i until you hit element A[i] > pivot
  - › Decrement j until you hit element A[j] < pivot
  - › Swap A[i] and A[j]
  - › Repeat until i and j cross
  - › Swap pivot (= A[N-2]) with A[i]

0	1	2	3	4	5	6	7	8	9
8	1	4	9	**0**	3	5	2	7	**6**

median-of-3

pivot = 6

0	1	4	9	7	3	5	2	6	8

median3 sorts 3 elements

i								j	
0	1	4	9	7	3	5	2	6	8

i=left; j=right-1

			i					j	
0	1	4	**9**	7	3	5	2	6	8

while(A[++i]<pivot){}

			i				j		
0	1	4	9	7	3	5	**2**	6	8

while(A[--j]>pivot){}

			i				j		
0	1	4	**2**	7	3	5	**9**	6	8

swap(&A[i],&A[j])

				i			j		
0	1	4	2	**7**	3	5	9	6	8

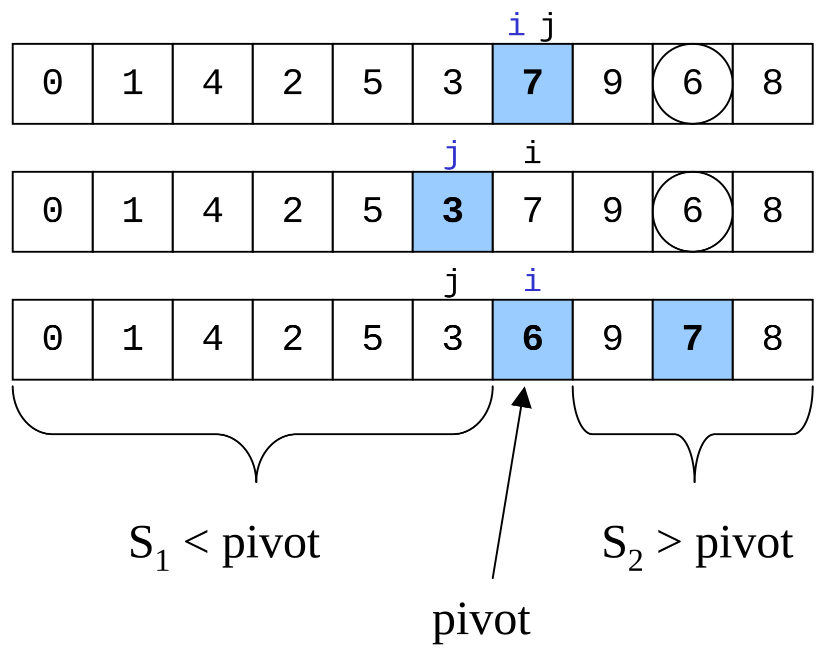
while(A[++i]<pivot){}

				i		j			
0	1	4	2	7	3	**5**	9	6	8

while(A[--j]>pivot){}

				i		j			
0	1	4	2	**5**	3	**7**	9	6	8

swap(&A[i],&A[j])



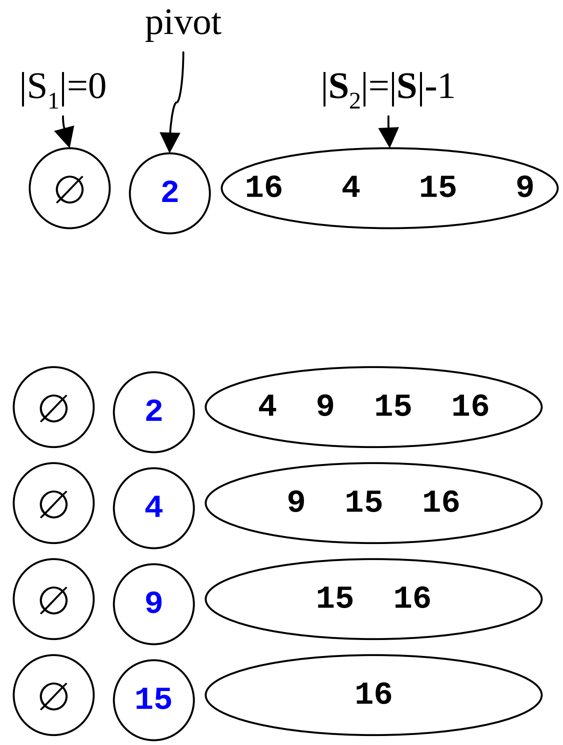
while(A[++i]<pivot){}

while(A[--j]>pivot){}

swap(&A[i],&A[right-1])

Choosing the Pivot (1)

- First (bad) Idea
  - › Pick the first element as pivot
  - › What if it is the smallest or largest?
  - › What if the array is sorted? How many recursive calls does quicksort make?
    - O(N) calls, and it does O(N) work for each call, so you do $O(N^2)$ work when the array is already sorted!



Choosing the Pivot (2)

- 2^{nd} (okay) Idea:
  - › Pick a *random* element to be the pivot
  - › Gets rid of asymmetry in left/right sizes
  - › Actually works pretty well
  - › But it requires calls to pseudo-random number generator
    - expensive in terms of time
    - many implementations are not particularly random

Choosing the Pivot (3a)

- Third idea
  - › Pick *median* element ($N/2^{th}$ largest element)
  - › This is great … it splits **S** exactly in two
  - › But it's hard to find the median element without sorting the entire array first, which is why we are here in the first place ...

Choosing the Pivot (3b)

- Find the median of the first, middle and last elements - “median of 3”
- If the data in the array is not sorted, median of 3 is similar to picking a random pivot
- If the data in the array is presorted, this will pick a value near the actual median of the entire array, which is good

Median-of-Three Pivot

- Find the median of the first, middle and last element

2 4 9 15 16

9

5 4 2 15 16

5

- Takes only O(1) time and not error-prone like the pseudo-random pivot choice
- Less chance of poor performance as compared to looking at only 1 element
- For sorted inputs, splits array nicely in half each recursion

A[i]==pivot?

- Stop and swap
  - `while(A[++i]<pivot){}`
  - `while(A[--j]>pivot){}`
- Although this seems a little odd, it moves i and j towards the middle
  - the benefit of balanced partitions when i and j cross in the middle outweighs the extra cost of swapping elements that are equal to the pivot

Quicksort Best Case Performance

- Algorithm always chooses best pivot and splits sub-arrays in half at each recursion
  - › $T(0) = T(1) = O(1)$
    - constant time if 0 or 1 element
  - › For $N > 1$, 2 recursive calls plus linear time for partitioning
  - › $T(N) = 2T(N/2) + O(N)$
    - Same recurrence relation as Mergesort
  - › $T(N) = O(N \log N)$

Quicksort Worst Case Performance

- Algorithm always chooses the worst pivot – one sub-array is empty at each recursion
  - › $T(0) = T(1) = O(1)$
  - › $T(N) = T(N-1) + O(N)$
  - › $\qquad = T(N-2) + O(N-1) + O(N)$
  - › $\qquad = T(0) + O(1) + \ldots + O(N)$
  - › $T(N) = O(N^2)$
- Fortunately, *average case performance* is O(N log N) (see text for proof)