

CSE 373 ASSIGNMENT 3 SOLUTIONS

1. (a) $f_S : \text{INDEX} \times \text{STRING} \times \text{ARRAY} \rightarrow \text{ARRAY}$
 $f_S : (i, s_i, [e_0, e_1, \dots, e_i, \dots, e_{n-1}]) \mapsto [e_0, e_1, \dots, s_i, \dots, e_{n-1}]$ where $0 \leq i \geq n - 1$

- (b) $f_S : \text{INDEX} \times \text{ARRAY} \rightarrow \text{STRING}$
 $f_S : (i, [e_0, e_1, \dots, e_i, \dots, e_{n-1}]) \mapsto e_i$ where $0 \leq i \geq n - 1$

2. (a)	5	$10\log_2 n$	\sqrt{n}	$\frac{n}{100}$	$50n$	$n\log_2 n$	$2n^2$	$100n^2$	1.01^n	$n!$
$f(n) = 5$	ΘO	O	O	O	O	O	O	O	O	O
$f(n) = 10\log_2 n$	$\Omega \Theta$	O	O	O	O	O	O	O	O	O
$f(n) = \sqrt{n}$	$\Omega \Omega$	Θ	O	O	O	O	O	O	O	O
$f(n) = \frac{n}{100}$	$\Omega \Omega$	Ω	Θ	Θ	O	O	O	O	O	O
$f(n) = 50n$	$\Omega \Omega$	Ω	Θ	Θ	O	O	O	O	O	O
$f(n) = n\log_2 n$	$\Omega \Omega$	Ω	Ω	Θ	O	O	O	O	O	O
$f(n) = 2n^2$	$\Omega \Omega$	Ω	Ω	Ω	Θ	Θ	Θ	Θ	O	O
$f(n) = 100n^2$	$\Omega \Omega$	Ω	Ω	Ω	Θ	Θ	Θ	O	O	O
$f(n) = 1.01^n$	$\Omega \Omega$	Ω	Ω	Ω	Ω	Ω	Ω	Θ	O	O
$f(n) = n!$	$\Omega \Omega$	Ω	Ω	Ω	Ω	Ω	Ω	Ω	Θ	Θ

- (b) The function $n\log_2 n$ is $O(2n^2)$

Justification: Using the knowledge that $\log n < n$ for $n \geq 1$ we can multiply both sides of the equation by n to get $n\log n < n^2$. Since we know this to be true we can multiply by any constant c (like 2 in our case) such that $n\log n$ will always be $O(2n^2)$.

- (c) The function $2n^2$ is $\Theta(100n^2)$

Justification: We need to find a c' and a c'' such that $c'g(n) \leq f(n) \leq c''g(n)$ for $n \geq n_0$. If we set c' to $\frac{1}{50}$ and c'' to 1 then we can show that indeed, $2n^2 \leq 2n^2 \leq 100n^2$ for $n \geq 1$. Therefore $2n^2$ is $\Theta(100n^2)$.