Graph Terminology

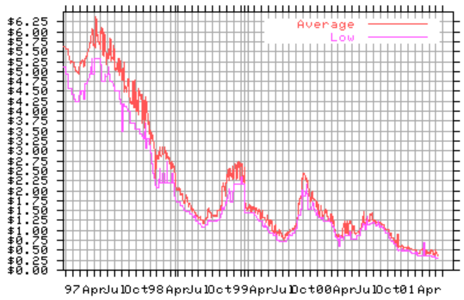
CSE 373
Data Structures

Reading

- Reading
  - › Goodrich and Tamassia, Chapter 12

What are graphs?

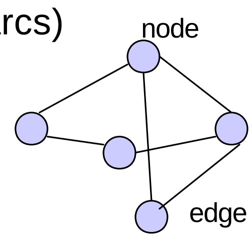
- Yes, this is a graph….



- But we are interested in a different kind of "graph"

Graphs

- Graphs are composed of
  - › Nodes (vertices)
  - › Edges (arcs)

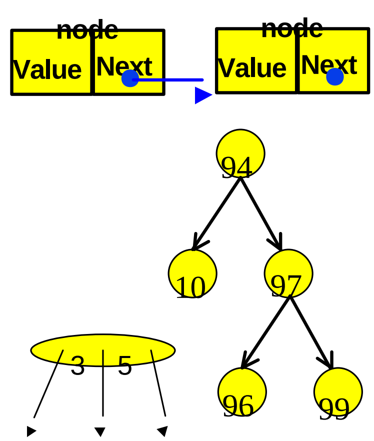


Varieties

- Nodes
  - › Labeled or unlabeled
- Edges
  - › Directed or undirected
  - › Labeled or unlabeled

Motivation for Graphs

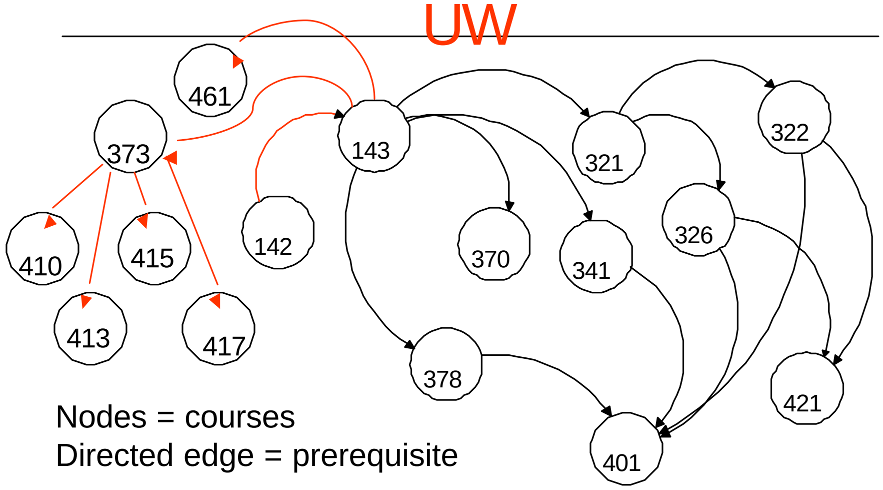
- Consider the data structures we have looked at so far…
- Linked list: nodes with 1 incoming edge + 1 outgoing edge
- Binary trees/heaps: nodes with 1 incoming edge + 2 outgoing edges
- B-trees: nodes with 1 incoming edge + multiple outgoing edges



Motivation for Graphs

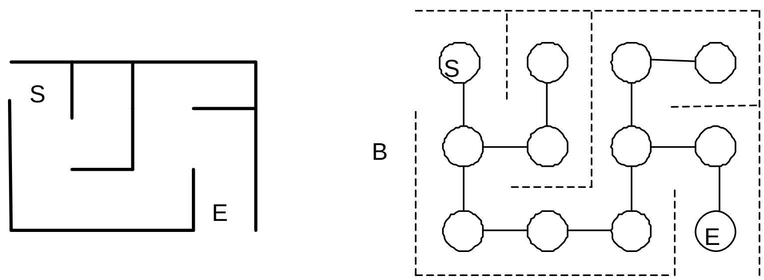
- How can you generalize these data structures?
- Consider data structures for representing the following problems…

CSE Course Prerequisites at UW



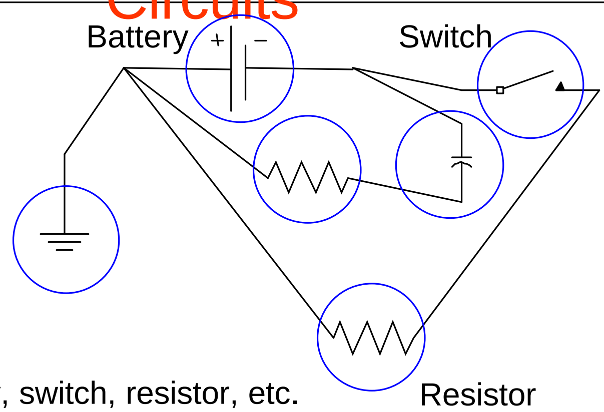
Nodes = courses
Directed edge = prerequisite

Representing a Maze



Nodes = rooms
Edge = door or passage

Representing Electrical Circuits



Nodes = battery, switch, resistor, etc.
Edges = connections

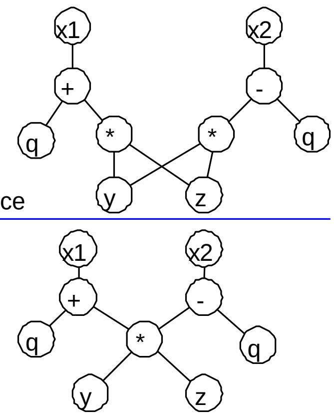
Program statements

```
x1=q+y*z
x2=y*z-q
```

Naive:

y*z calculated twice

common subexpression eliminated:

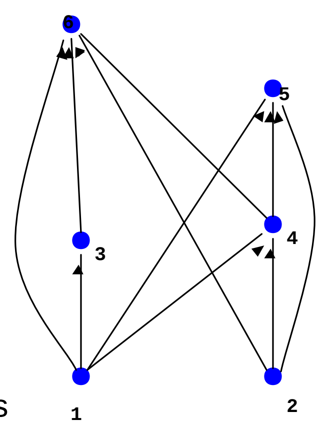


Nodes = symbols/operators
Edges = relationships

Precedence

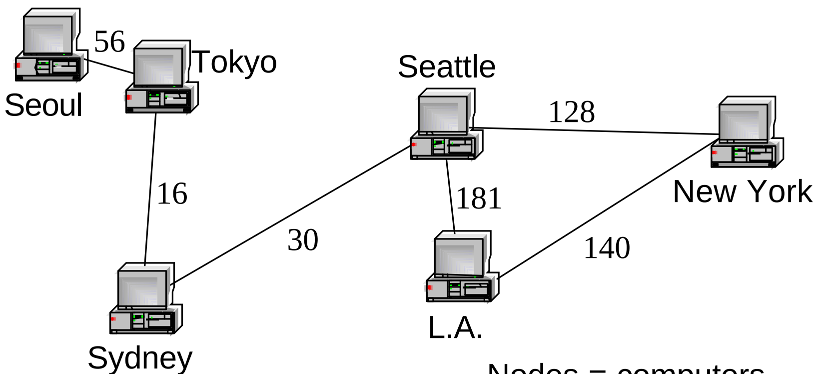
S_1	`a=0;`
S_2	`b=1;`
S_3	`c=a+1`
S_4	`d=b+a;`
S_5	`e=d+1;`
S_6	`e=c+d;`

Which statements must execute before S_6?
S_1, S_2, S_3, S_4



Nodes = statements
Edges = precedence requirements

Information Transmission in a Computer Network



Nodes = computers
Edges = transmission rates

Traffic Flow on Highways

Nodes = cities
Edges = # vehicles on connecting highway

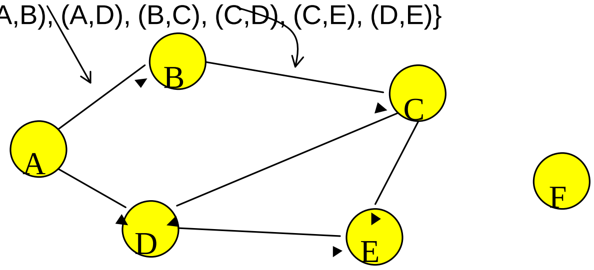
Graph Definition

- A graph is simply a collection of nodes plus edges
  - › Linked lists, trees, and heaps are all special cases of graphs
- The nodes are known as vertices (node = "vertex")
- Formal Definition: A graph *G* is a pair (*V*, *E*) where
  - › *V* is a set of vertices or nodes
  - › *E* is a set of edges that connect vertices

Graph Example

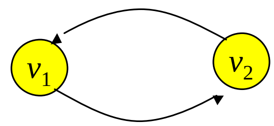
- Here is a directed graph *G* = (*V*, *E*)
  - › Each edge is a pair (v_1, v_2), where v_1, v_2 are vertices in *V*
  - › *V* = {A, B, C, D, E, F}

E = {(A,B), (A,D), (B,C), (C,D), (C,E), (D,E)}

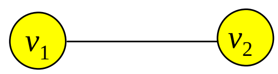


Directed vs Undirected Graphs

- If the order of edge pairs (v_1, v_2) matters, the graph is directed (also called a digraph): $(v_1, v_2) \neq (v_2, v_1)$



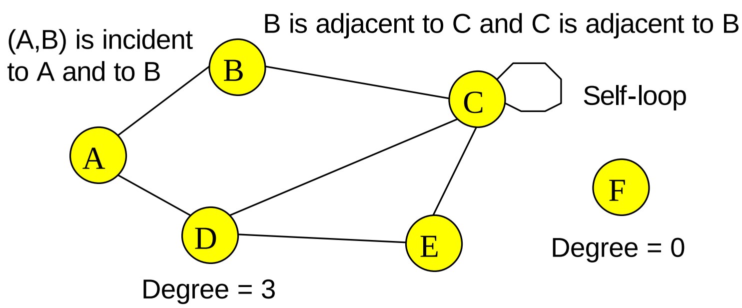
- If the order of edge pairs (v_1, v_2) does not matter, the graph is called an undirected graph: in this case, $(v_1, v_2) = (v_2, v_1)$



Undirected Terminology

- Two vertices u and v are adjacent in an undirected graph G if {u,v} is an edge in G
  - › edge e = {u,v} is incident with vertex u and vertex v
- The degree of a vertex in an undirected graph is the number of edges incident with it
  - › a self-loop counts twice (both ends count)
  - › denoted with deg(v)

Undirected Terminology

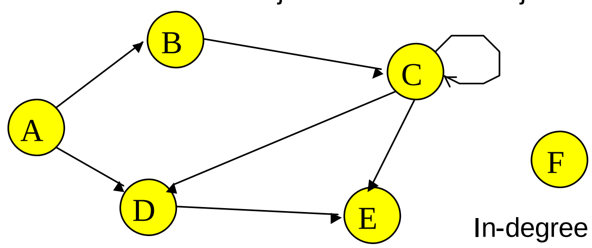


Directed Terminology

- Vertex u is adjacent to vertex v in a directed graph G if (u,v) is an edge in G
  - › vertex u is the initial vertex of (u,v)
- Vertex v is adjacent from vertex u
  - › vertex v is the terminal (or end) vertex of (u,v)
- Degree
  - › in-degree is the number of edges with the vertex as the terminal vertex
  - › out-degree is the number of edges with the vertex as the initial vertex

Directed Terminology

B adjacent to C and C adjacent from B



In-degree = 2
Out-degree = 1

In-degree = 0
Out-degree = 0

Handshaking Theorem

- Let G=(V,E) be an undirected graph with |E|=e edges. Then

$$2e = \sum_{v \in V} \deg(v)$$

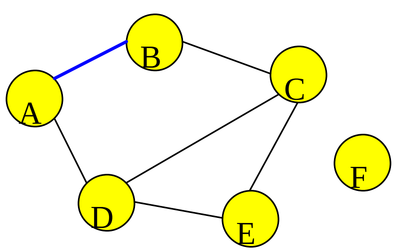
Add up the degrees of all vertices.

- Every edge contributes +1 to the degree of each of the two vertices it is incident with
  - › number of edges is exactly half the sum of deg(v)
  - › the sum of the deg(v) values must be even

Graph Representations

- Space and time are analyzed in terms of:
  - Number of vertices = $|V|$ and
  - Number of edges = $|E|$
- There are at least two ways of representing graphs:
  - The *adjacency matrix* representation
  - The *adjacency list* representation

Adjacency Matrix

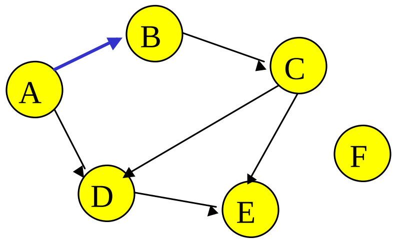


	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	0	0
C	0	1	0	1	1	0
D	1	0	1	0	1	0
E	0	0	1	1	0	0
F	0	0	0	0	0	0

$$M(v, w) = \begin{cases} 1 \text{ if } (v, w) \text{ is in E} \\ 0 \text{ otherwise} \end{cases}$$

Space = $|V|^2$

Adjacency Matrix for a Digraph



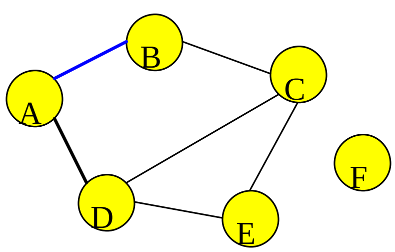
	A	B	C	D	E	F
A	0	1	0	1	0	0
B	0	0	1	0	0	0
C	0	0	0	1	1	0
D	0	0	0	0	1	0
E	0	0	0	0	0	0
F	0	0	0	0	0	0

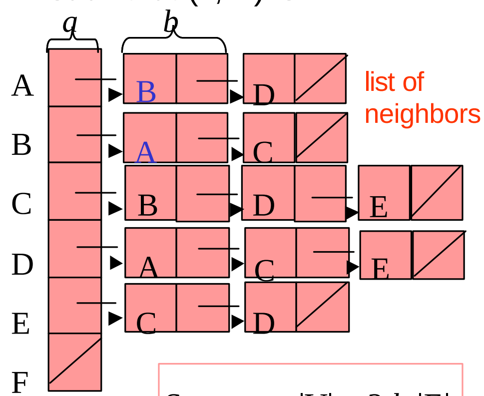
$$M(v, w) = \begin{cases} 1 \text{ if } (v, w) \text{ is in E} \\ 0 \text{ otherwise} \end{cases}$$

Space = $|V|^2$

Adjacency List

For each *v* in *V*, *L*(*v*) = list of *w* such that (*v*, *w*) is in *E*

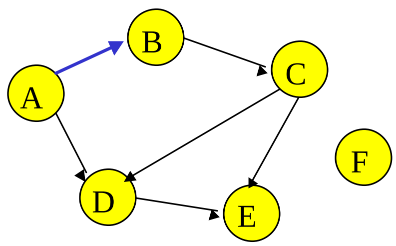


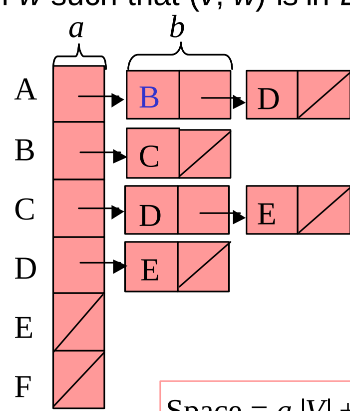


Space = $a\,|V| + 2\,b\,|E|$

Adjacency List for a Digraph

For each v in V, $L(v)$ = list of w such that (v, w) is in E





Space = $a\,|V| + b\,|E|$