Heap Sort

CSE 373
Data Structures
Winter 2007

Heap Sort

Robert Floyd 1937-2002

- Recall Selection Sort:

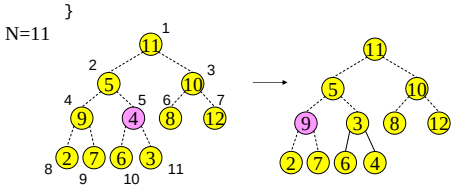
```
While !S.isEmpty(){
  k := S.DeleteMin();
  T.addlast(k); // An easy
        simplification of Insert(k)
```

- Let S be a heap and T be the target
  - › O(n log n) since DeleteMin is O(log n)
  - › But how do we build a heap?

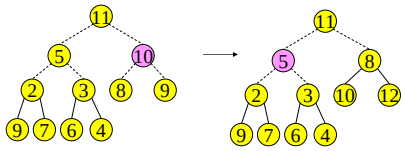
Build Heap

```
BuildHeap {
  for i = N/2 to 1 by -1 PercDown(i,A[i])
}
```

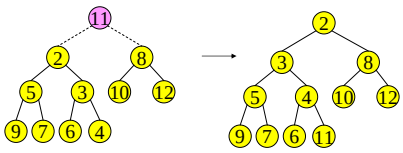
N=11



Build Heap



Build Heap



Analysis of Build Heap

- Each node can percolate down at most its own height
- Let $N = 2^{k+1} - 1$ (height of complete heap is k)
- Then sum of heights is

$$\sum_{i=0}^{k} 2^i (k-i) = 2^{k+1} - 1 - (k+1) = N - (k+1)$$

Other Heap Operations

- Find(X, H): Find the element X in heap H of N elements
  - What is the running time? O(N)
- FindMax(H): Find the maximum element in H where FindMin is O(1)
  - What is the running time? O(N)
- We sacrificed performance of these operations in order to get O(1) performance for FindMin

Other Heap Operations

- DecreaseKey(P,Δ,H): Decrease the key value of node at position P by a positive amount Δ, e.g., to increase priority
  - First, subtract Δ from current value at P
  - Heap order property may be violated
  - so percolate up to fix
  - Running Time: O(log N)

Other Heap Operations

- IncreaseKey(P,Δ,H): Increase the key value of node at position P by a positive amount Δ, e.g., to decrease priority
  - First, add Δ to current value at P
  - Heap order property may be violated
  - so percolate down to fix
  - Running Time: O(log N)

Other Heap Operations

- Delete(P,H): E.g. Delete a job waiting in queue that has been preemptively terminated by user
  - Use DecreaseKey(P,∞,H) followed by DeleteMin
  - Running Time: O(log N)

Other Heap Operations

- Merge(H1,H2): Merge two heaps H1 and H2 of size O(N). H1 and H2 are stored in two arrays.
  - Can do O(N) Insert operations: O(N log N) time
  - Better: Copy H2 at the end of H1 and use BuildHeap. Running Time: O(N)