CSE 373, Autumn 2008, Assignment 2 Solutions

October 18, 2008

1. (8 points)

   (a) $n = ((795 - (-10))/7) + 1 = 116$
   Sum $= (((-10) + 795)116)/2 = 45530$

   (b) Sum $= (256 * (1 - (1/2)^9))/(1 - (1/2)) = 511$

   (c) Sum $= (1 * (3^9 - 1))/(3 - 1) = 9841$

   (d) Sum $= 144/(1 - (1/4)) = 192$

2. (6 points)

   (a) 10^{x+y+z}

   (b) xy

   (c) $1 + 2\log_2 x + 3\log_2 y$

3. (7 points)
   Basis Step:
   $n = 1$, $(1 + 1) = 1 * (1 + 3)/2 = 2$.
   Induction hypothesis:
   $\sum_{i=1}^{k}(i + 1) = \frac{k(k+3)}{2}$, for some k.
   Induction step:
   $\sum_{i=1}^{k+1}(i + 1) = \sum_{i=1}^{k}(i + 1) + ((k + 1) + 1) = \frac{k(k+3)}{2} + ((k + 1) + 1) = \frac{(k+1)((k+1)+3)}{2}$
   This represents the proposition to be proved for the case $n = k + 1$, and completes the proof.

4. (6 points)

   (a) $\{\}, \{0\}, \{1\}, \{0, 1\}$

   (b) $(0, 0), (0, 1), (1, 0), (1, 1)$

   (c) $(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)$

5. (18 points)

	$R1$	$R2$	$R3$	$R4$	$R5$	$R6$
Reflexive	N	N	N	Y	Y	Y
Symmetric	Y	Y	Y	Y	N	Y
Transitive	Y	Y	N	Y	Y	Y
Antisymmetric	Y	Y	N	Y	Y	N
Equivalence Relation	N	N	N	Y	N	Y
Partial Order	N	N	N	Y	Y	N

6. (20 points, 15 for table entries and 5 for explanations)

	100	$2n+5$	$\log_2 n$	$5n^2$	$n \log_2 n$
$3n+1$	Ω	Θ	Ω	O	O
$0.001 * 2^{n-10}$	Ω	Ω	Ω	Ω	Ω
$\log_{10} n^n$	Ω	Ω	Ω	O	Θ

$0.001 * 2^{n-10} \geq 5n^2$ for $n \geq 33$ as can be verified by taking base 2 logs on both sides.

$\log_{10} n^n = n \log_{10} n = n \log_2 n / \log_2 10 = \Theta(n \log_2 n)$

7. (20 points)

   (a) (12 points) We will use stack Sa for enqueueing, Sb for dequeueing, and a boolean variable *enQmode* for storing the current operating mode. The methods are shown below.

```
boolean isEmpty(){
  if(Sa.isEmpty() && Sb.isEmpty())
     return true;
  else
     return false; }

void enqueue(Object obj){
  if(!enQmode){
    while(!Sb.isEmpty())
       Sa.push(Sb.pop()); }
  Sa.push(obj); }

Object dequeue(){
  if(enQMode){
    while(!Sa.isEmpty())
       Sb.push(Sa.pop()); }
  return Sb.pop(); }
```

(b) (4 points) The isEmpty method is constant time. The enqueue and dequeue operations take $O(m)$ time in the worst case where m is the current size of the queue. This is because we may need to move all m objects from one stack to another.

(c) (4 points) The total time complexity is $O(n^2)$. There are $2n$ operations each of which takes $O(n)$ time.

8. (15 points)

(a) (5 points) The algorithm goes thru the following steps.
poly $= 4$
poly $= 4 * 3 + 8 = 20$
poly $= 20 * 3 + 0 = 60$
poly $= 60 * 3 + 1 = 181$
poly $= 181 * 3 + 2 = 545$

(b) (5 points) Observe that the polynomial $a_0 + a_1x + a_2x^2 + \ldots$ can be rewritten as $a_0 + x(a_1 + x(a_2 + \ldots$ by repeatedly factoring out x. The algorithm computes the polynomial using this equivalent form.

(c) (5 points) The running time is $\Theta(n)$.