CSE 373: Data Structures and Algorithms

Lecture 3: Math Review/Asymptotic Analysis

Motivation

- So much data!!
  - Human genome: $3.2 * 10^9$ base pairs
    - If there are $6.8 * 10^9$ on the planet, how many base pairs of human DNA?
  - Earth surface area: $1.49 * 10^8$ km^2
    - How many photos if taking a photo of each m^2?
    - For every day of the year ($3.65 * 10^2$)?

- But aren't computers getting faster and faster?

Why algorithm analysis?

- As problem sizes get bigger, analysis is becoming *more* important.

- The difference between good and bad algorithms is getting bigger.

- Being able to analyze algorithms will help us identify good ones without having to program them and test them first.

Measuring Performance: Empirical Approach

- Implement it, run it, time it (averaging trials)
  - Pros?
  - Cons?

Measuring Performance: Empirical Approach

- Implement it, run it, time it (averaging trials)
  - Pros?
    - Find out how the system effects performance
    - Stress testing – how does it perform in dynamic environment
    - No math!
  - Cons?
    - Need to implement code
    - Can be hard to estimate performance
    - When comparing two algorithms, all other factors need to be held constant (e.g., same computer, OS, processor, load)

Measuring Performance: Analytical Approach

- Use a simple model for basic operation costs

- Computational Model
  - has all the basic operations:
    +, -, *, / , =, comparisons
  - fixed sized integers (e.g., 32-bit)
  - infinite memory
  - all basic operations take exactly one time unit (one CPU instruction) to execute

Measuring Performance: Analytical Approach

- Analyze steps of algorithm, estimating amount of work each step takes
  - Pros?
    - Independent of system-specific configuration
    - Good for estimating
    - Don't need to implement code
  - Cons?
    - Won't give you info exact runtimes optimizations made by the architecture (i.e. cache)
    - Only gives useful information for large problem sizes
    - In real life, not all operations take exactly the same time and have memory limitations

Analyzing Performance

- General "rules" to help measure how long it takes to do things:

Basic operations	Constant time
Consecutive statements	Sum of number of statements
Conditionals	Test, plus larger branch cost
Loops	Sum of iterations
Function calls	Cost of function body
Recursive functions	Solve recurrence relation…

Efficiency examples

```
statement1;
statement2;
statement3;
```
?

```
for (int i = 1; i <= N; i++) {
    statement4;
}
```
?

```
for (int i = 1; i <= N; i++) {
    statement5;
    statement6;
    statement7;
}
```
?

?

Efficiency examples

```
statement1;
statement2;
statement3;

for (int i = 1; i <= N; i++) {
    statement4;
}

for (int i = 1; i <= N; i++) {
    statement5;
    statement6;
    statement7;
}
```

statement1–statement3: 3

first loop: N

second loop: 3N

total: 4N + 3

Efficiency examples 2

```
for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {        ?
        statement1;
    }
}
                                                  ?
for (int i = 1; i <= N; i++) {
    statement2;
    statement3;
    statement4;                            ?
    statement5;
}
```

Efficiency examples 2

```
for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {
        statement1;
    }
}

for (int i = 1; i <= N; i++) {
    statement2;
    statement3;
    statement4;
    statement5;
}
```

First loop: N^2

Second loop: $4N$

Total: $N^2 + 4N$

- How many statements will execute if N = 10? If N = 1000?

Relative rates of growth

- most algorithms' runtime can be expressed as a *function* of the input size *N*

- **rate of growth**: measure of how quickly the graph of a function rises

- goal: distinguish between fast- and slow-growing functions
  - we only care about very large input sizes
    (for small sizes, most any algorithm is fast enough)
  - this helps us discover which algorithms will run more quickly or slowly, for large input sizes

- most of the time interested in worst case performance; sometimes look at best or average performance

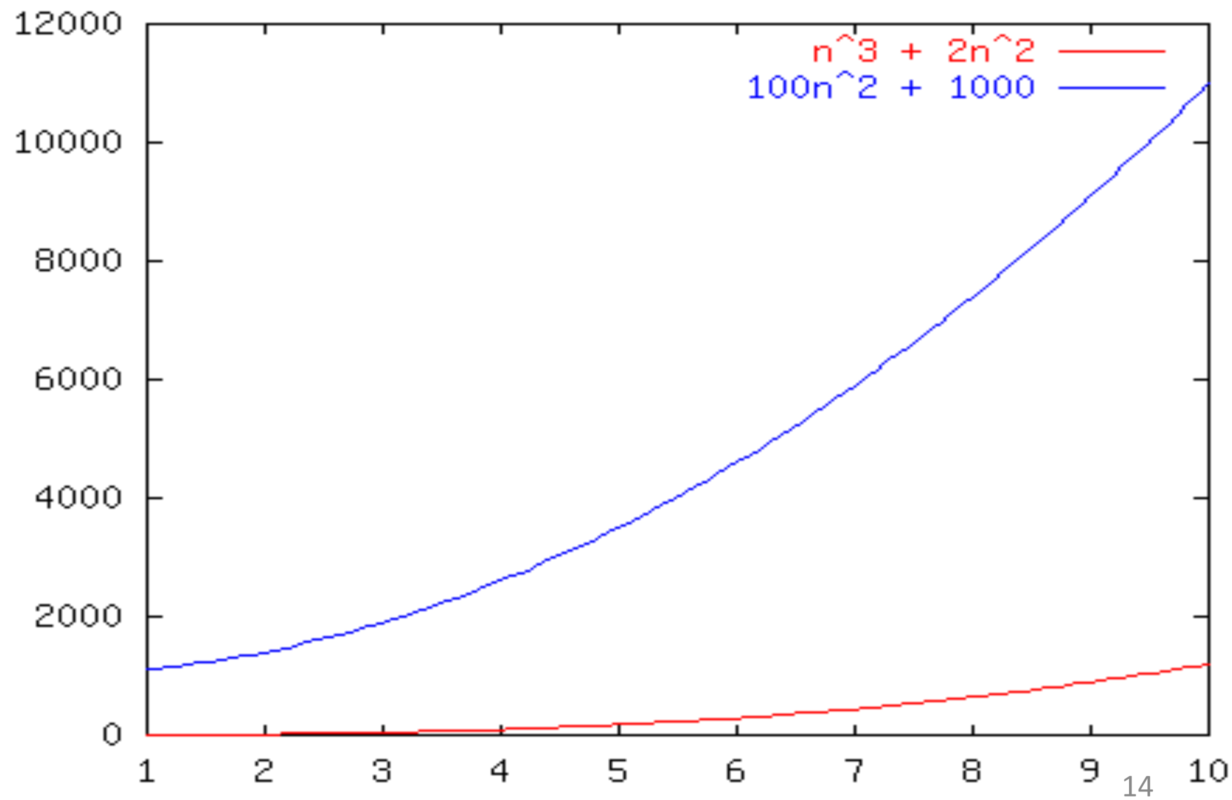
Growth rate example

Consider these graphs of functions.
Perhaps each one represents an algorithm:

$n^3 + 2n^2$

$100n^2 + 1000$

- Which grows faster?



Growth rate example

- How about now?