CSE 373: Data Structures and Algorithms

Lecture 6: Searching

Searching and recursion

- Problem: Given a sorted array *a* of integers and an integer *i*, find the index of any occurrence of *i* if it appears in the array. If not, return -1.
  - We could solve this problem using a standard iterative search; starting at the beginning, and looking at each element until we find *i*
  - What is the runtime of an iterative search?

- However, in this case, the array is sorted, so does that help us solve this problem more intelligently? Can recursion also help us?

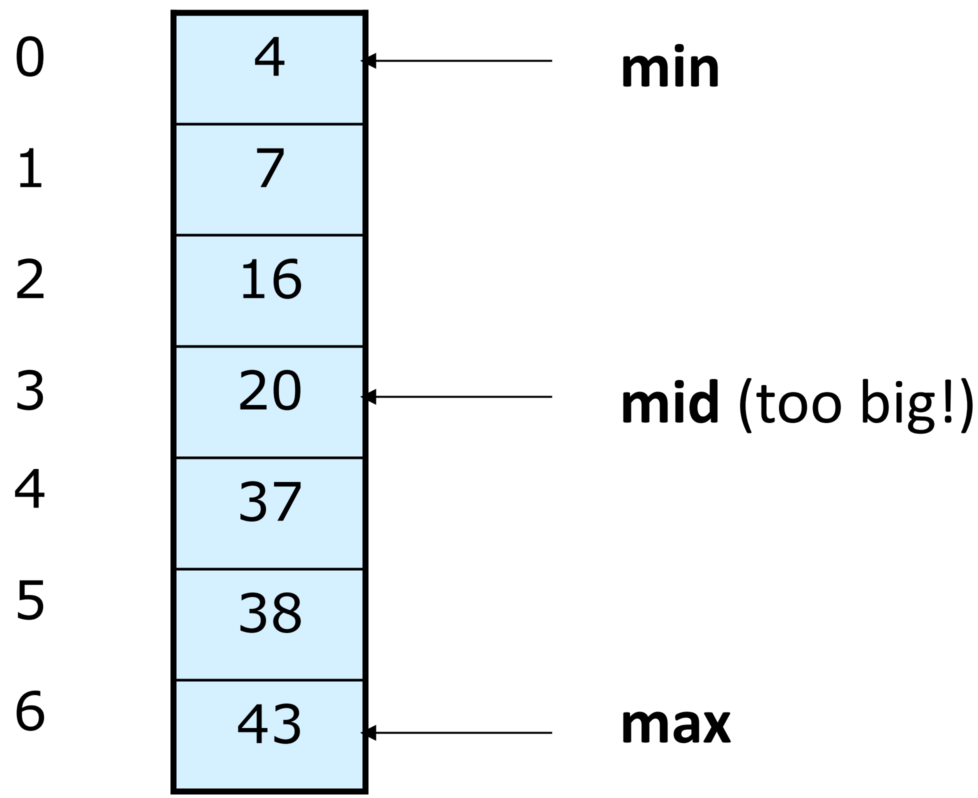
Binary search algorithm

- Algorithm idea: Start in the middle, and only search the portions of the array that might contain the element *i*. Eliminate half of the array from consideration at each step.
  - can be written iteratively, but is harder to get right

- called **binary search** because it chops the area to examine in half each time
  - implemented in Java as method `Arrays.binarySearch` in `java.util` package

Binary search example

i = 16



Binary search example

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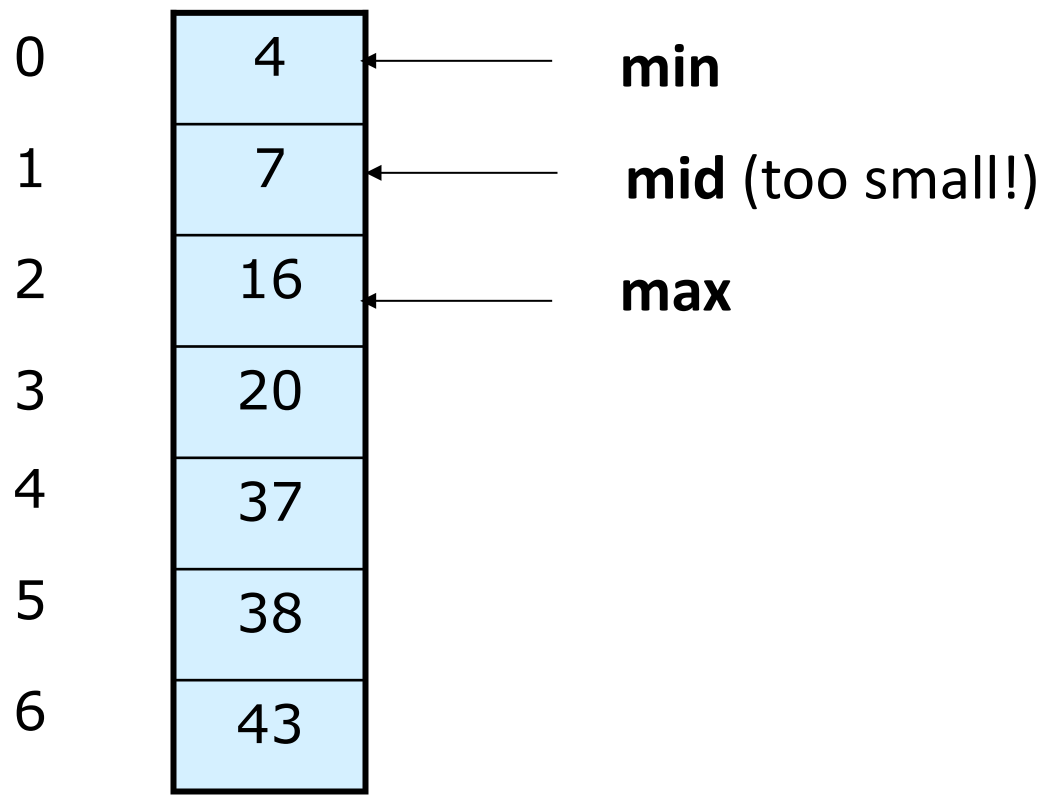

Binary search example

i = 16

0	4
1	7
2	**16**
3	20
4	37
5	38
6	43

min, mid, max (found it!)

Binary search pseudocode

binary search array *a* for value *i*:
    if all elements have been searched,
        result is -1.
    examine middle element *a*[*mid*].
    if a[*mid*] equals *i*,
        result is *mid*.
    if *a*[*mid*] is greater than *i*,
        binary search left half of *a* for *i*.
    if *a*[*mid*] is less than *i*,
        binary search right half of *a* for *i*.

Runtime of binary search

- How do we analyze the runtime of binary search and recursive functions in general?

- binary search either exits immediately,
when input size <= 1 or value found (base case),
or executes itself on 1/2 as large an input (rec. case)
  - T(1) = c
  - T(2) = T(1) + c
  - T(4) = T(2) + c
  - T(8) = T(4) + c
  - ...
  - T(n) = T(n/2) + c

- How many times does this division in half take place?

Divide-and-conquer

- **divide-and-conquer algorithm**: a means for solving a problem that first separates the main problem into 2 or more smaller problems, then solves each of the smaller problems, then uses those sub-solutions to solve the original problem
  - 1: "divide" the problem up into pieces
  - 2: "conquer" each smaller piece
  - 3: (if necessary) combine the pieces at the end to produce the overall solution

  - binary search is one such algorithm

Recurrences, in brief

- How can we prove the runtime of binary search?
- Let's call the runtime for a given input size *n*, T(*n*). At each step of the binary search, we do a constant number c of operations, and then we run the same algorithm on 1/2 the original amount of input. Therefore:
  - $T(n) = T(n/2) + c$
  - $T(1) = c$
- Since T is used to define itself, this is called a **recurrence relation**.

Solving recurrences

- **Master Theorem**:
  A recurrence written in the form
  T(*n*) = *a* * T(*n* / b) + f(*n*)

  (where f(*n*) is a function that is O(n^k) for some power *k*)
  has a solution such that

$$T(n) = \begin{cases} O(n^{\log_b a}), & a > b^k \\ O(n^k \log n), & a = b^k \\ O(n^k), & a < b^k \end{cases}$$

- This form of recurrence is very common for divide-and-conquer algorithms

Runtime of binary search

- Binary search is of the correct format:
  $T(n) = a * T(n / b) + f(n)$
  - $T(n) = T(n/2) + c$
  - $T(1) = c$

  - $f(n) = c = O(1) = O(n^0)$ ... therefore $k = 0$
  - $a = 1$, $b = 2$

- $1 = 2^0$, therefore:
  $T(n) = O(n^0 \log n) =$ **$O(\log n)$**

- (recurrences not needed for our exams)