CSE 373: Data Structures and Algorithms

Lecture 15: Priority Queues (Heaps) III

Generic Collections

Generics and arrays

```
public class Foo<T> {
    private T myField;                    // ok

    public void method1(T param) {
        myField = new T();                // error
        T[] a = new T[10];                // error
    }
}
```

– You cannot create objects or arrays of a parameterized type.

Generics/arrays, fixed

```
public class Foo<T> {
    private T myField;                          // ok

    public void method1(T param) {
        myField = param;                        // ok
        T[] a2 = (T[]) (new Object[10]);        // ok
    }
}
```

– But you can create variables of that type, accept them as parameters, return them, or create arrays by casting `Object[]`.

The `compareTo` method

- The standard way for a Java class to define a comparison function for its objects is to define a `compareTo` method.
  - Example: in the String class, there is a method:

    ```
    public int compareTo(String other)
    ```

- A call of `A.compareTo(B)` will return:

a value <	0	if A comes "before" B in the ordering,
a value >	0	if A comes "after" B in the ordering,
or	0	if A and B are considered "equal" in the ordering.

Comparable

```
public interface Comparable<E> {
    public int compareTo(E other);
}
```

- A class can implement the `Comparable` interface to define a natural ordering function for its objects.

- A call to the `compareTo` method should return:

  a value < 0 if the `other` object comes "before" `this` one,

  a value > 0 if the `other` object comes "after" `this` one,

  or 0 if the `other` object is considered "equal" to `this`.

`Comparable` template

```
public class name implements Comparable<name> {

    ...

    public int compareTo(name other) {
        ...
    }
}
```

- Exercise: Add a `compareTo` method to the `PrintJob` class such that `PrintJobs` are ordered according to their priority (ascending – lower priorities are more important than higher ones).

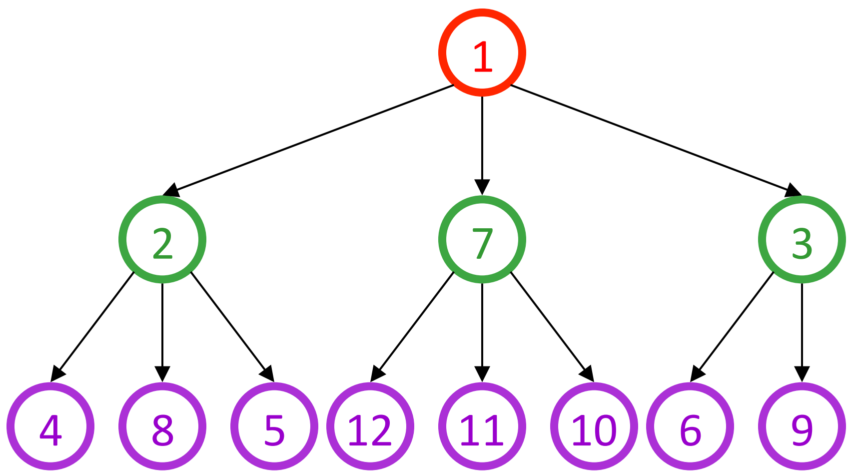
Comparable example

```
public class PrintJob implements Comparable<PrintJob> {
    private String user;
    private int number;
    private int priority;

    public PrintJob(int number, String user, int priority)
    {
        this.number = number;
        this.user = user;
        this.priority = priority;
    }

    public int compareTo(PrintJob otherJob) {
        return this.priority - otherJob.priority;
    }

    public String toString() {
        return this.number + " (" + user + "):" + this.priority;
    }
}
```

d-Heaps

Generalization: *d*-Heaps

- Each node has *d* children
- Still can be represented by array
- Good choices for *d* are a power of 2
  - How does height compare to binary heap?



1	2	7	3	4	8	5	12	11	10	6	9

Operations on *d*-Heap

- insert: runtime =

depth of tree decreases, $\Theta(\log_d n)$ worst

- remove: runtime =

bubbleDown requires comparison to find min, $\Theta(d \log_d n)$, worst/ave

Does this help insert or remove more?

Other Priority Queue Operations

More Min-Heap Operations

- **decreasePriority**
  - given a reference of an element in the queue, reduce its priority value

  Solution: change priority and ______________________________
- **increasePriority**
  - given a reference of an element in the queue, increase its priority value

  Solution: change priority and _______________________________

Why do we need a *reference*? Why not simply data value?

More Min-Heap Operations

- **remove**
  - given a reference to an object in the queue, remove the object from the queue

  Solution: set priority to negative infinity, percolate up to root and deleteMin

- **findMax**

Building a Heap

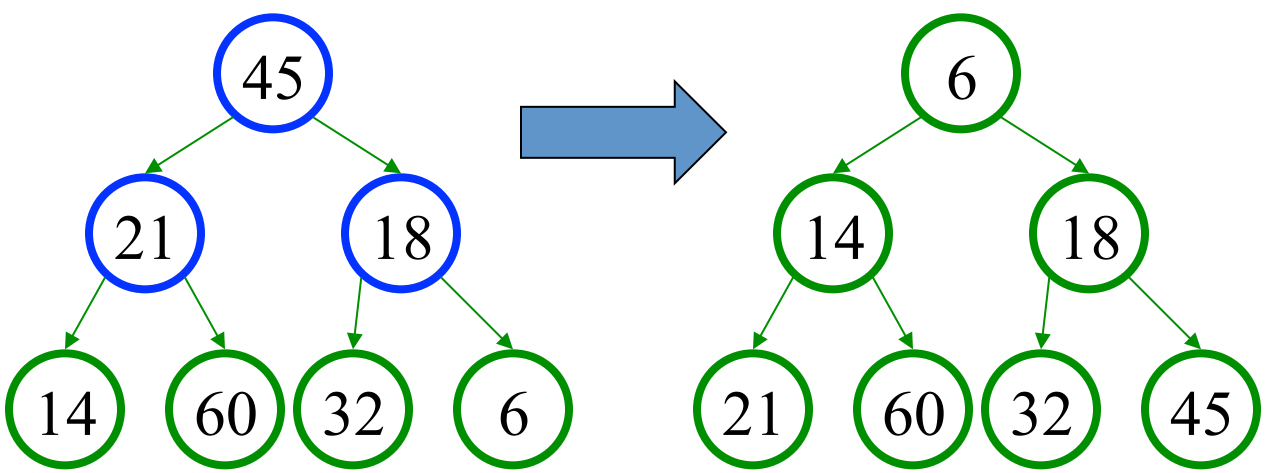
- At every point, the new item may need to percolate all the way through the heap

- Adding the items one at a time is $\Theta(n \log n)$ in the worst case (what is the average case?)

- A more sophisticated algorithm does it in $\Theta(n)$

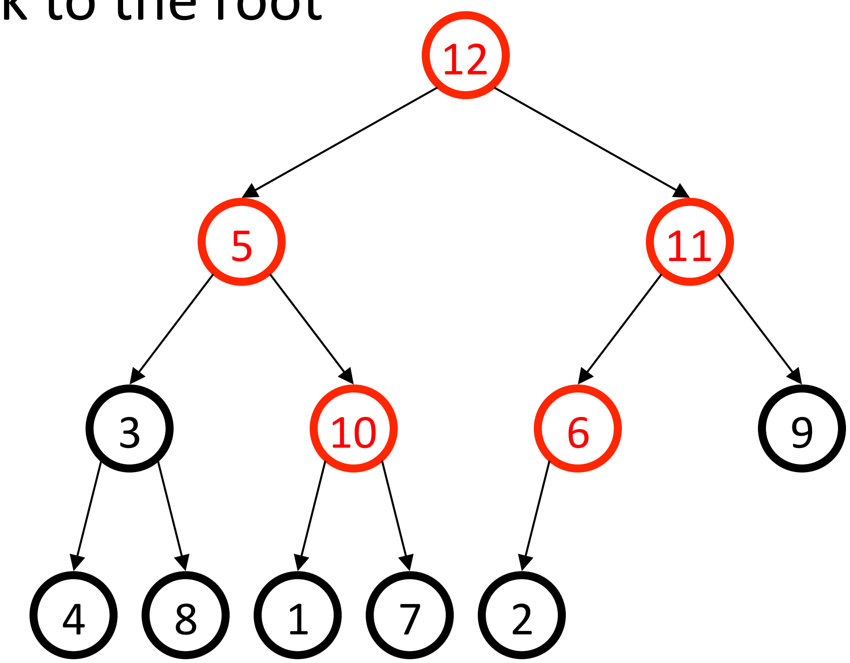
O(*N*) buildHeap

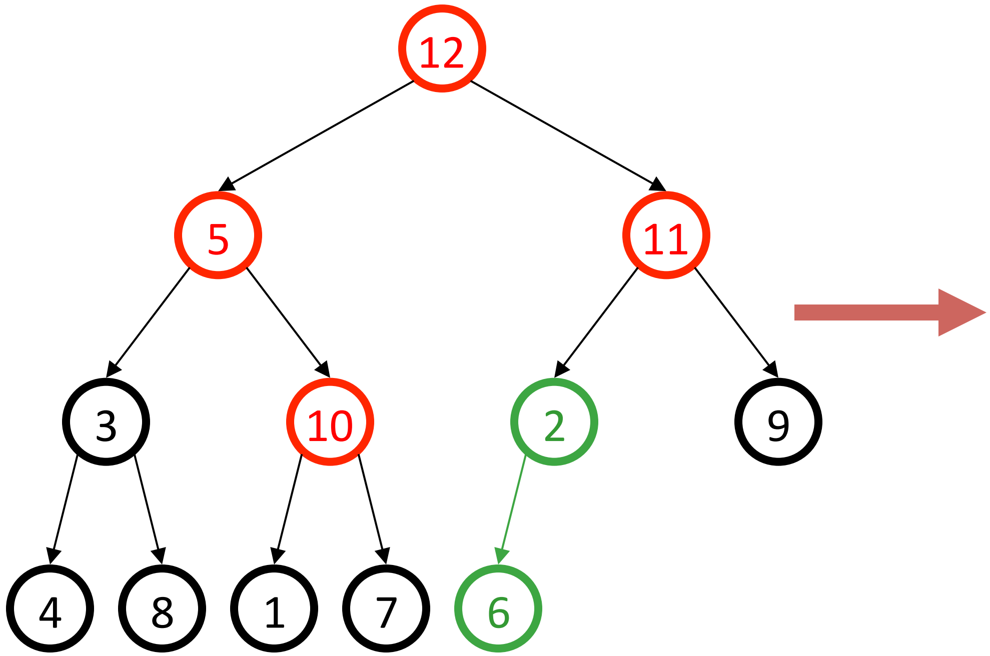
- **Algorithm idea**
  - First, add all elements arbitrarily maintaining the completeness property
  - Then fix the heap order property by performing a "bubble down" operation on every node that is not a leaf, starting from the rightmost internal node and working back to the root
    - why does this `buildHeap` operation work?



buildHeap practice problem

- Each element in the list [12, 5, 11, 3, 10, 6, 9, 4, 8, 1, 7, 2] has been inserted into a heap such that the completeness property has been maintained.
- Now, fix the heap's order property by "bubbling down" every internal node, starting from the rightmost internal node working back to the root

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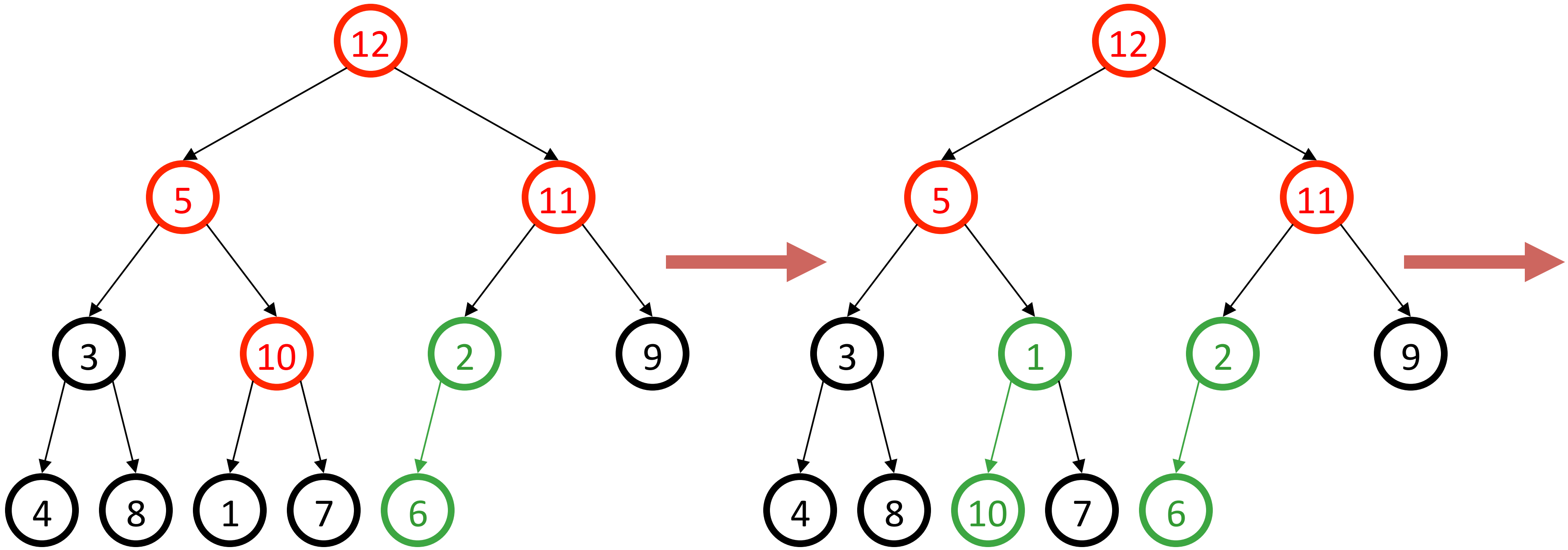
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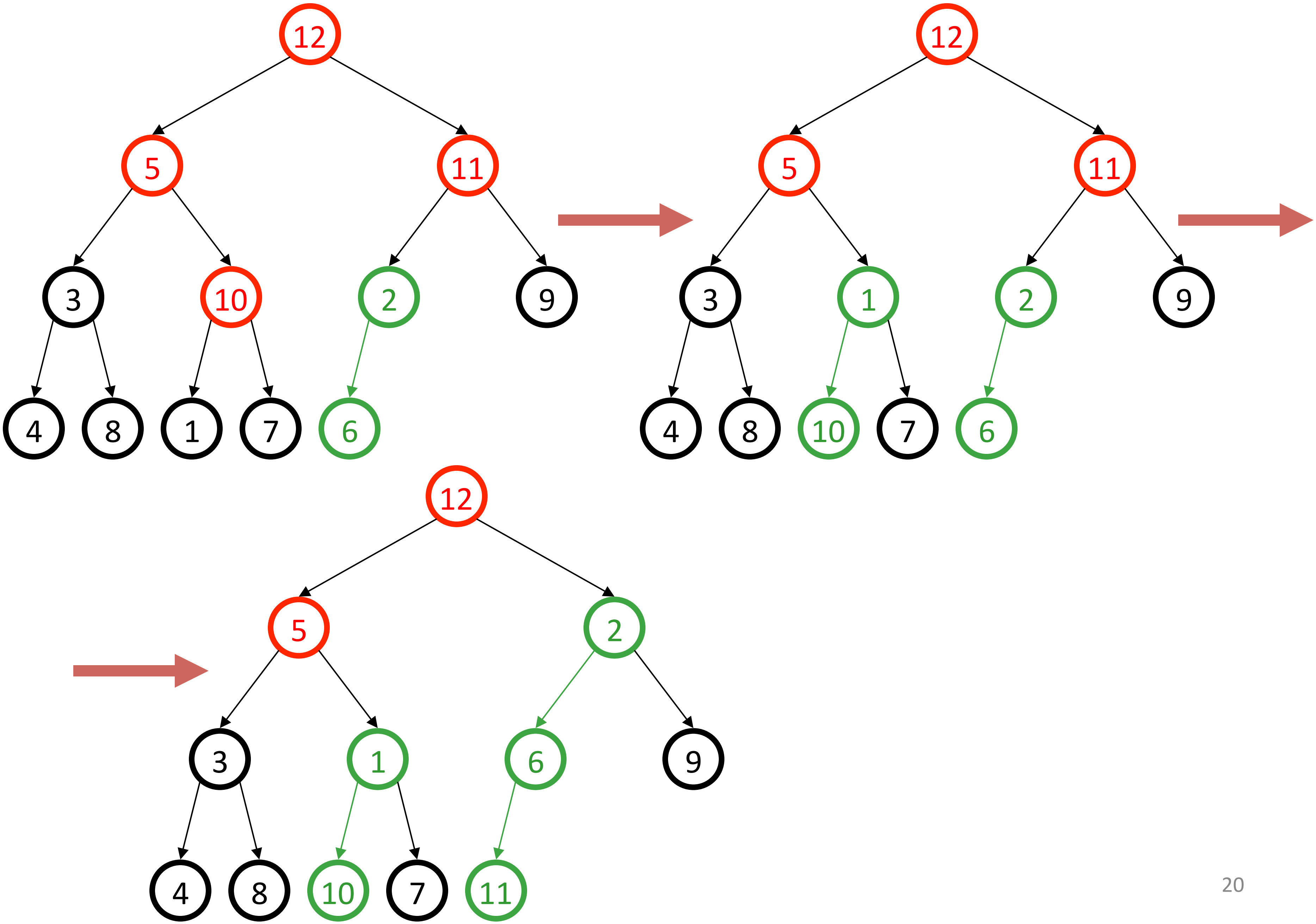
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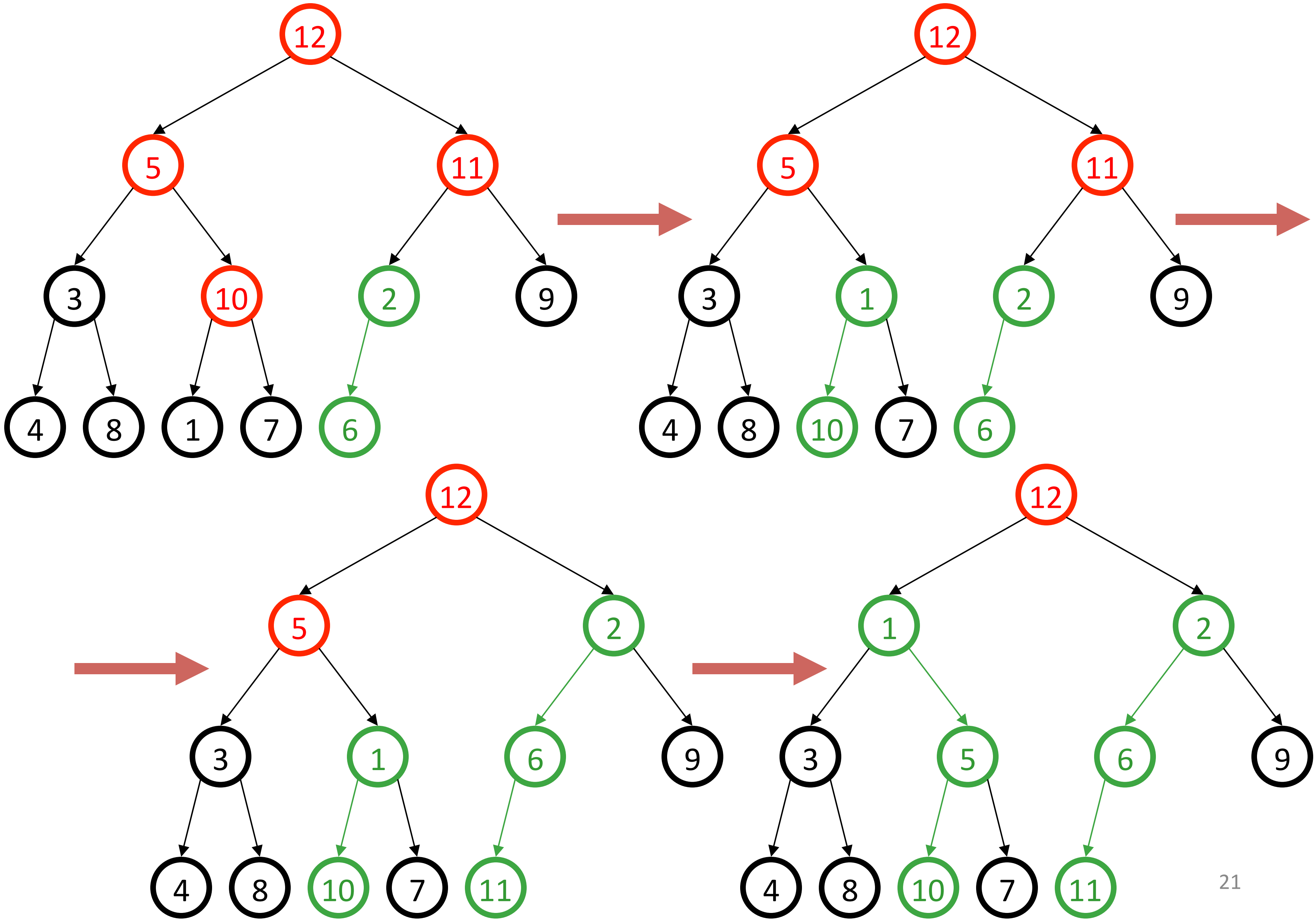
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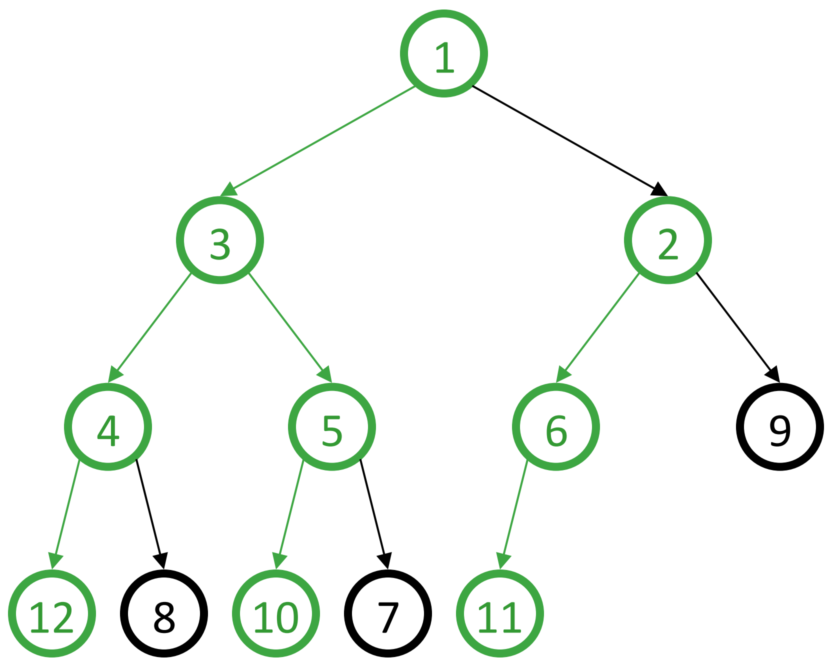
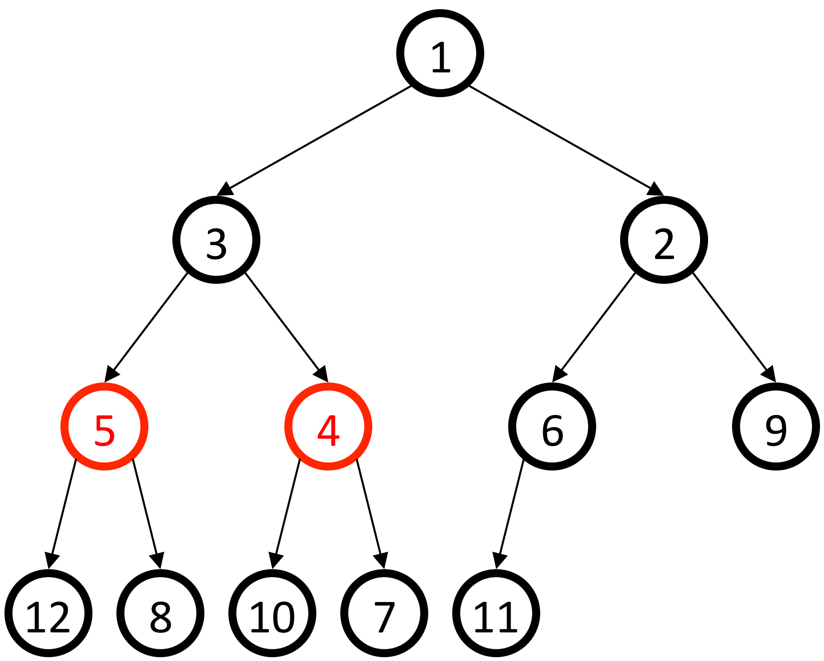
Final State of the Heap

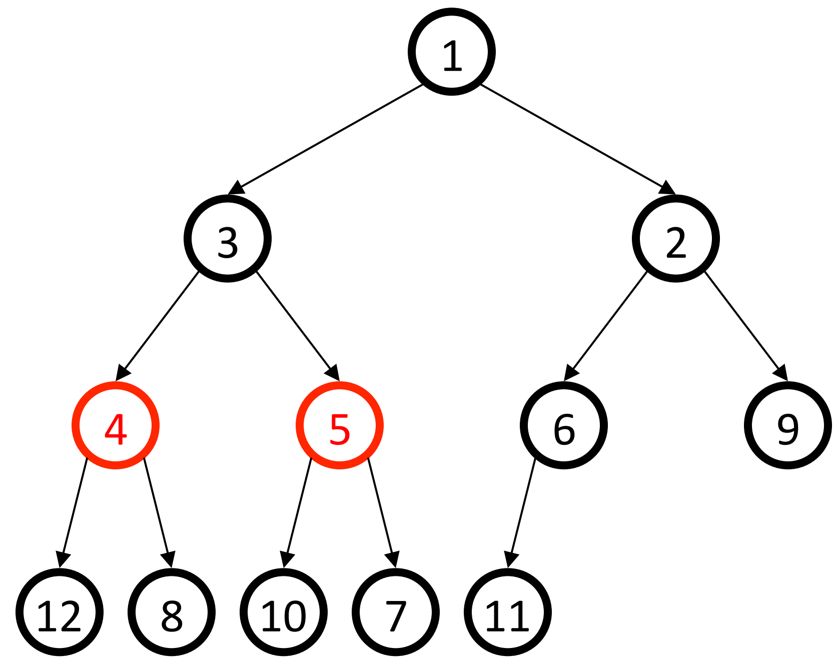


Different Heaps

Successive inserts Θ(*n* log *n*):



buildHeap Θ(*n*):

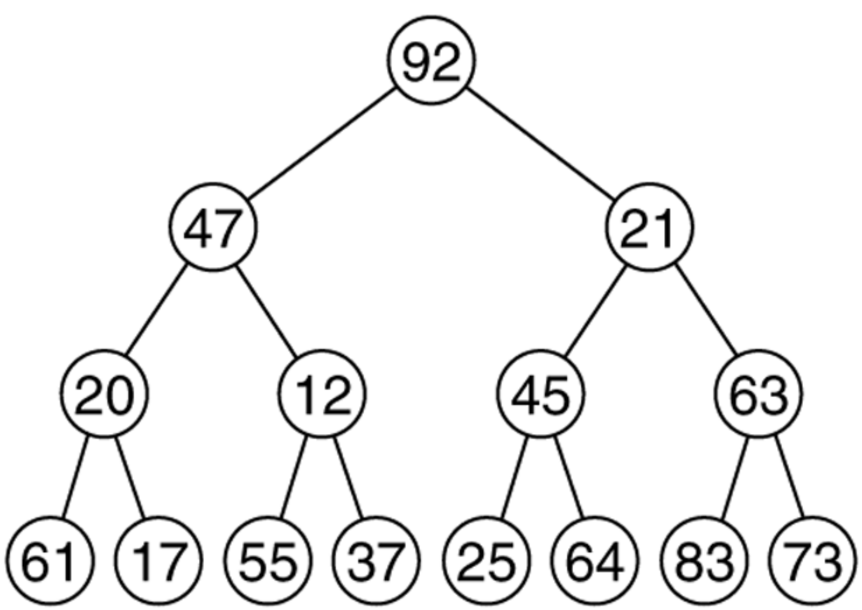


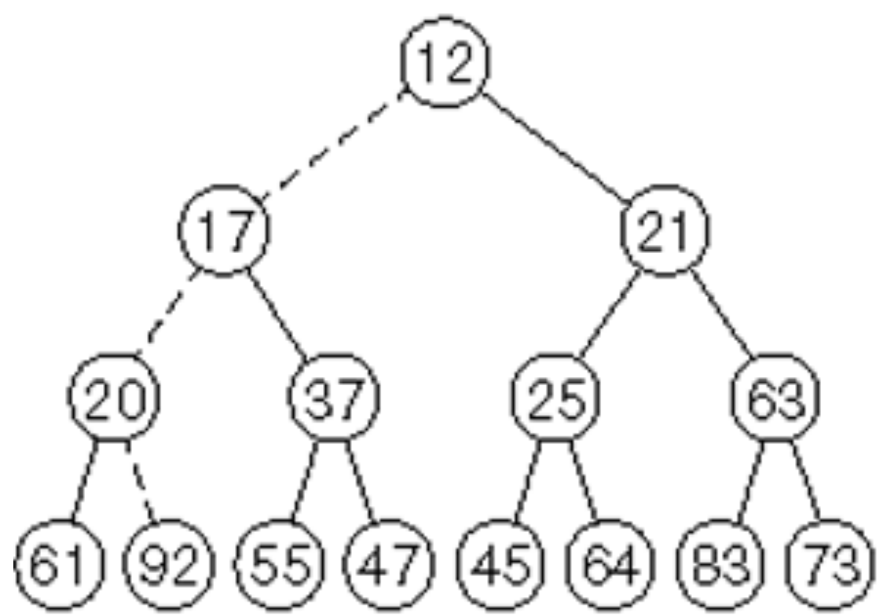
But it doesn't matter because they are both heaps.

Heap Sort

Heap sort

- **heap sort**: an algorithm to sort an array of N elements by turning the array into a heap, then doing a `remove` N times
  - the elements will come out in sorted order!
  - we can put them into a new sorted array



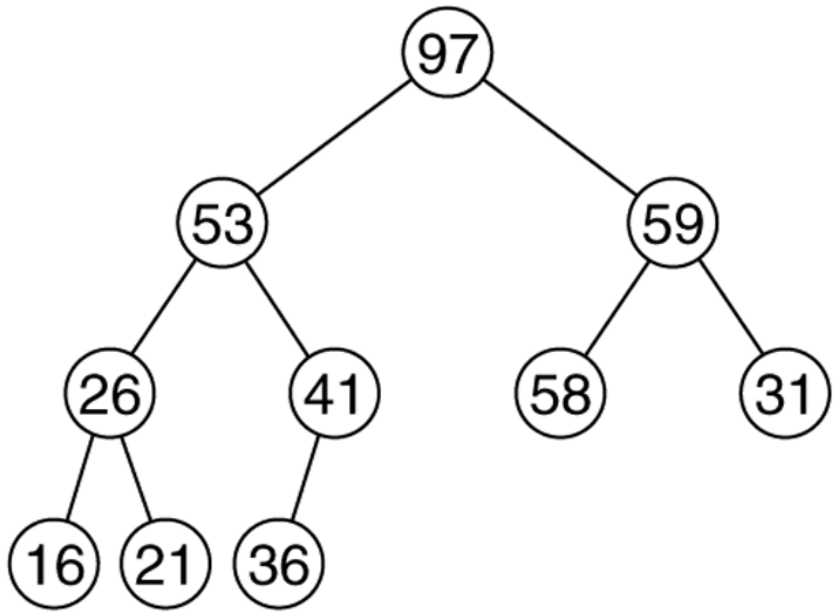


Improved heap sort

- the heap sort shown requires a second array
- we can use a max-heap to implement an improved version of heap sort that needs no extra storage
  - O(*n* log *n*) runtime
  - no external storage required!
  - useful on low-memory devices
  - elegant

Improved heap sort 1

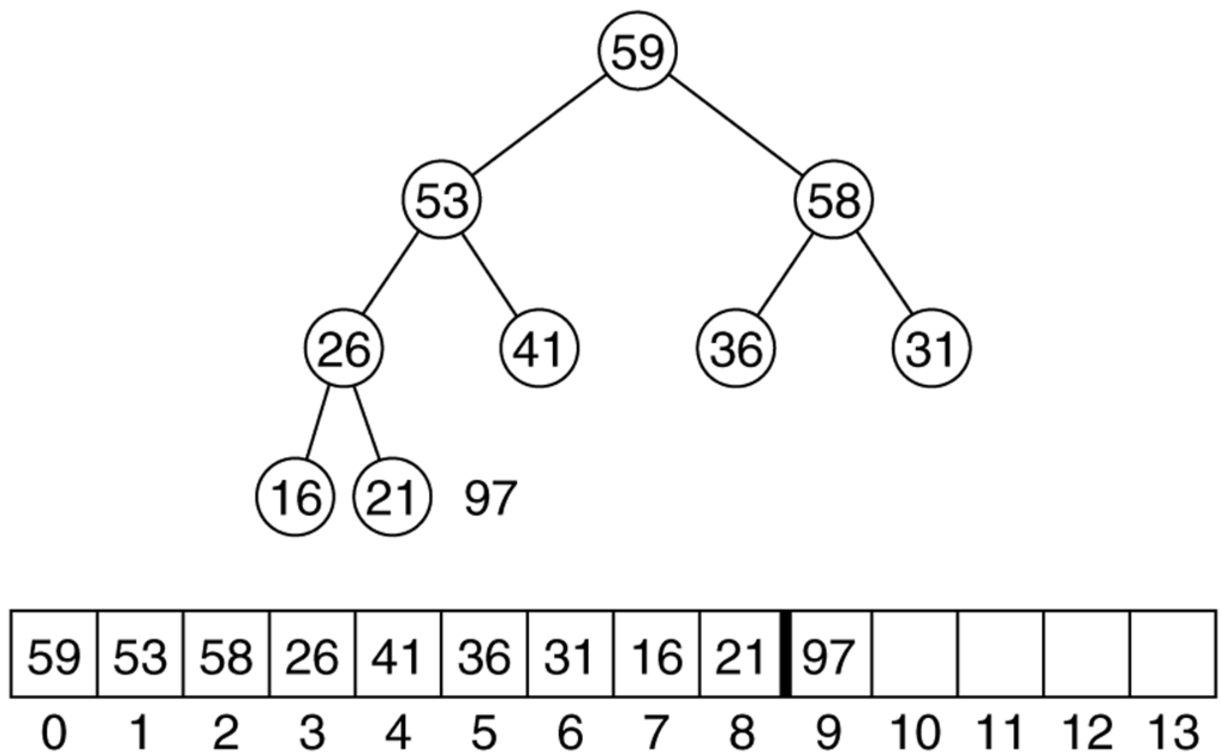
- use an array heap, but with 0 as the root index
- max-heap state after `buildHeap` operation:



97	53	59	26	41	58	31	16	21	36				
0	1	2	3	4	5	6	7	8	9	10	11	12	13

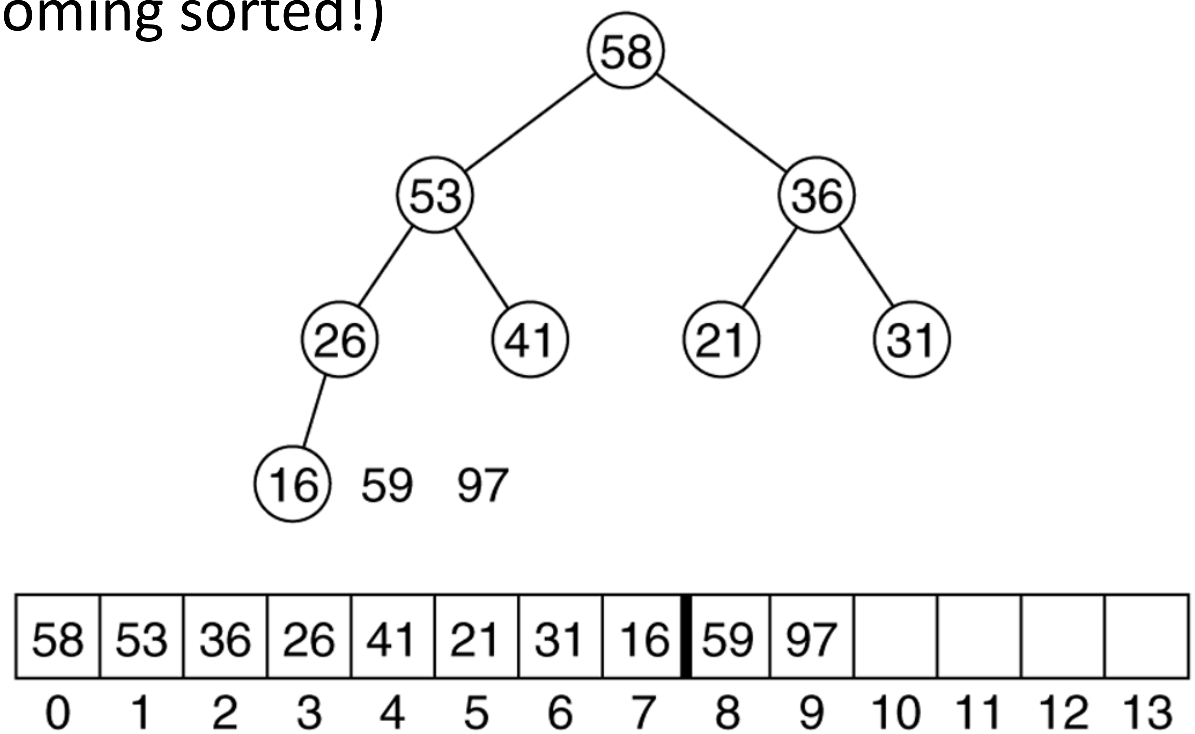
Improved heap sort 2

- state after one `remove` operation:
  - modified `remove` that moves element to end



Improved heap sort 3

- state after two `remove` operations:
  - notice that the largest elements are at the end (becoming sorted!)



Sorting algorithms review

	Best case	*Average* case (†)	*Worst* case
Selection sort	n^2	n^2	n^2
Bubble sort	n	n^2	n^2
Insertion sort	n	n^2	n^2
Mergesort	$n \log_2 n$	$n \log_2 n$	$n \log_2 n$
Heapsort	$n \log_2 n$	$n \log_2 n$	$n \log_2 n$
Treesort	$n \log_2 n$	$n \log_2 n$	n^2
Quicksort	$n \log_2 n$	$n \log_2 n$	n^2

† According to Knuth, the **average growth rate** of Insertion sort is about 0.9 times that of Selection sort and about 0.4 times that of Bubble Sort. Also, the **average growth rate** of Quicksort is about 0.74 times that of Mergesort and about 0.5 times that of Heapsort.