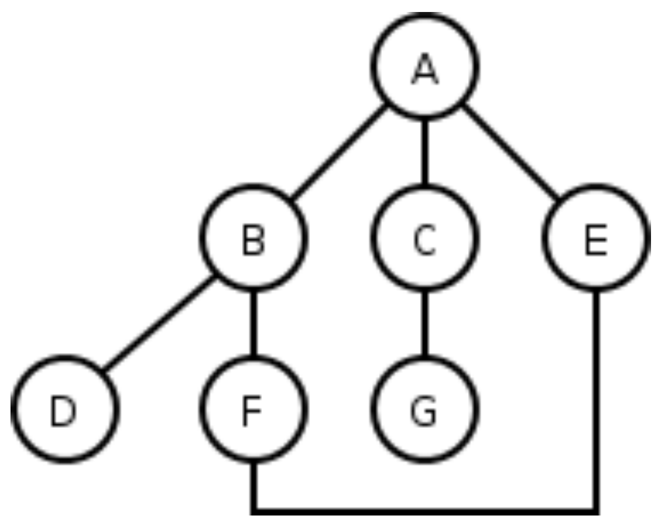
CSE 373: Data Structures and Algorithms

Lecture 21: Graphs III

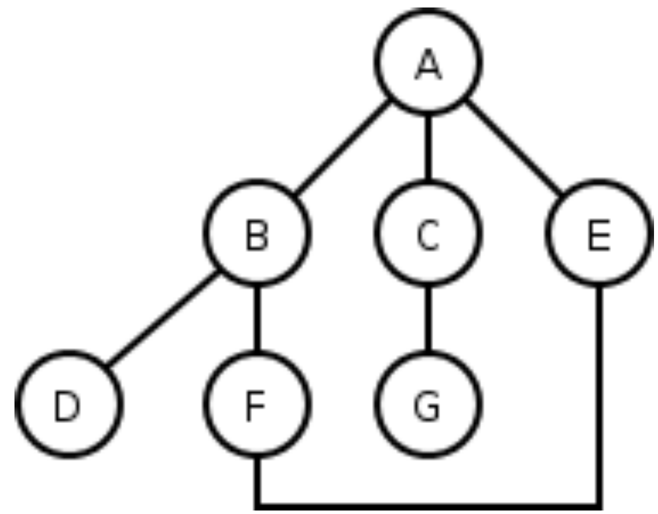
Depth-first search

- **depth-first search (DFS)**: finds a path between two vertices by exploring each possible path as many steps as possible before backtracking
  - often implemented recursively



DFS example

- All DFS paths from A to others (assumes ABC edge order)
  - A
  - A -> B
  - A -> B -> D
  - A -> B -> F
  - A -> B -> F -> E
  - A -> C
  - A -> C -> G



- What are the paths that DFS did not find?

DFS pseudocode

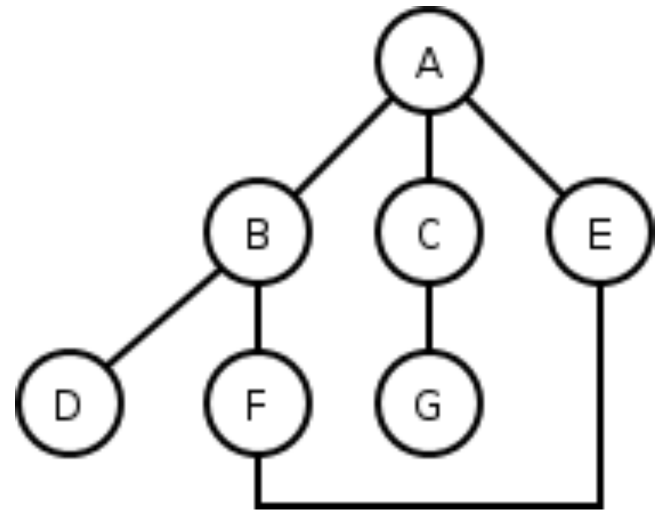
- Pseudo-code for depth-first search:

dfs(v1, v2):
    dfs(v1, v2, {})

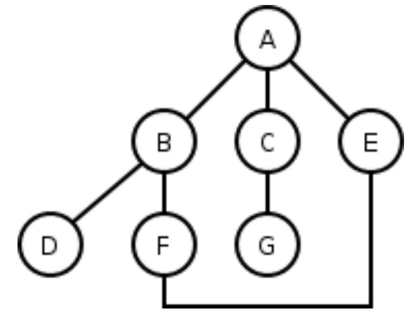
dfs(v1, v2, path):
    path += v1.
    mark v1 as visited.
    if v1 is v2:
        path is found.

    for each unvisited neighbor v_i of v1
    where there is an edge from v1 to v_i:
        if dfs(v_i, v2, path) finds a path, path is found.

    path -= v1.   path is not found.



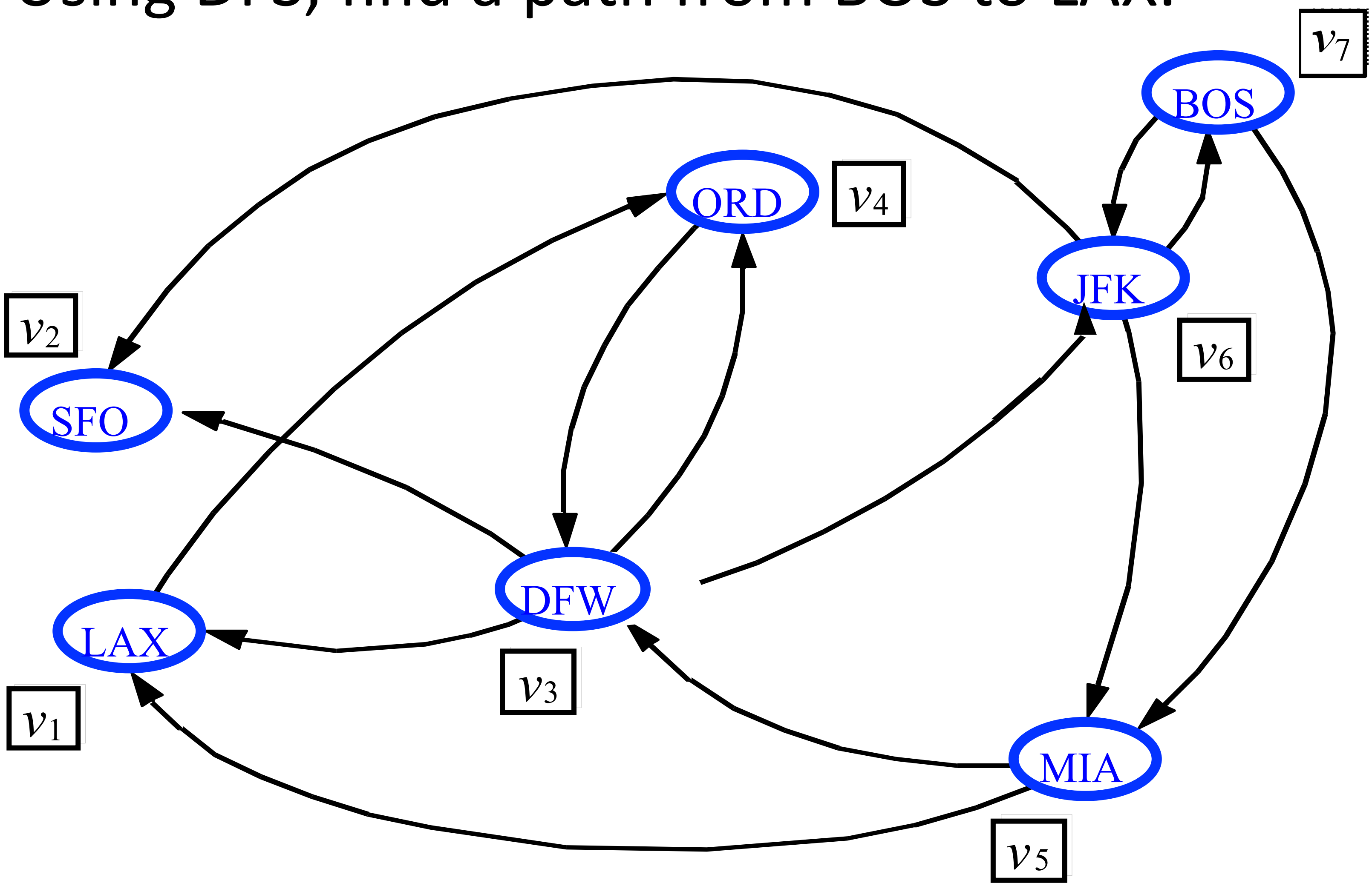
DFS observations



- guaranteed to find a path if one exists
- easy to retrieve exactly what the path is (to remember the sequence of edges taken) if we find it
- *optimality*: not optimal. DFS is guaranteed to find a path, not necessarily the best/shortest path
  - Example: DFS(A, E) may return A -> B -> F -> E

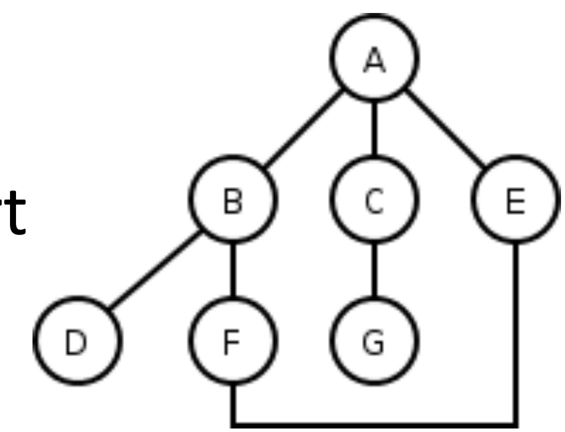
Another DFS example

- Using DFS, find a path from BOS to LAX.



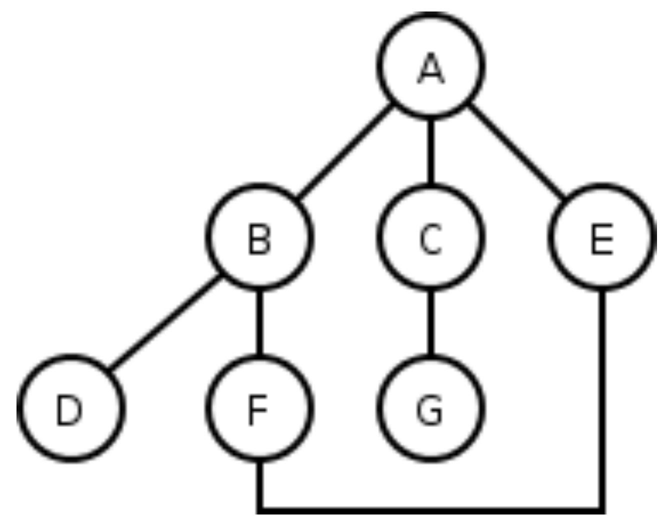
Breadth-first search

- **breadth-first search (BFS)**: finds a path between two nodes by taking one step down all paths and then immediately backtracking
  - often implemented by maintaining a list or queue of vertices to visit
  - BFS always returns the path with the fewest edges between the start and the goal vertices



BFS example

- All BFS paths from A to others (assumes ABC edge order)
  - A
  - A -> B
  - A -> C
  - A -> E
  - A -> B -> D
  - A -> B -> F
  - A -> C -> G



- What are the paths that BFS did not find?

BFS pseudocode

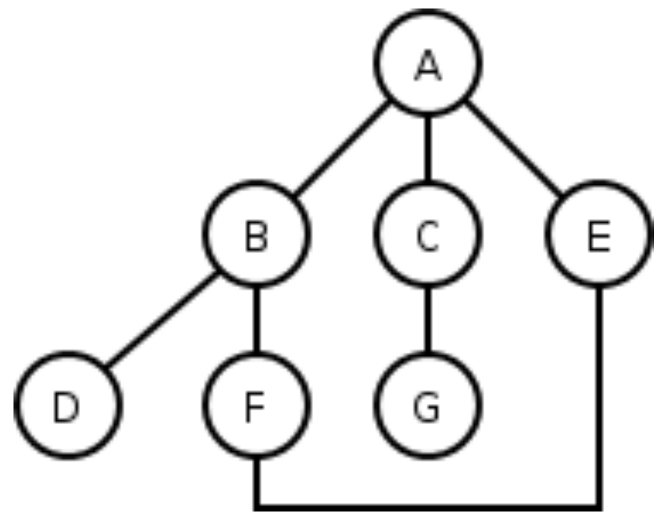
- Pseudo-code for breadth-first search:

```
bfs(v1, v2):
   List := {v1}.
   mark v1 as visited.

   while List not empty:
      v := List.removeFirst().
      if v is v2:
         path is found.

      for each unvisited neighbor v_i of v
      where there is an edge from v to v_i:
         mark v_i as visited
         List.addLast(v_i).

   path is not found.
```



BFS observations

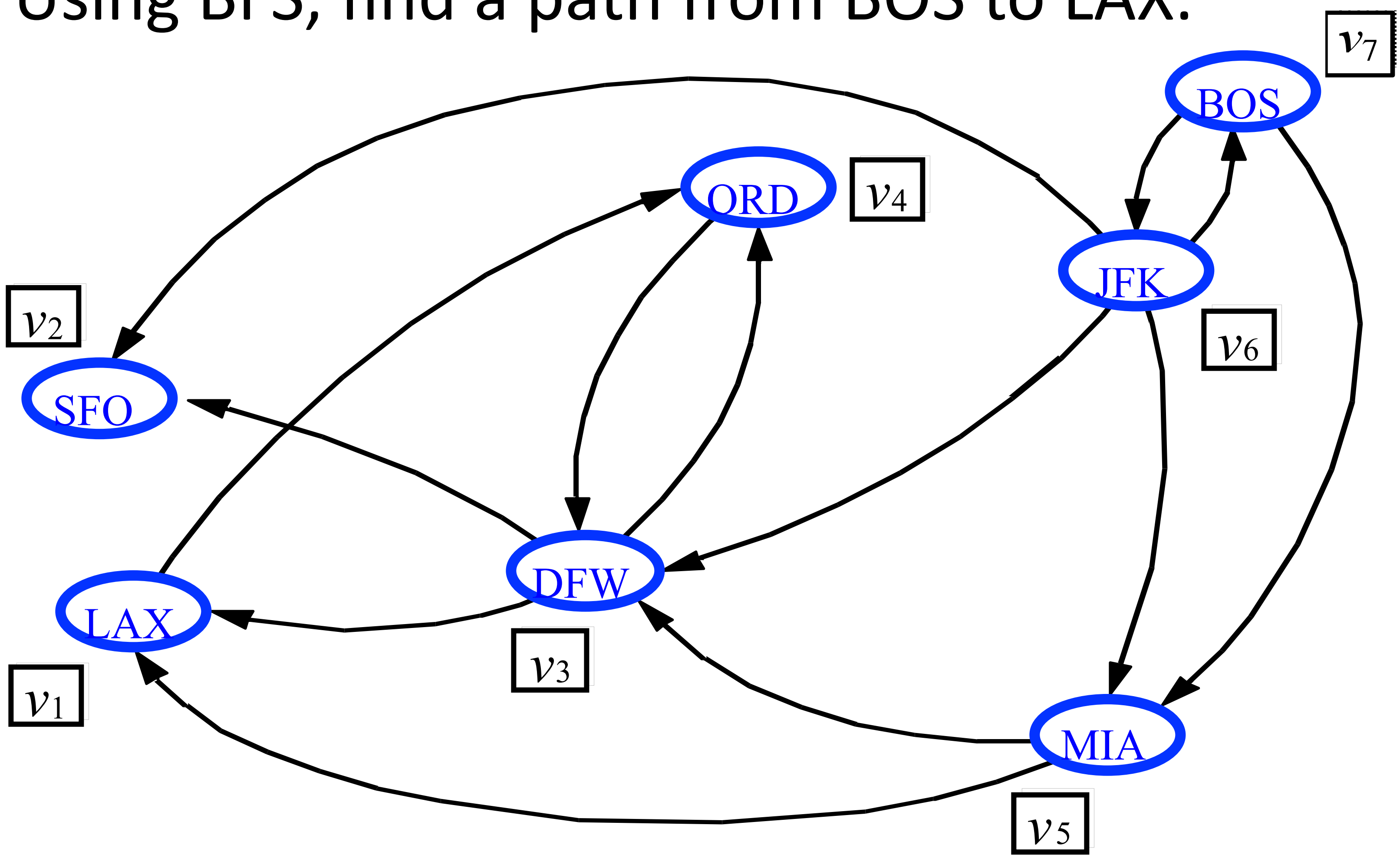
- *optimality*:
  - in unweighted graphs, optimal. (fewest edges = best)
  - In weighted graphs, not optimal.
    (path with fewest edges might not have the lowest weight)

- *disadvantage*: harder to reconstruct what the actual path is once you find it
  - conceptually, BFS is exploring many possible paths in parallel, so it's not easy to store a Path array/list in progress

- observation: any particular vertex is only part of one partial path at a time
  - We can keep track of the path by storing *predecessors* for each vertex (references to the previous vertex in that path)

Another BFS example

- Using BFS, find a path from BOS to LAX.



DFS, BFS runtime

- What is the expected runtime of DFS, in terms of the number of vertices V and the number of edges E ?

- What is the expected runtime of BFS, in terms of the number of vertices V and the number of edges E ?

- Answer: $O(|V| + |E|)$
  – each algorithm must potentially visit every node and/or examine every edge once.
  – why not $O(|V| * |E|)$?

- What is the space complexity of each algorithm?

VertexInfo class

```
public class VertexInfo<V> {
   public V v;
   public boolean visited;

   public VertexInfo(V v) {
      this.v = v;
      this.clear();
   }

   public void clear() {
      this.visited = false;
   }
}
```