CSE373: Data Structures & Algorithms

Lecture 6: Binary Search Trees

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Spring 2014

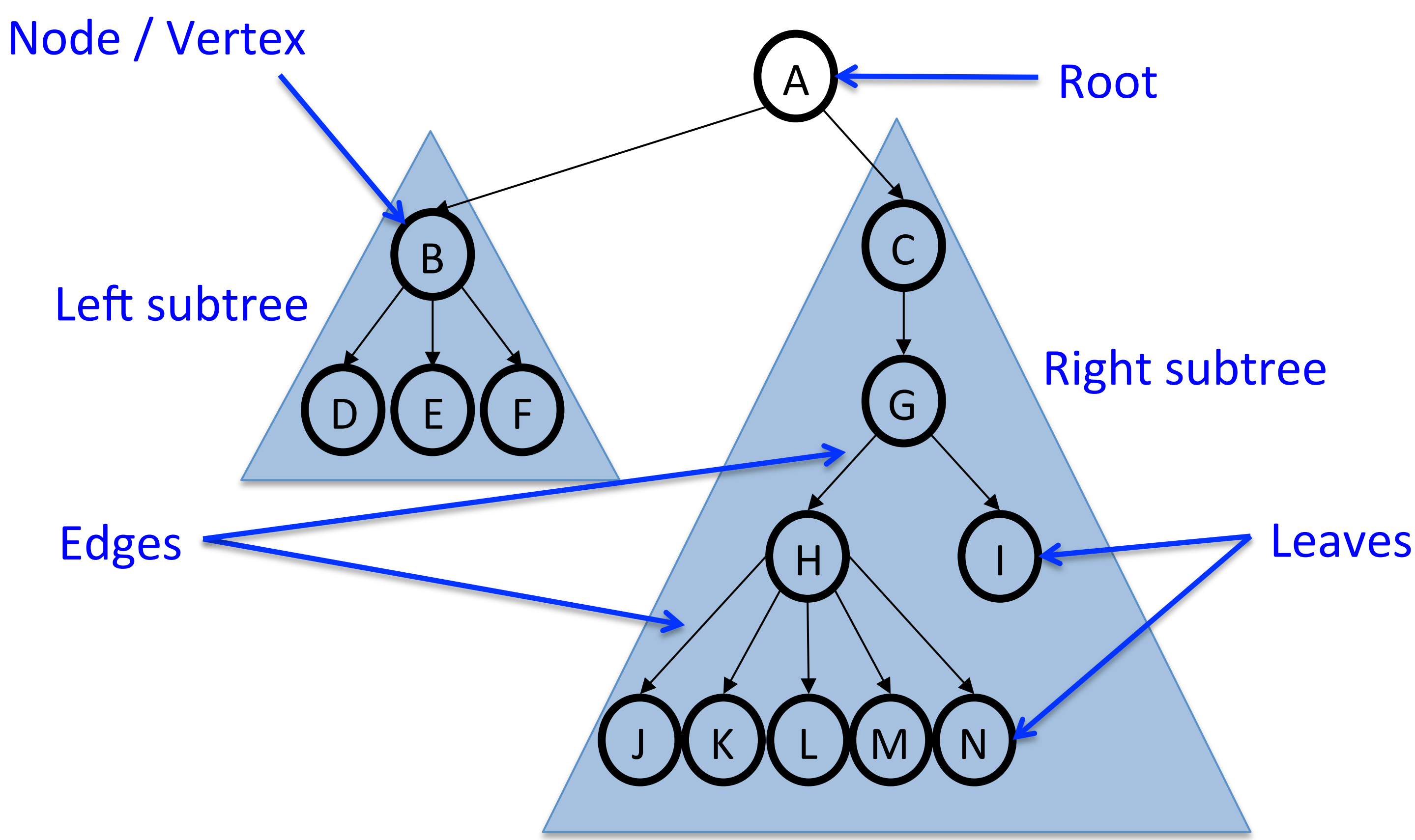
Announcements

- HW2 due start of class Wednesday 16th April
- TA sessions next week
  - Tuesday: More help with homework 2
  - Thursday: Binary Search Trees and AVL Trees

Previously on CSE 373

- Dictionary ADT
  - stores (key, value) pairs
  - **`find`**, **`insert`**, **`delete`**
- Trees
  - Terminology
  - Binary Trees

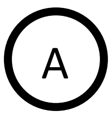
Reminder: Tree terminology



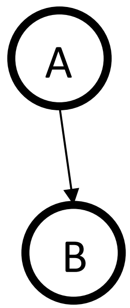
Example Tree Calculations

Recall: Height of a tree is the maximum number of edges from the root to a leaf.

What is the height of this tree?

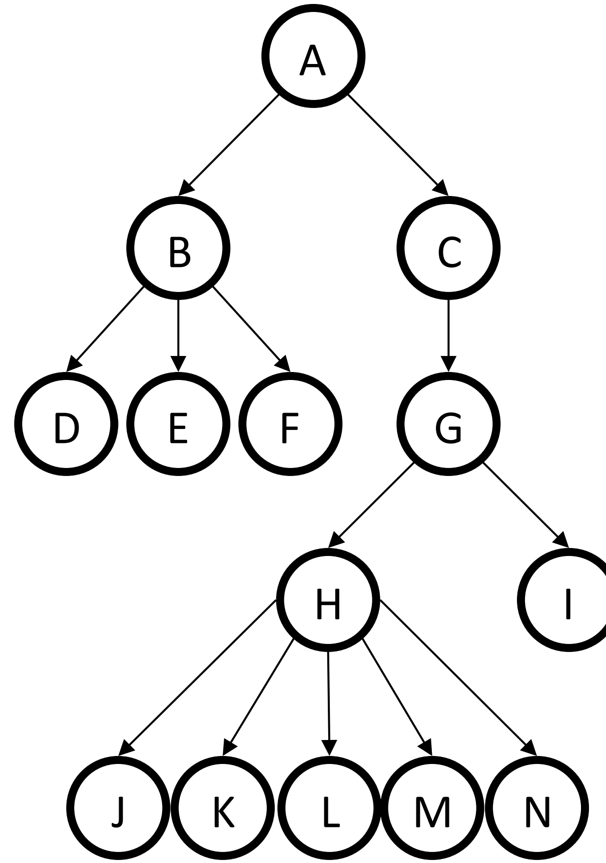


Height = 0



Height = 1

Height = 4



What is the depth of node G?

Depth = 2

What is the depth of node L?

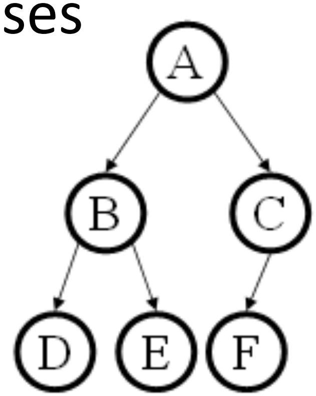
Depth = 4

Binary Trees

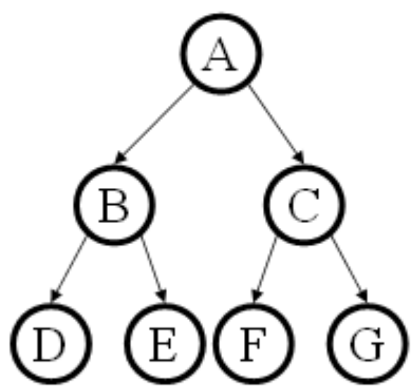
- Binary tree: Each node has at most 2 children (branching factor 2)

- Binary tree is
  - A root *(with data)*
  - A left subtree *(may be empty)*
  - A right subtree *(may be empty)*

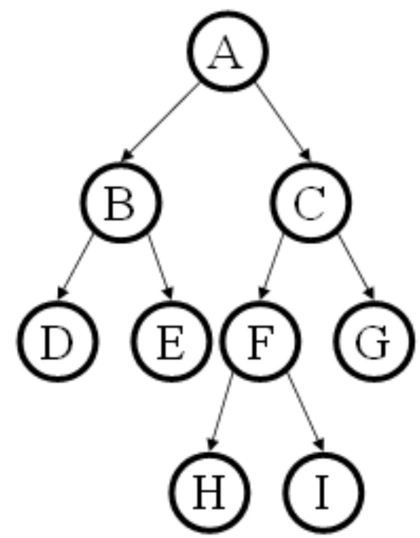
- Special Cases



Complete Tree



Perfect Tree

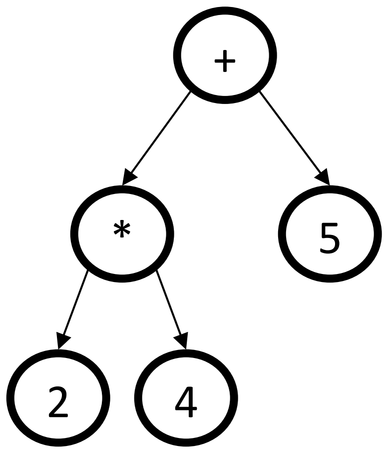


Full Tree

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

(an expression tree)



- *Pre-order*: root, left subtree, right subtree

  \+ * 2 4 5

- *In-order*: left subtree, root, right subtree

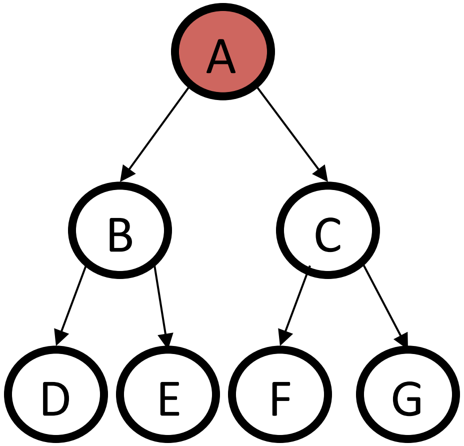
  2 * 4 + 5

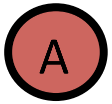
- *Post-order*: left subtree, right subtree, root

  2 4 * 5 +

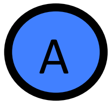
More on traversals

```
void inOrderTraversal(Node t){
  if(t != null) {
    inOrderTraversal(t.left);
    process(t.element);
    inOrderTraversal(t.right);
  }
}
```





= current node



= processing (on the call stack)

= completed node

✓ = element has been processed

More on traversals

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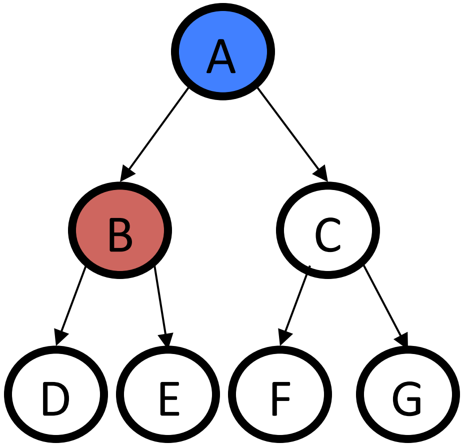

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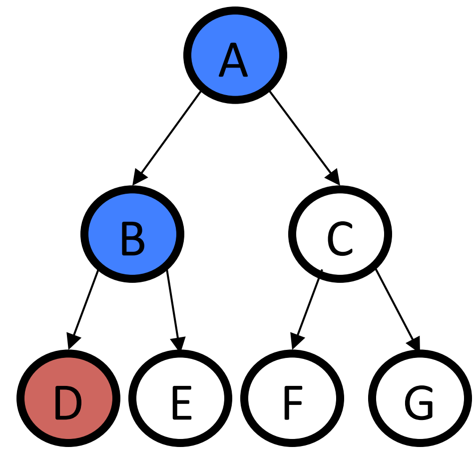

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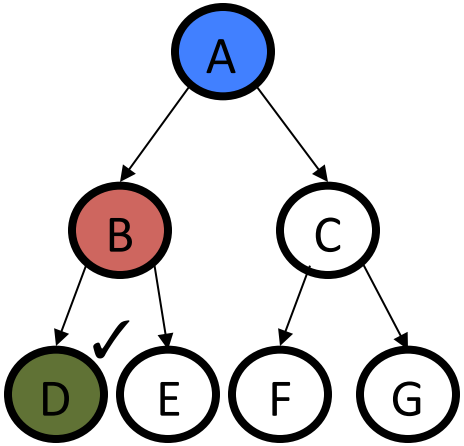

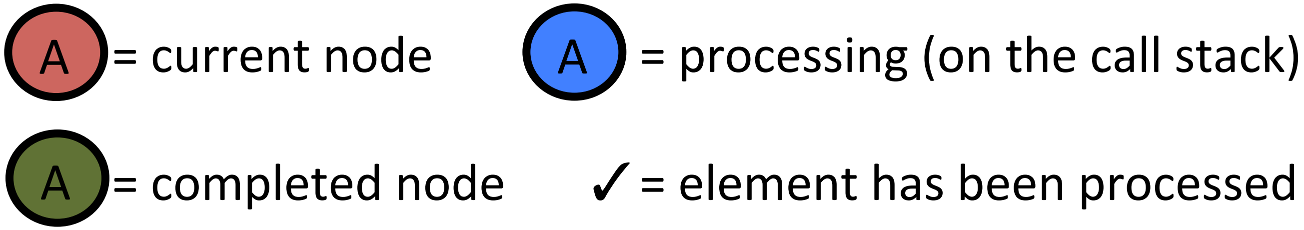

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More on traversals

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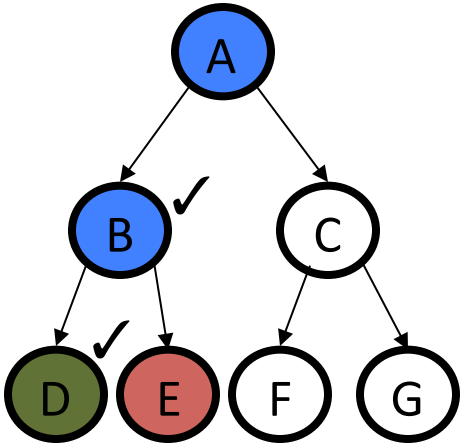

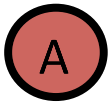

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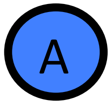

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D B

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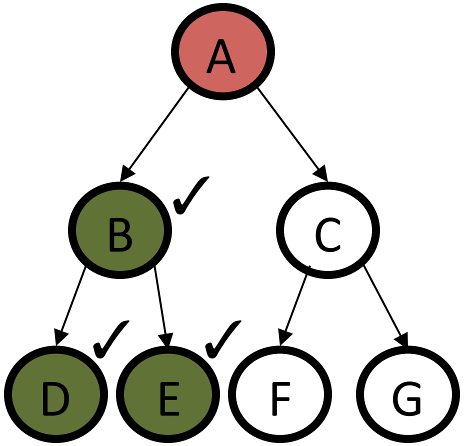

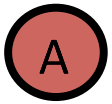

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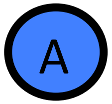

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D B E

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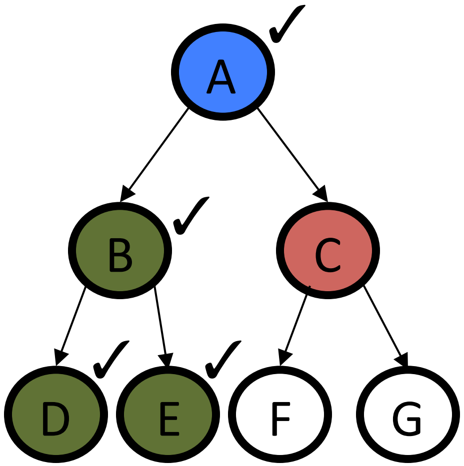

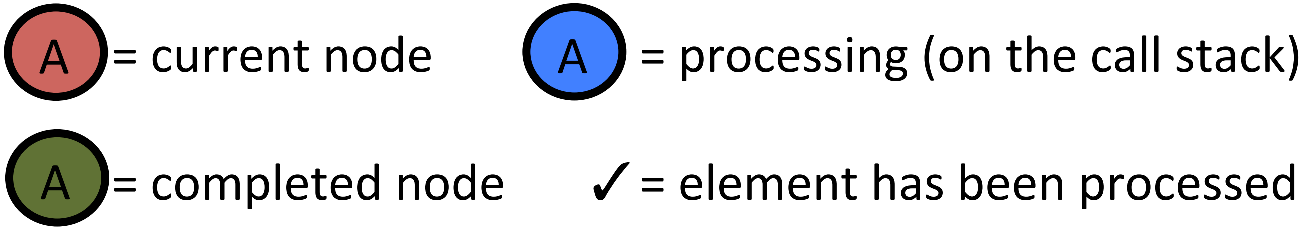

D B E A

More on traversals

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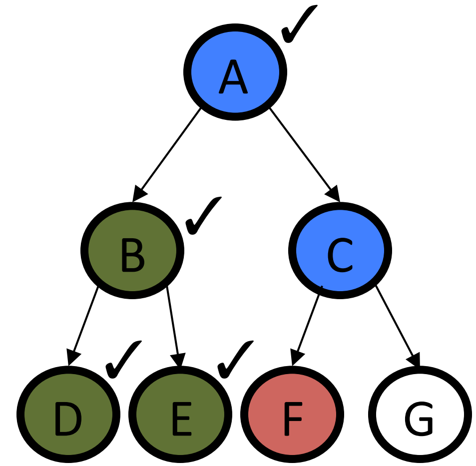

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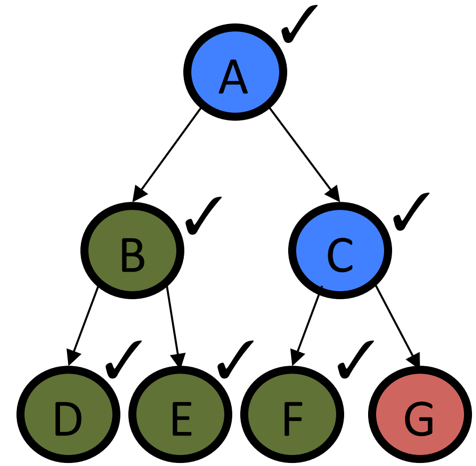

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D B E A F C

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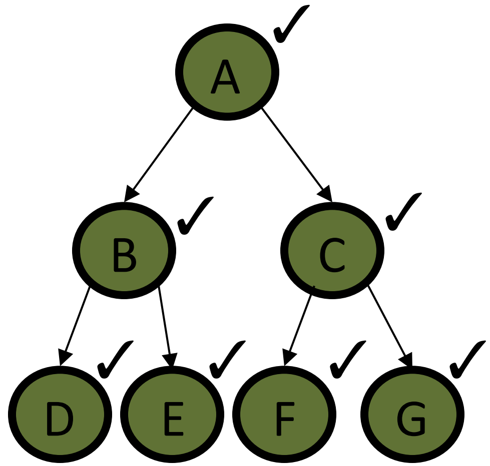

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D B E A F C G

More on traversals

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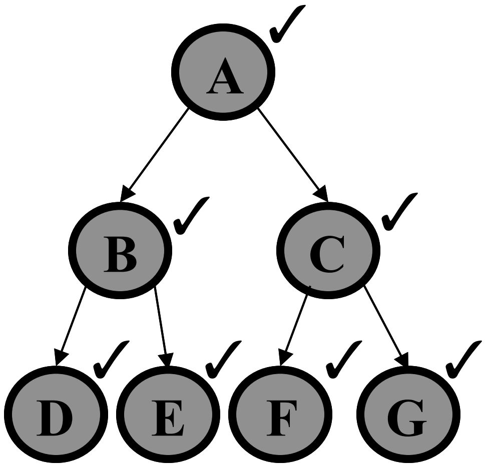


Sometimes order doesn't matter

- Example: sum all elements

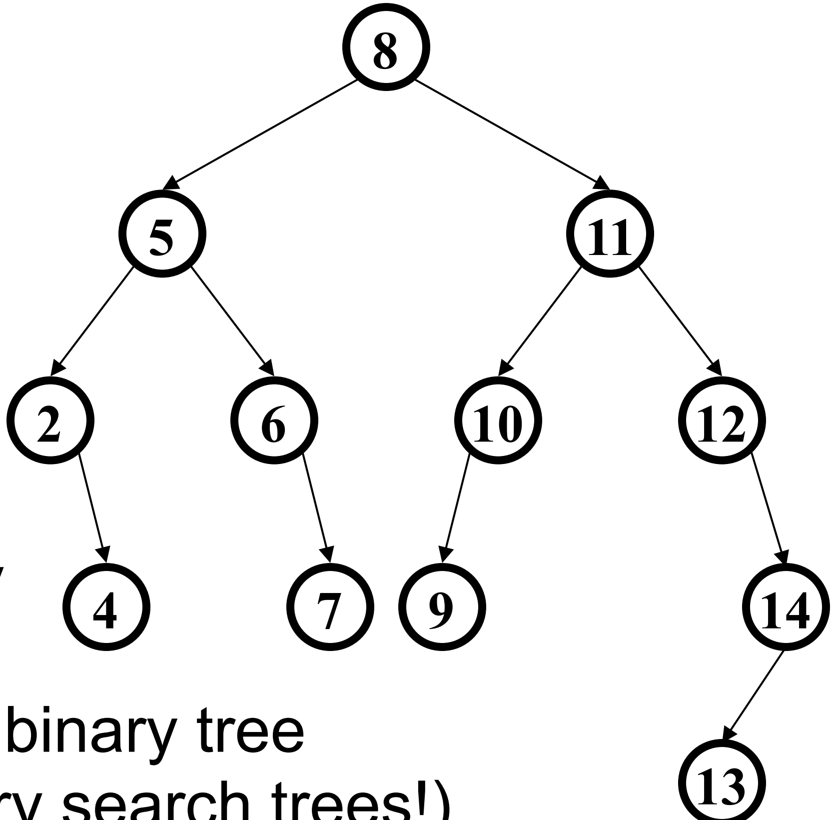
Sometimes order matters

- Example: evaluate an expression tree

Binary Search Tree (BST) Data Structure

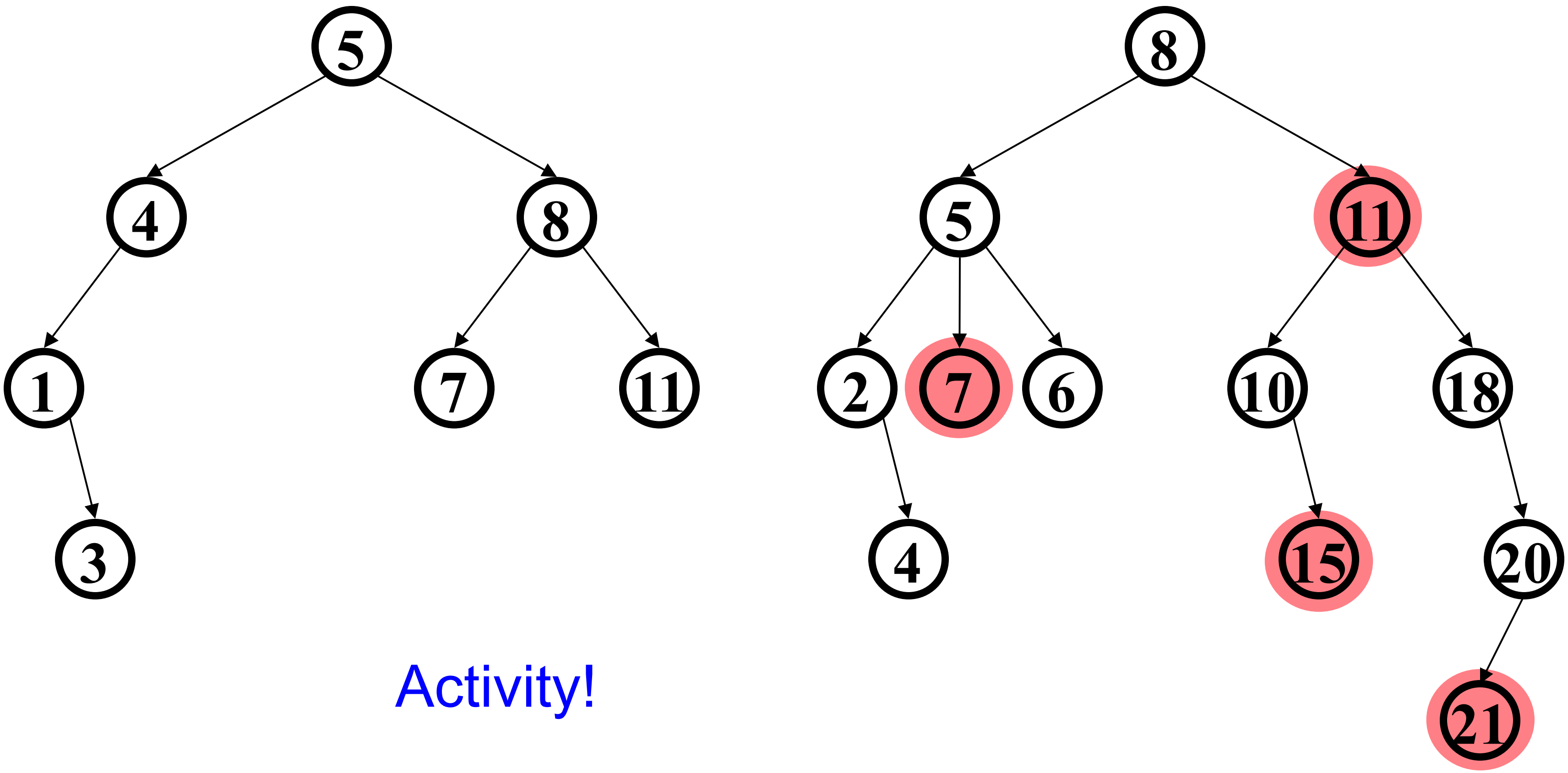
- Structure property (binary tree)
  - Each node has ≤ 2 children
  - Result: keeps operations simple

- Order property
  - All keys in left subtree smaller than node's key
  - All keys in right subtree larger than node's key
  - Result: easy to find any given key



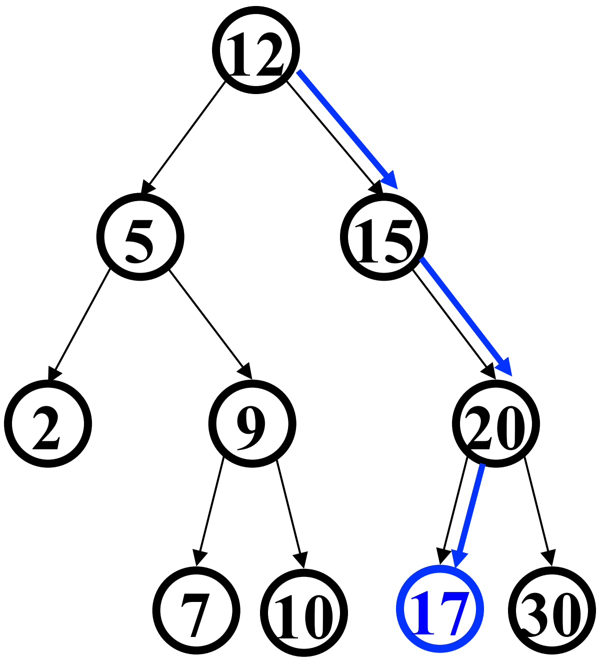
A binary search tree is a type of binary tree
(but not all binary trees are binary search trees!)

Are these BSTs?



Activity!

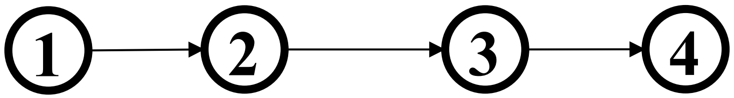
Find in BST, Recursive



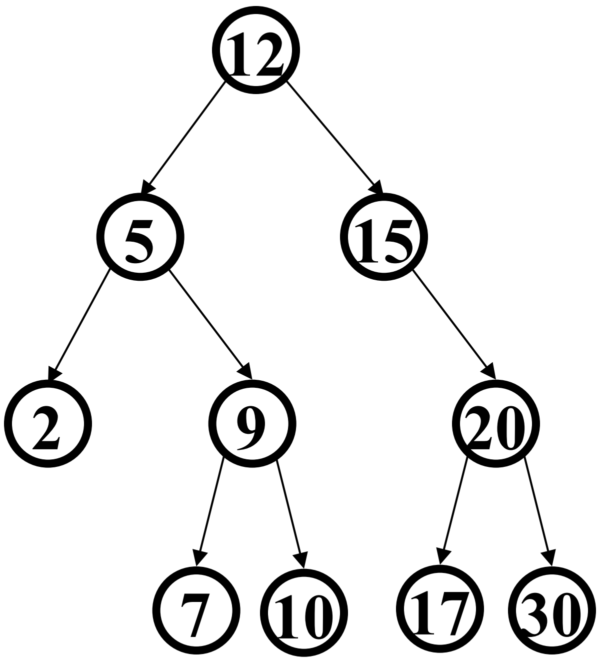

```
Data find(Key key, Node root){
 if(root == null)
   return null;
 if(key < root.key)
   return find(key,root.left);
 if(key > root.key)
   return find(key,root.right);
 return root.data;
}
```

What is the running time?

Worst case running time is O(n).
- Happens if the tree is very lopsided (e.g. list)



Find in BST, Iterative

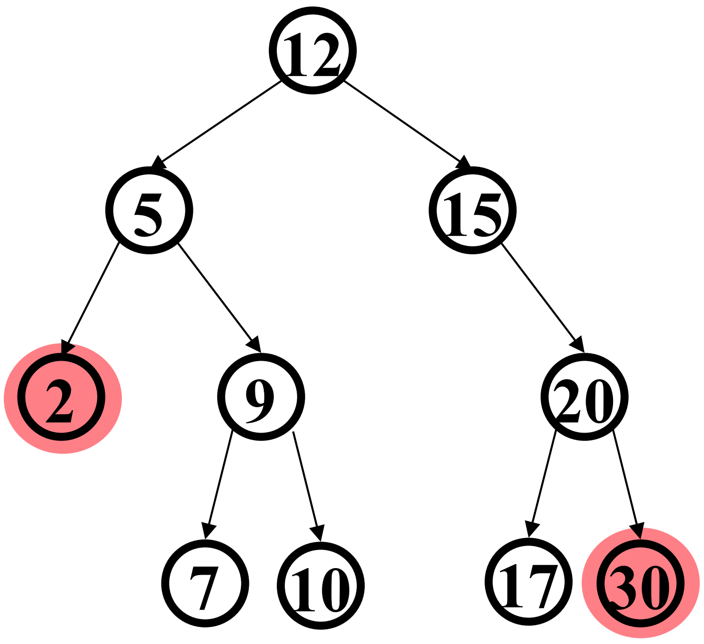



```
Data find(Key key, Node root){
 while(root != null
        && root.key != key) {
  if(key < root.key)
    root = root.left;
  else(key > root.key)
    root = root.right;
 }
 if(root == null)
    return null;
 return root.data;
}
```

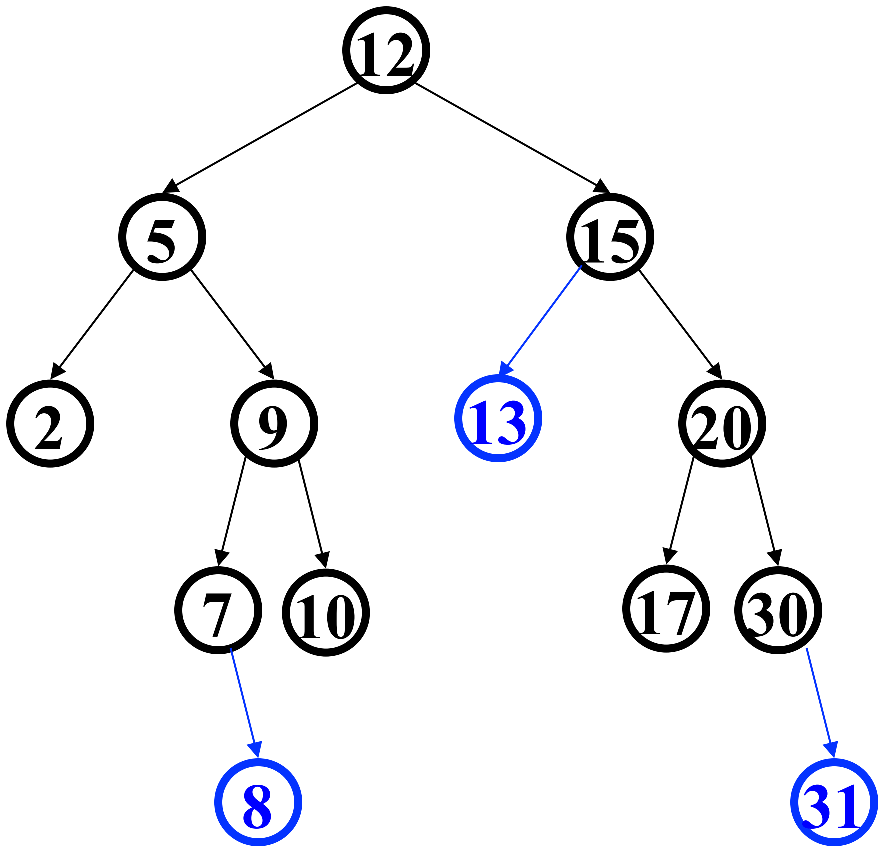
Worst case running time is O(n).
- Happens if the tree is very lopsided (e.g. list)

Bonus: Other BST "Finding" Operations

- **`FindMin`**: Find *minimum* node
  - Left-most node
- **`FindMax`**: Find *maximum* node
  - Right-most node



Insert in BST

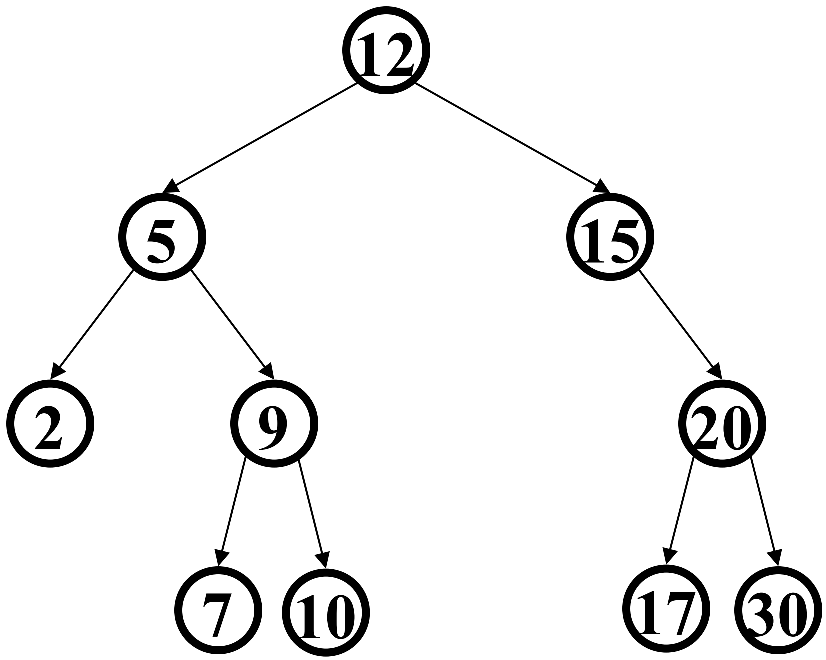



```
insert(13)
insert(8)
insert(31)
```

(New) insertions happen only at leaves – easy!

Again… worst case running time is O(n), which may happen if the tree is not balanced.

Deletion in BST



Why might deletion be harder than insertion?

Removing an item may disrupt the tree structure!

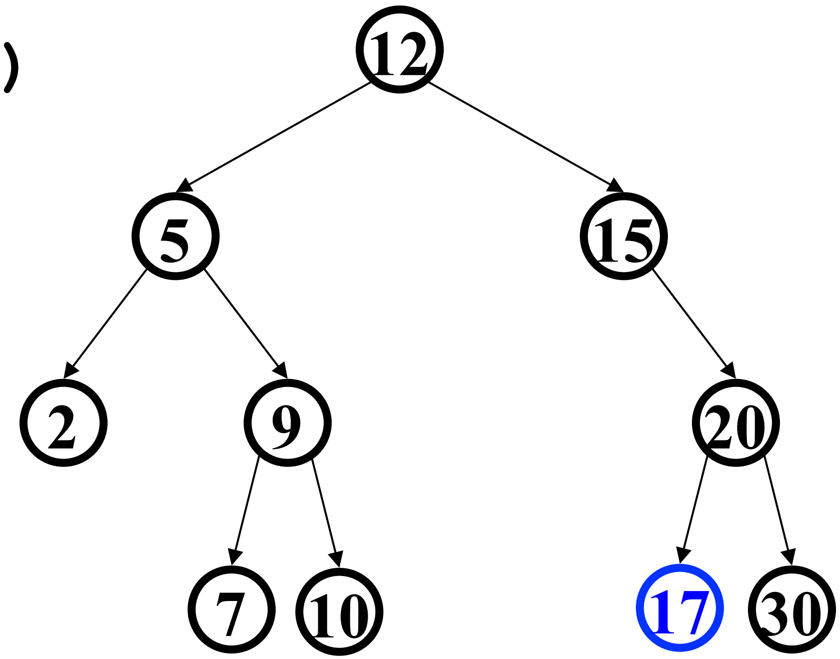
Deletion in BST

- Basic idea: `find` the node to be removed, then "fix" the tree so that it is still a binary search tree

- Three potential cases to fix:
  - Node has no children (leaf)
  - Node has one child
  - Node has two children

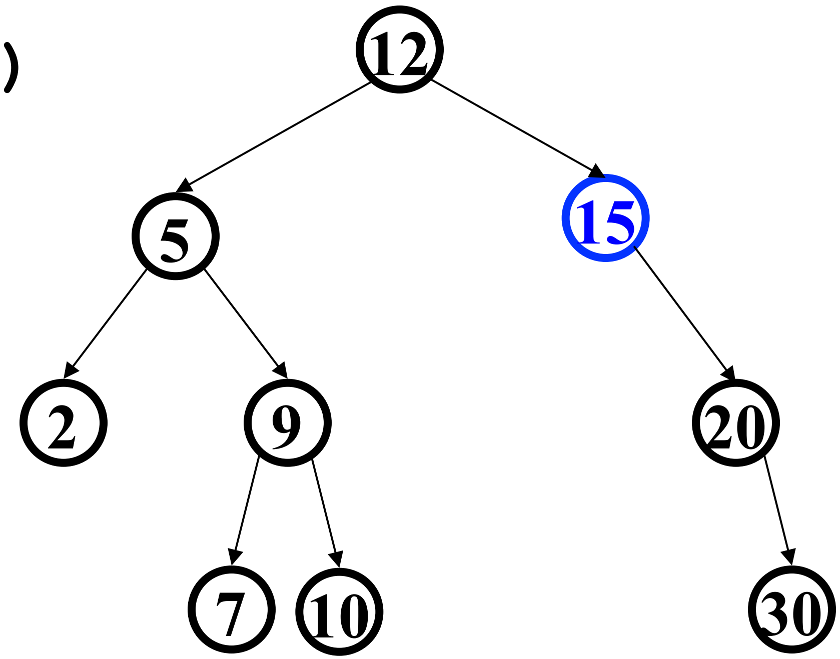
Deletion – The Leaf Case

`delete(17)`



Deletion – The One Child Case

`delete(15)`



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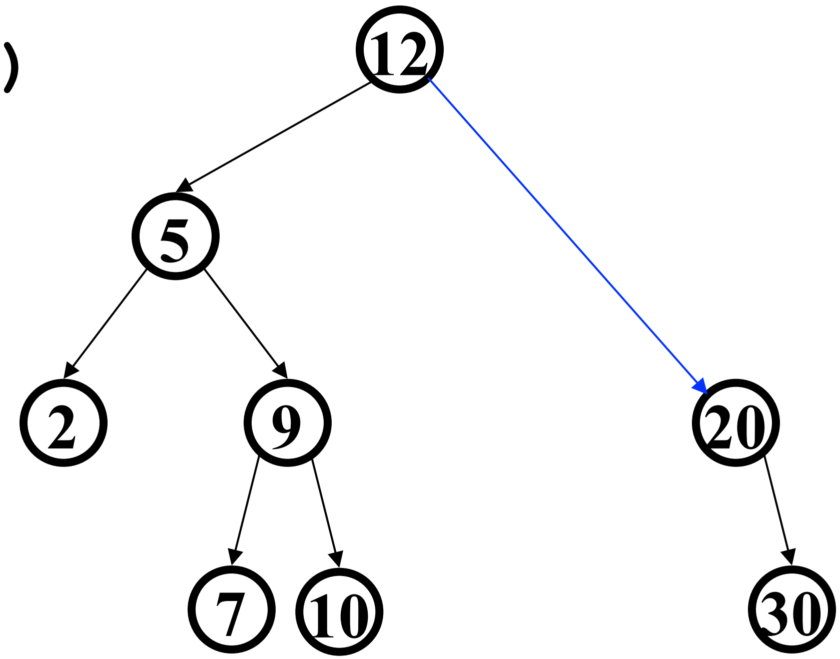
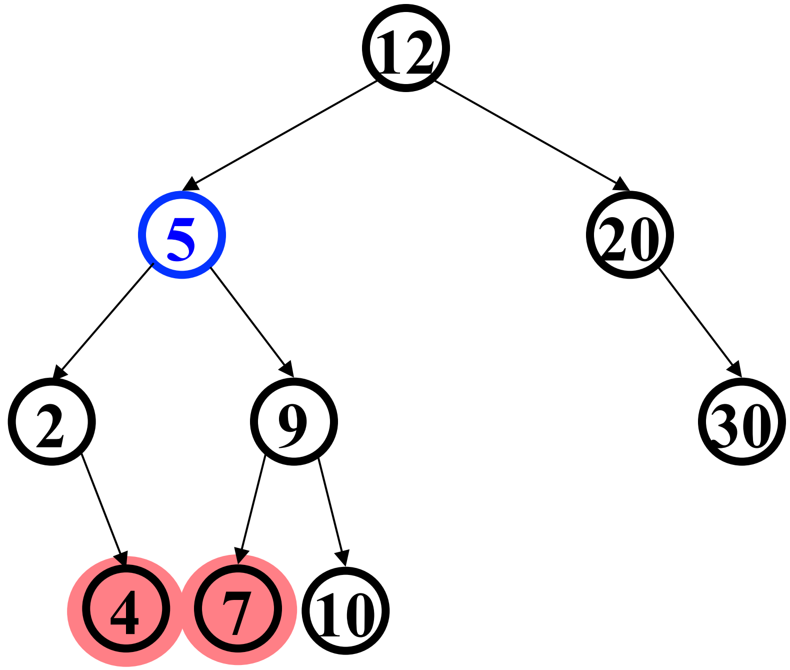


Deletion – The Two Child Case

`delete(5)`



What can we replace 5 with?

Deletion – The Two Child Case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

Options:

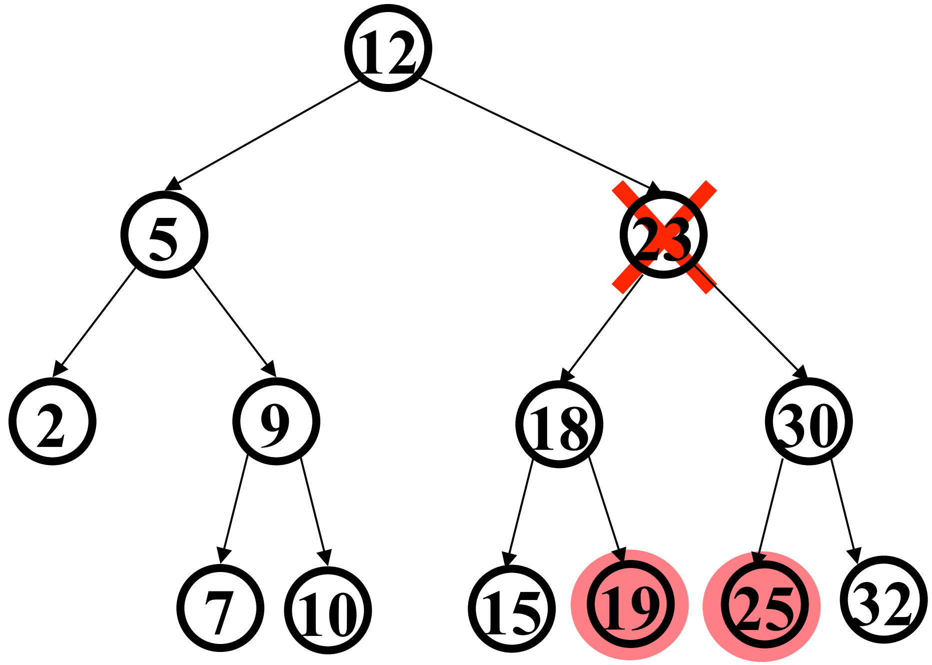
- *successor* minimum node from right subtree
  `findMin(node.right)`

- *predecessor* maximum node from left subtree
  `findMax(node.left)`

Now delete the original node containing *successor* or *predecessor*

Deletion: The Two Child Case (example)

`delete(23)`



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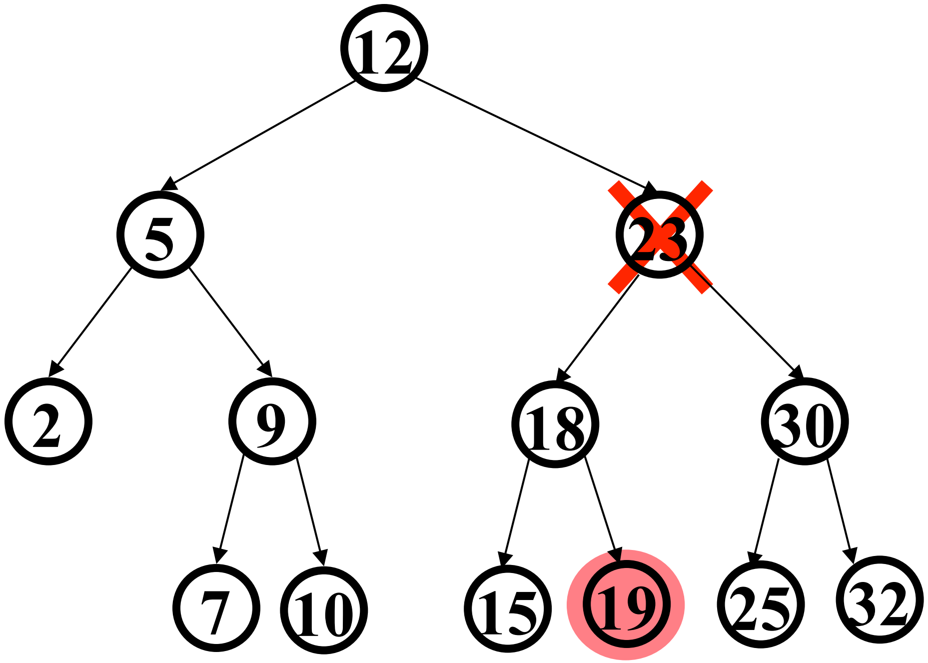

Deletion: The Two Child Case (example)

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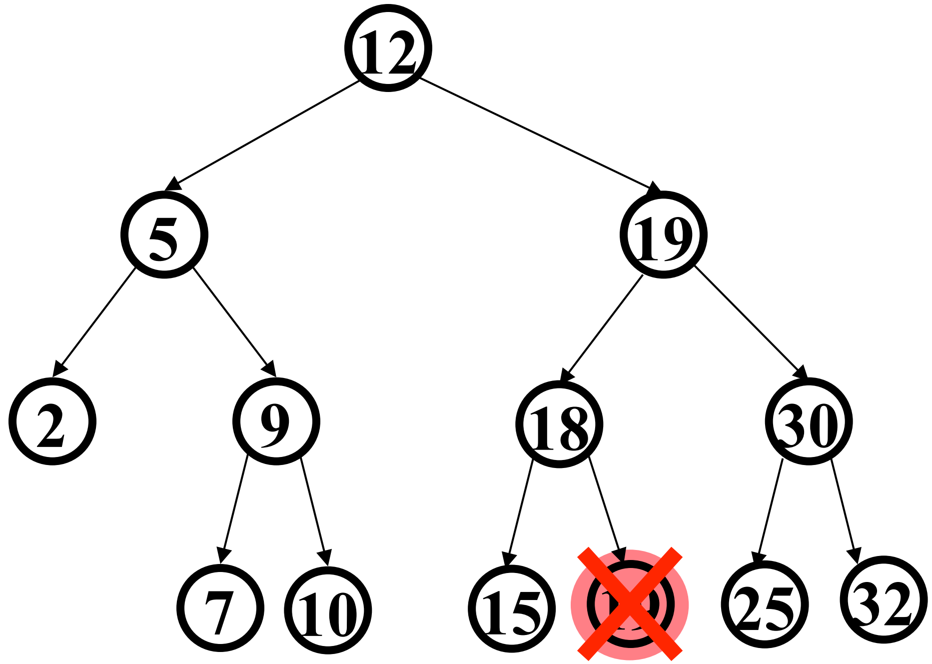

Deletion: The Two Child Case (example)

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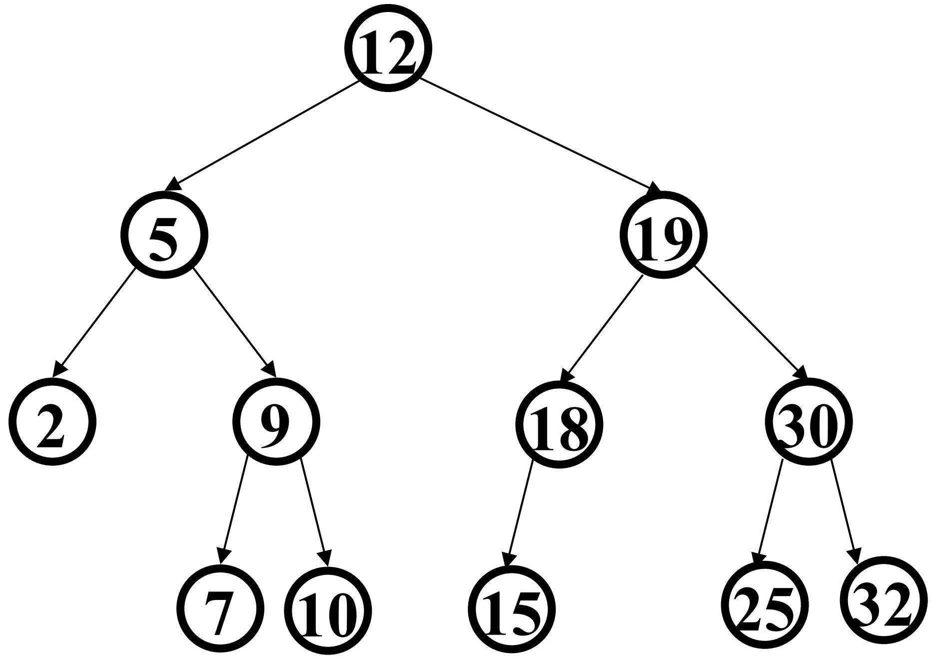


Success! ☺

Lazy Deletion

- Lazy deletion can work well for a BST
  - Simpler
  - Can do "real deletions" later as a batch
  - Some inserts can just "undelete" a tree node

- But
  - Can waste space and slow down find operations
  - Make some operations more complicated:
    - e.g., **`findMin`** and **`findMax`**?

BuildTree for BST

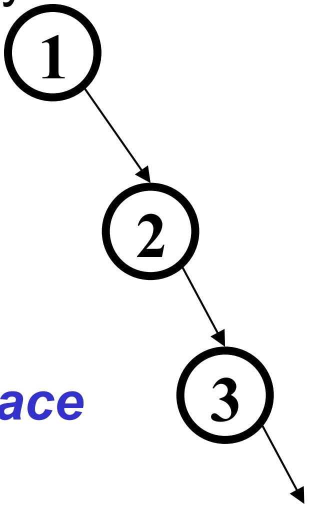
- Let's consider **`buildTree`**
  - Insert all, starting from an empty tree

- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST

  - If inserted in given order,
    what is the tree?

  - What big-O runtime for
    this kind of sorted input? ***$O(n^2)$*** ***Not a happy place***

  - Is inserting in the reverse order
    any better?



BuildTree for BST

- Insert keys 1, 2, 3, 4, 5, 6, 7, 8, 9 into an empty BST
- What we if could somehow re-arrange them
  - median first, then left median, right median, etc.
  - 5, 3, 7, 2, 1, 4, 8, 6, 9
  - What tree does that give us?
  - What big-O runtime?

O(n* log *n), definitely better