CSE373: Data Structures & Algorithms

Lecture 5: Dictionaries; Binary Search Trees

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Where we are

Studying the absolutely essential ADTs of computer science and classic data structures for implementing them

ADTs so far:

1. Stack: **`push`**, **`pop`**, **`isEmpty`**, …
2. Queue: **`enqueue`**, **`dequeue`**, **`isEmpty`**, …

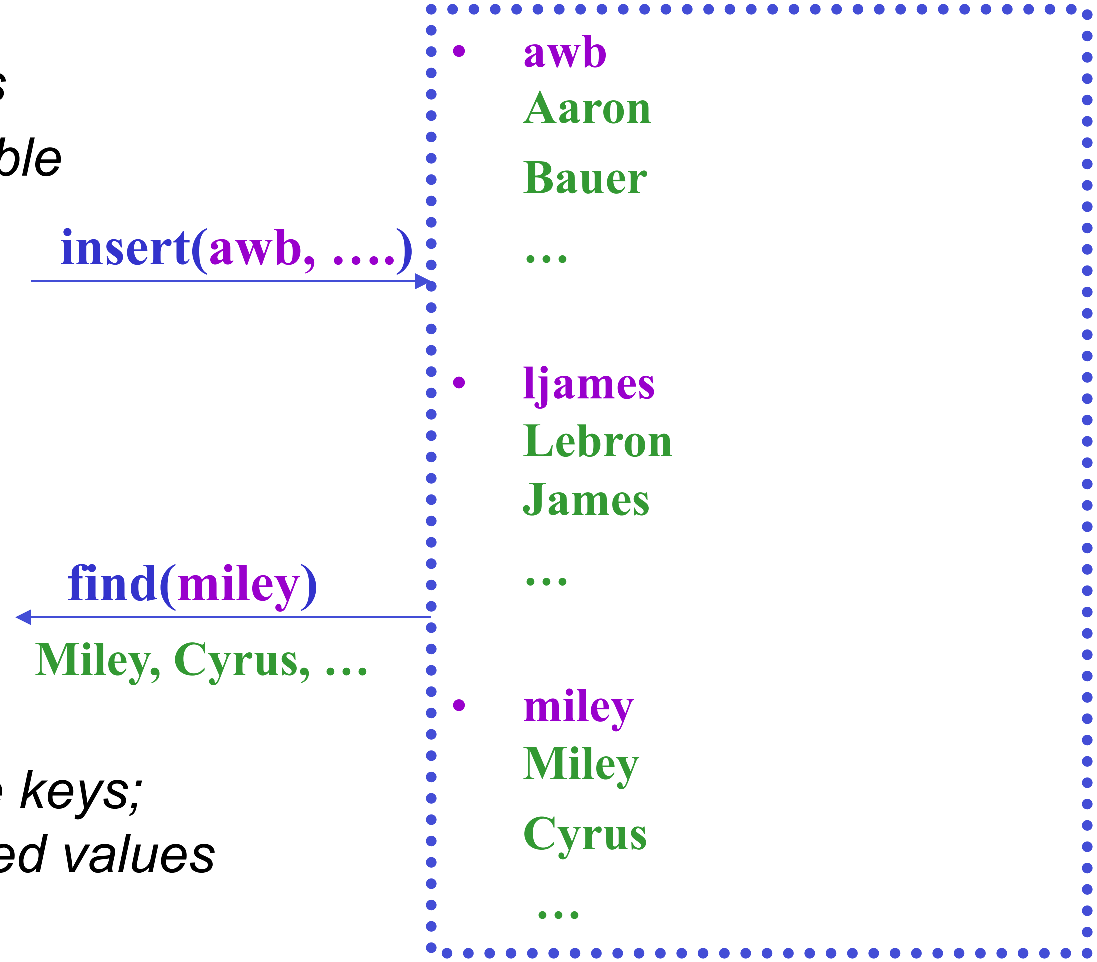
Next:

3. Dictionary (a.k.a. Map): associate keys with values
   – Extremely common

The Dictionary (a.k.a. Map) ADT

- Data:
  - *set* of (key, value) *pairs*
  - keys must be *comparable*

- Operations:
  - **`insert(key,value)`**
  - **`find(key)`**
  - **`delete(key)`**
  - …



Will tend to emphasize the keys;
don't forget about the stored values

Comparison: The Set ADT

The *Set* ADT is like a Dictionary without any values

- A key is *present* or not (no repeats)

For **`find`**, **`insert`**, **`delete`**, there is little difference

- In dictionary, values are “just along for the ride”
- So *same data-structure ideas* work for dictionaries and sets

But if your Set ADT has other important operations this may not hold

- **`union`**, **`intersection`**, **`is_subset`**
- Notice these are binary operators on sets

Dictionary data structures

There are many good data structures for (large) dictionaries

1. AVL trees
   – Binary search trees with *guaranteed balancing*

2. B-Trees
   – Also always balanced, but different and shallower
   – B!=Binary; B-Trees generally have large branching factor

3. Hashtables
   – Not tree-like at all

Skipping: Other balanced trees (e.g., red-black, splay)

But first some applications and less efficient implementations…

A Modest Few Uses

Any time you want to store information according to some key and be able to retrieve it efficiently

– Lots of programs do that!

• Search:	inverted indexes, phone directories, …
• Networks:	router tables
• Operating systems:	page tables
• Compilers:	symbol tables
• Databases:	dictionaries with other nice properties
• Biology:	genome maps
• …	

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	**`find`**	**`delete`**
• Unsorted linked-list			
• Unsorted array			
• Sorted linked list			
• Sorted array			

Simple implementations

For dictionary with n key/value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$*	$O(n)$	$O(n)$
• Unsorted array	$O(1)$*	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\texttt{log}\ n)$	$O(n)$

* Unless we need to check for duplicates

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

Lazy Deletion

10	12	24	30	41	42	44	45	50
✓	✗	✓	✓	✓	✓	✗	✓	✓

A *general technique* for making **`delete`** as fast as **`find`**:

- Instead of actually removing the item just mark it deleted

Plusses:

- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

Minuses:

- Extra *space* for the “is-it-deleted” flag
- Data structure full of deleted nodes wastes *space*
- **`find`** $O($**`log`** $m)$ *time* where m is data-structure size (okay)
- May complicate other operations

Tree terms (review?)

root(tree)

leaves(tree)

children(node)

parent(node)

siblings(node)

ancestors(node)

descendents(node)

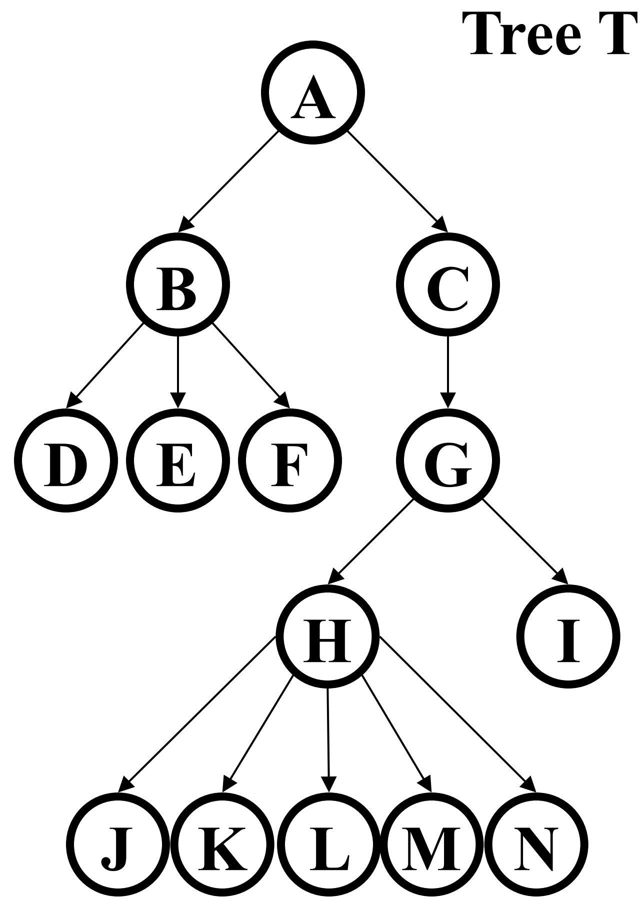
subtree(node)

depth(node)

height(tree)

degree(node)

branching factor(tree)



Some tree terms (mostly review)

- There are many kinds of trees
  - Every binary tree is a tree
  - Every list is kind of a tree (think of “next” as the one child)

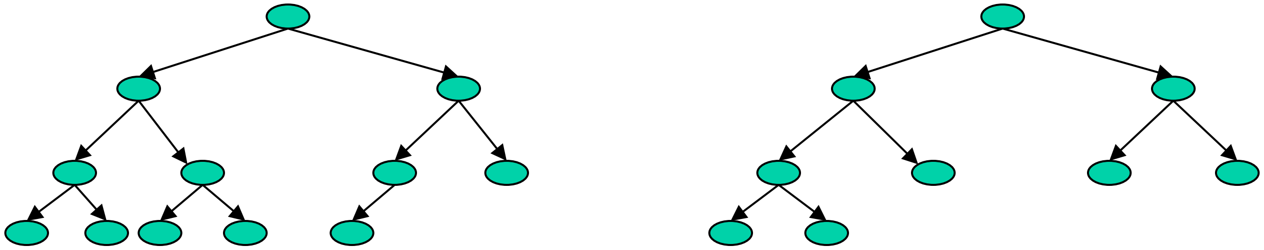
- There are many kinds of binary trees
  - Every binary search tree is a binary tree
  - Later: A binary heap is a different kind of binary tree

- A tree can be balanced or not
  - A balanced tree with *n* nodes has a height of $O(\mathtt{log}\ n)$
  - Different tree data structures have different “balance conditions” to achieve this

Kinds of trees

Certain terms define trees with specific structure

- Binary tree: Each node has at most 2 children (branching factor 2)
- *n*-ary tree: Each node has at most *n* children (branching factor *n*)
- Perfect tree: Each row completely full
- Complete tree: Each row completely full except maybe the bottom row, which is filled from left to right



What is the height of a perfect binary tree with n nodes?

A complete binary tree?

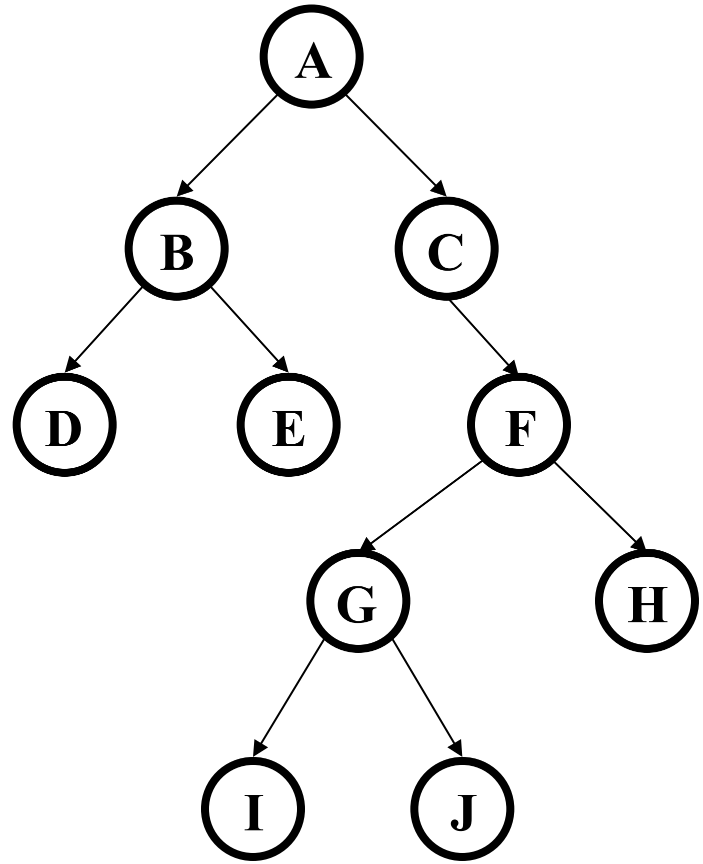
Binary Trees

- Binary tree is empty or
  - A root *(with data)*
  - A left subtree *(may be empty)*
  - A right subtree *(may be empty)*

- Representation:

Data	
left pointer	**right pointer**

- For a dictionary, data will include a key and a value



Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:

- max # of leaves:
- max # of nodes:
- min # of leaves:
- min # of nodes:

Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:

- max # of leaves: 2^h
- max # of nodes:
- min # of leaves:
- min # of nodes:

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For binary tree of height *h*:

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- min # of leaves: 1
- min # of nodes: $h + 1$

For n nodes, we cannot do better than O(`log` n) height, and we want to avoid O(n) height

Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {

            ???


}
```

Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {
  if(root == null)
    return -1;
  return 1 + max(treeHeight(root.left),
                 treeHeight(root.right));
}
```

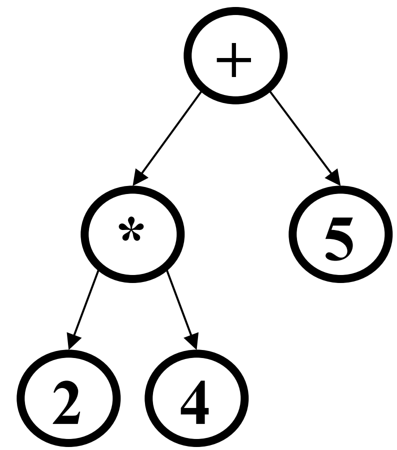
Running time for tree with n nodes: $O(n)$ – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion's call stack

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree
- *In-order*: left subtree, root, right subtree
- *Post-order*: left subtree, right subtree, root



(an expression tree)

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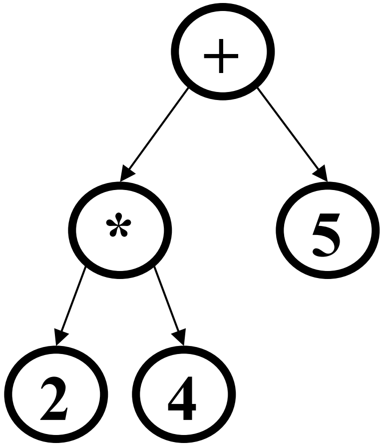

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- *Pre-order*: root, left subtree, right subtree
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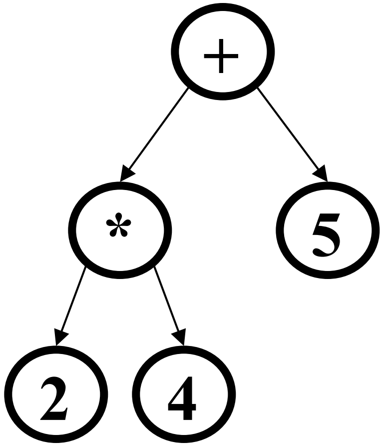

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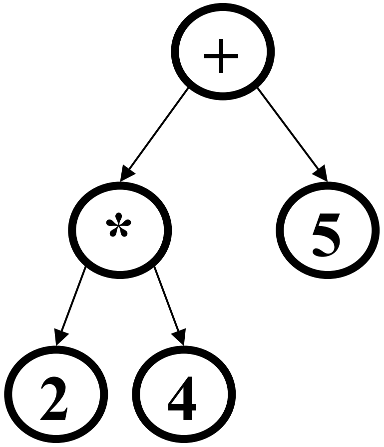
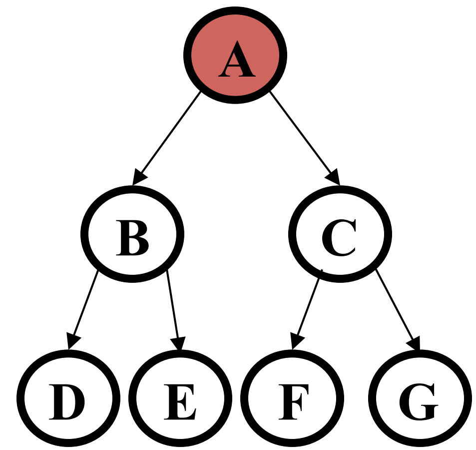


(an expression tree)

More on traversals

```
void inOrderTraversal(Node t){
  if(t != null) {
    inOrderTraversal(t.left);
    process(t.element);
    inOrderTraversal(t.right);
  }
}
```



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= processing (on the call stack)

= completed node

✓ = element has been processed

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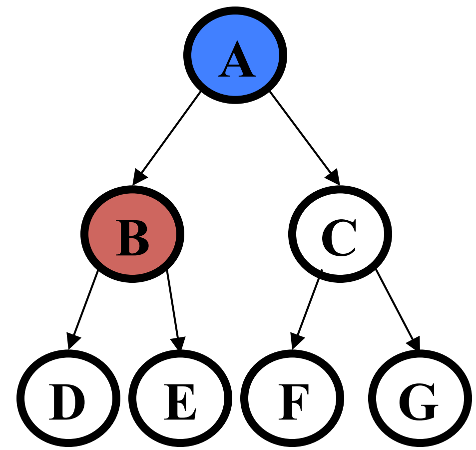

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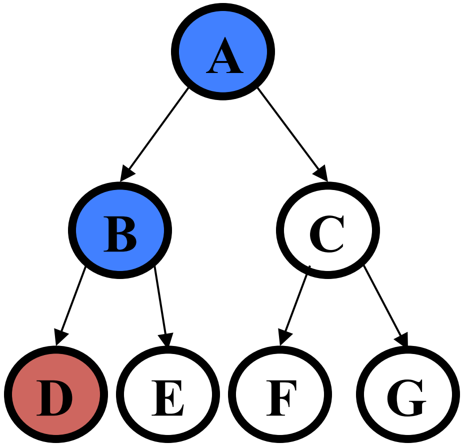

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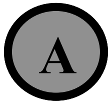

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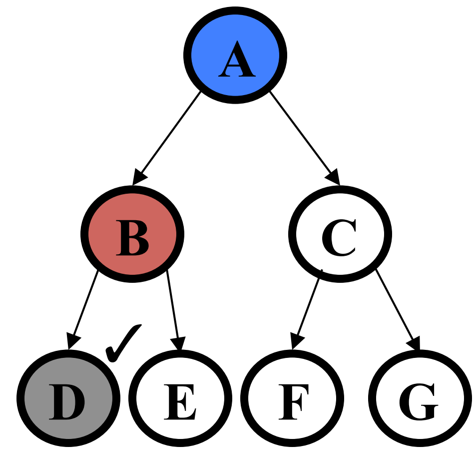

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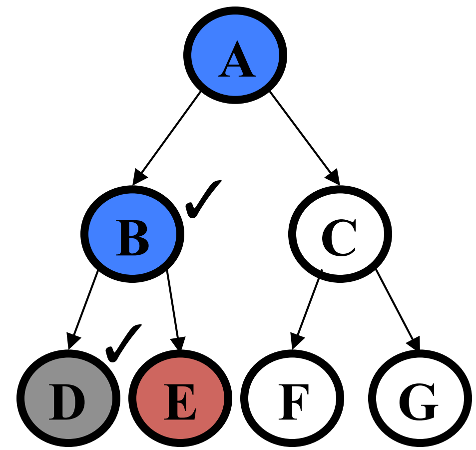

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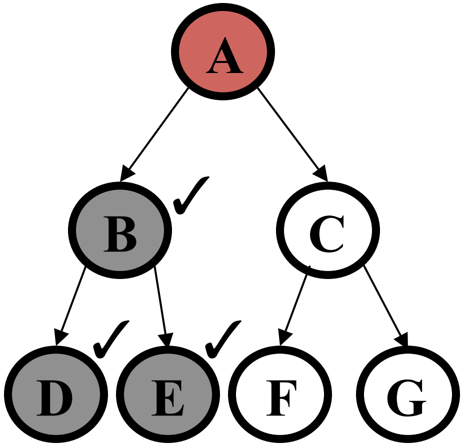

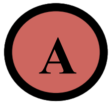

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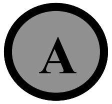

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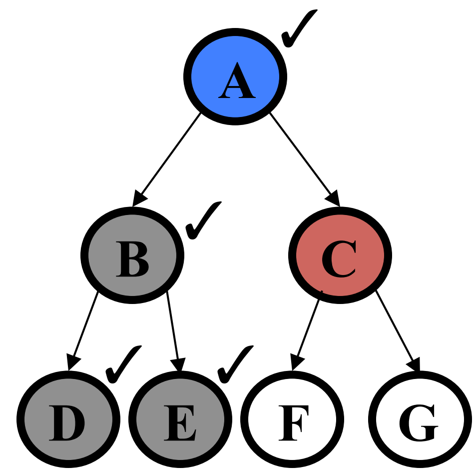

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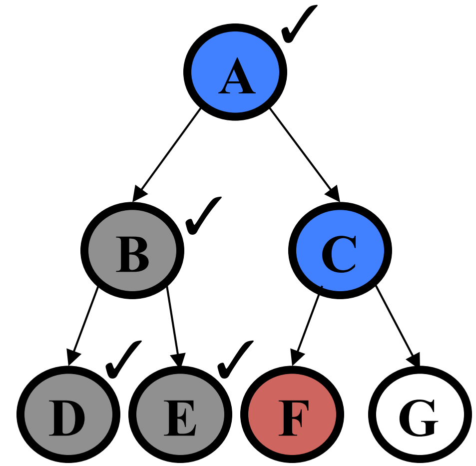

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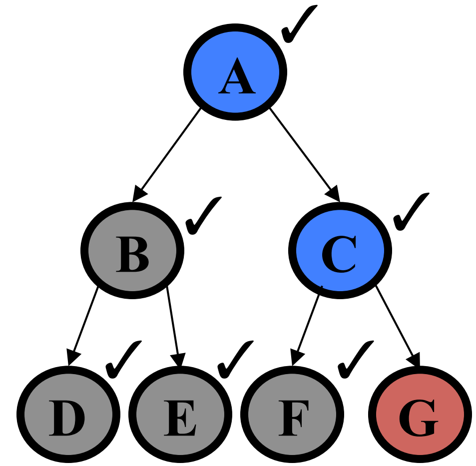

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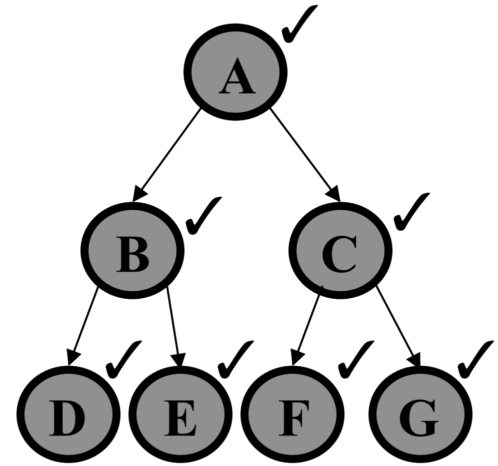

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