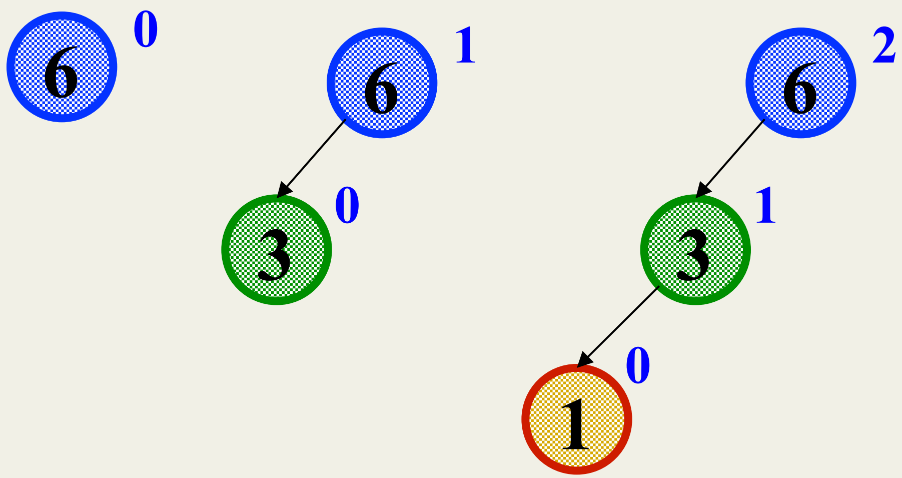
Case #1: Example



Insert(6)

Insert(3)

Insert(1)

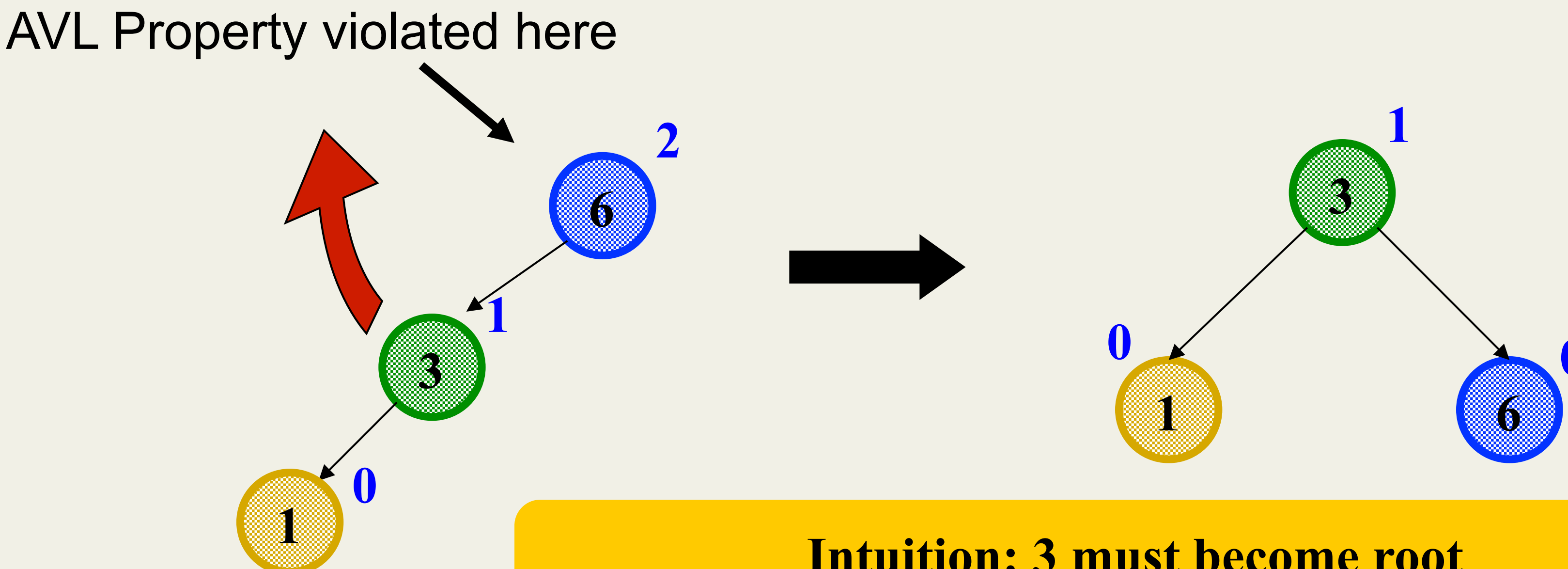
Third insertion violates balance property

- happens to be at the root

What is the only way to fix this?

Fix: Apply "Single Rotation"

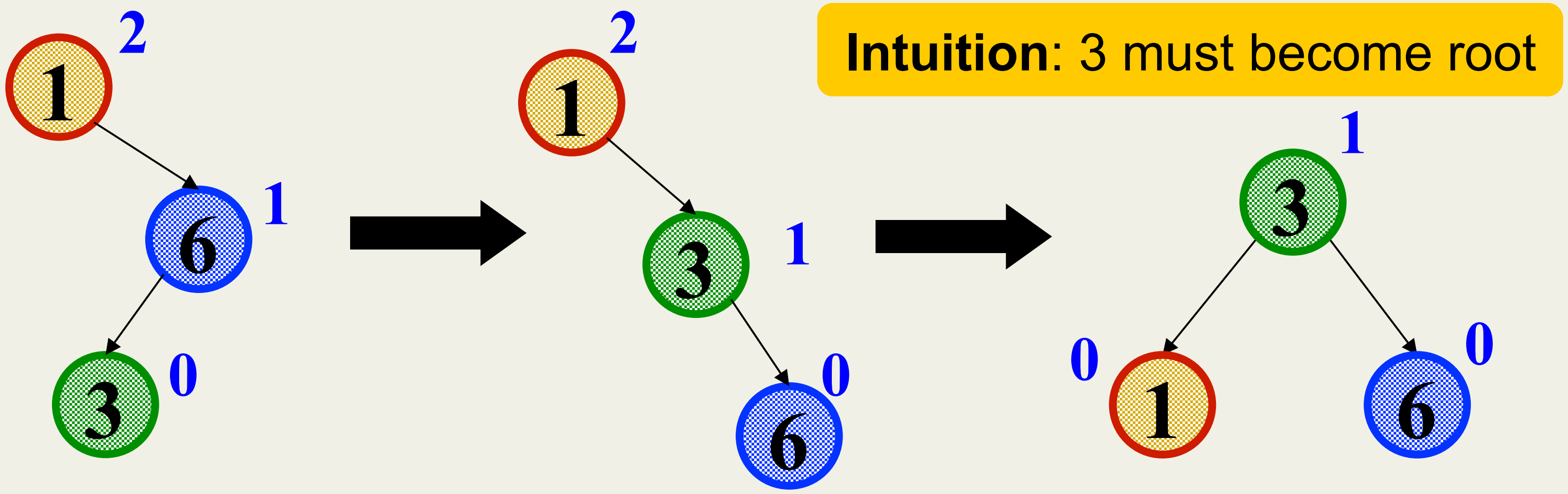
- *Single rotation:* The basic operation we'll use to rebalance
  - Move child of unbalanced node into parent position
  - Parent becomes the "other" child (always okay in a BST!)
  - Other subtrees move in only way BST allows (next slide)



Intuition: 3 must become root
New parent height is now the old parent's height before insert

Sometimes two wrongs make a right

- First idea violated the BST property
- Second idea didn't fix balance
- But if we do both single rotations, starting with the second, it works! (And not just for this example.)
- Double rotation:
  1. Rotate problematic child and grandchild
  2. Then rotate between self and new child



Insert, summarized

- Insert as in a BST
- Check back up path for imbalance, which will be 1 of 4 cases:
  - Node's left-left grandchild is too tall (**left-left single rotation**)
  - Node's left-right grandchild is too tall (**left-right double rotation**)
  - Node's right-left grandchild is too tall (**right-left double rotation)**
  - Node's right-right grandchild is too tall (**right-right double rotation**)
- Only one case occurs because tree was balanced before insert
- After the appropriate single or double rotation, the smallest-unbalanced subtree has the same height as before the insertion
  - So all ancestors are now balanced

Now efficiency

- Worst-case complexity of **`find`**: $O($**`log`** $n)$
  - Tree is balanced
- Worst-case complexity of **`insert`**: $O($**`log`** $n)$
  - Tree starts balanced
  - A rotation is $O(1)$ and there's an $O($**`log`** $n)$ path to root
  - (Same complexity even without one-rotation-is-enough fact)
  - Tree ends balanced
- Worst-case complexity of **`buildTree`**: $O(n$ **`log`** $n)$

Takes some more rotation action to handle **`delete`**…

Pros and Cons of AVL Trees

Arguments for AVL trees:

1. All operations logarithmic worst-case because trees are *always* balanced
2. Height balancing adds no more than a constant factor to the speed of **`insert`** and **`delete`**

Arguments against AVL trees:

1. Difficult to program & debug [but done once in a library!]
2. More space for height field
3. Asymptotically faster but rebalancing takes a little time
4. Most large searches are done in database-like systems on disk and use other structures (e.g., *B*-trees, a data structure in the text)
5. If *amortized* (later, I promise) logarithmic time is enough, use splay trees (also in text)

CSE373: Data Structures & Algorithms
Lecture 6: Hash Tables

Kevin Quinn

Fall 2015

Motivating Hash Tables

For a **dictionary** with *n* key, value pairs

	`insert`	`find`	`delete`
• Unsorted linked-list	$O(1)$	$O(n)$	$O(n)$
• Unsorted array	$O(1)$	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O(\texttt{log}\ n)$	$O(n)$
• *Balanced* tree	$O(\texttt{log}\ n)$	$O(\texttt{log}\ n)$	$O(\texttt{log}\ n)$
• Magic array	$O(1)$	$O(1)$	$O(1)$

Sufficient "magic":

- Use key to compute array index for an item in $O(1)$ time [doable]
- Have a different index for every item [magic]

Motivating Hash Tables

- Let's say you are tasked with counting the frequency of integers in a text file. You are guaranteed that only the integers 0 through 100 will occur:

  For example: 5, 7, 8, 9, 9, 5, 0, 0, 1, 12

  Result: 0 → 2 1 → 1 5 → 2 7 → 1 8 → 1 9 → 2

  What structure is appropriate?

  Tree?

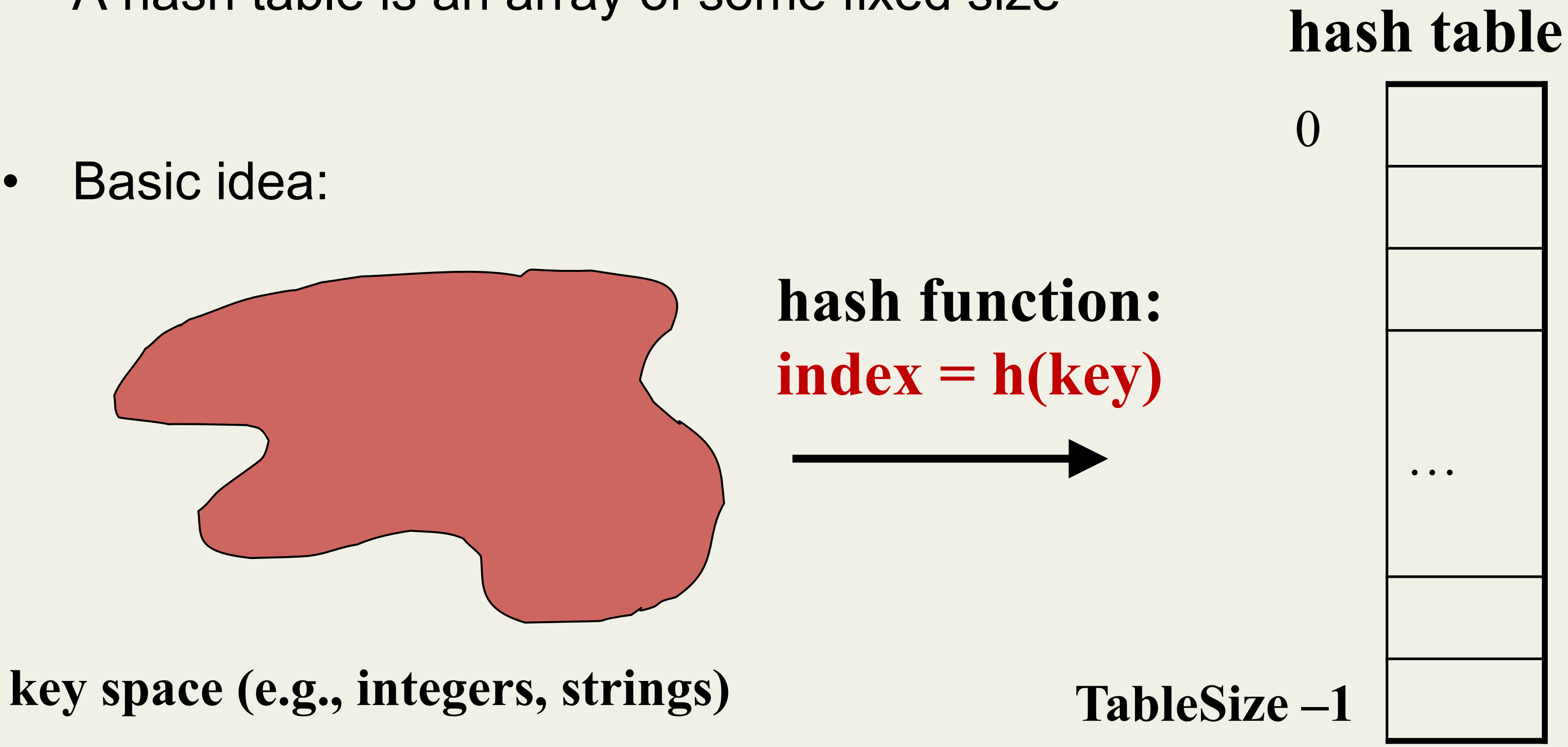
  List?

  Array?

0	1	2	3	4	5	6	7	8	9
2	**1**				**2**		**1**	**1**	**2**

Hash Tables

- Aim for constant-time (i.e., *O*(1)) **`find`**, **`insert`**, and **`delete`**
  - "On average" under some often-reasonable assumptions
- A hash table is an array of some fixed size
- Basic idea:



Hash Tables vs. Balanced Trees

- In terms of a Dictionary ADT for just **`insert`**, **`find`**, **`delete`**, hash tables and balanced trees are just different data structures
  - Hash tables *O*(1) on average (*assuming* we follow good practices)
  - Balanced trees *O*(**`log`** *n*) worst-case

- Constant-time is better, right?
  - Yes, but you need "hashing to behave" (must avoid collisions)
  - Yes, but **`findMin`**, **`findMax`**, **`predecessor`**, and **`successor`** go from *O*(**`log`** *n*) to *O*(*n*), **`printSorted`** from *O*(*n*) to *O*(*n* **`log`** *n*)
    - Why your textbook considers this to be a different ADT

Hash Tables

- There are ***m*** possible keys (*m* typically large, even infinite)
- We expect our table to have only ***n*** items
- ***n*** is much less than ***m*** (often written ***n* << *m***)

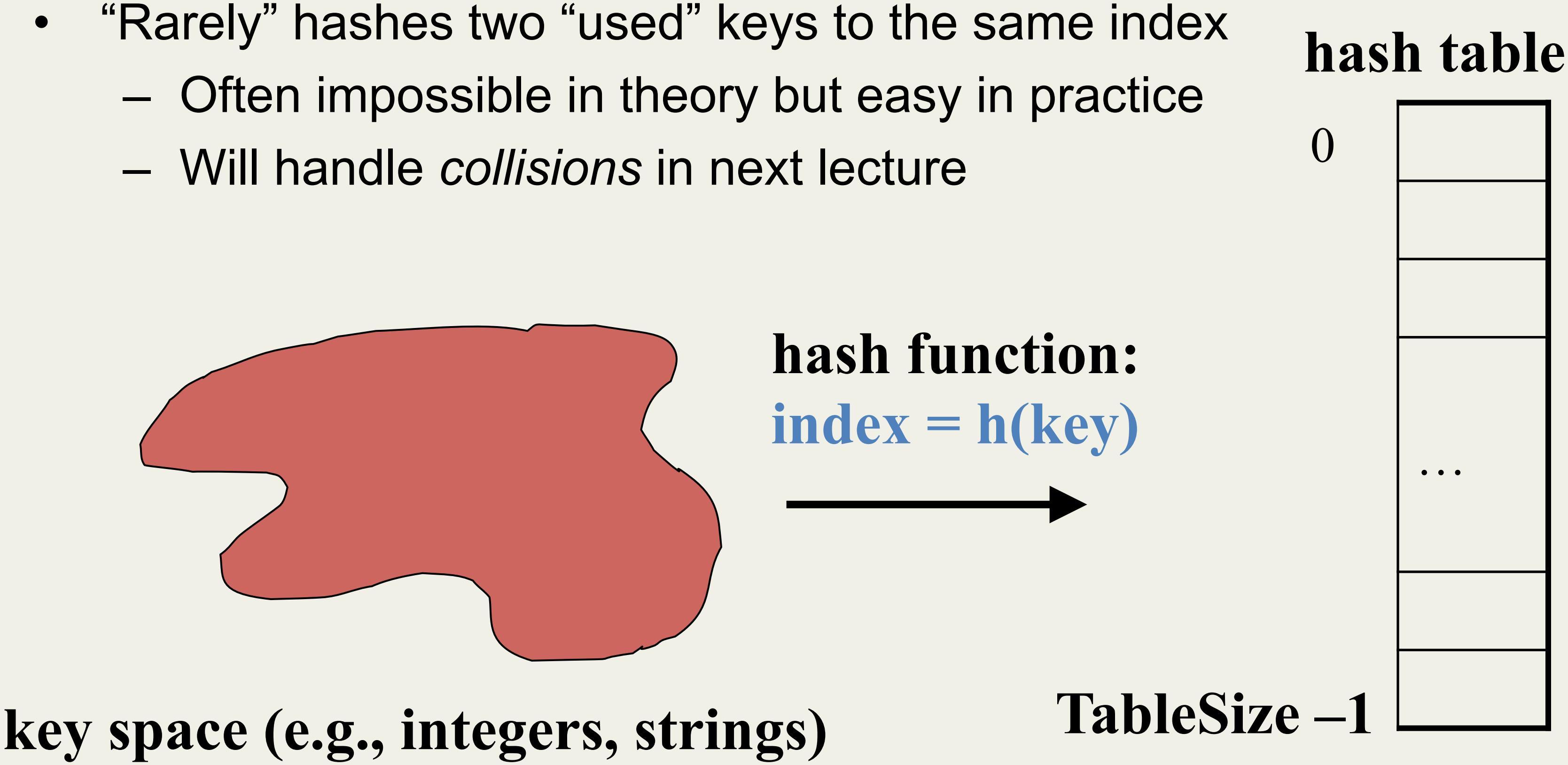
Many dictionaries have this property

- Compiler: All possible identifiers allowed by the language vs. those used in some file of one program
- Database: All possible student names vs. students enrolled
- AI: All possible chess-board configurations vs. those considered by the current player
- …

Hash functions

An ideal hash function:

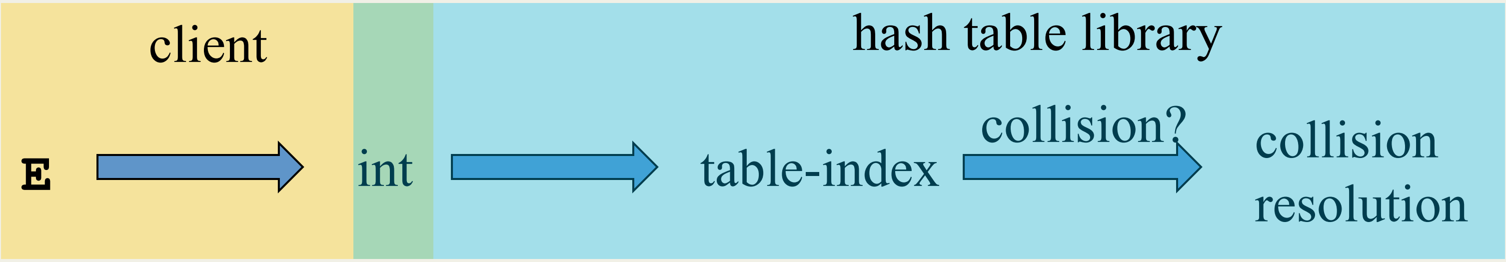
- Fast to compute
- "Rarely" hashes two "used" keys to the same index
  - Often impossible in theory but easy in practice
  - Will handle *collisions* in next lecture



Who hashes what?

- Hash tables can be generic
  - To store elements of type **`E`**, we just need **`E`** to be:
    1. *Comparable*: order any two **`E`** (as with all dictionaries)
    2. *Hashable*: convert any **`E`** to an **`int`**

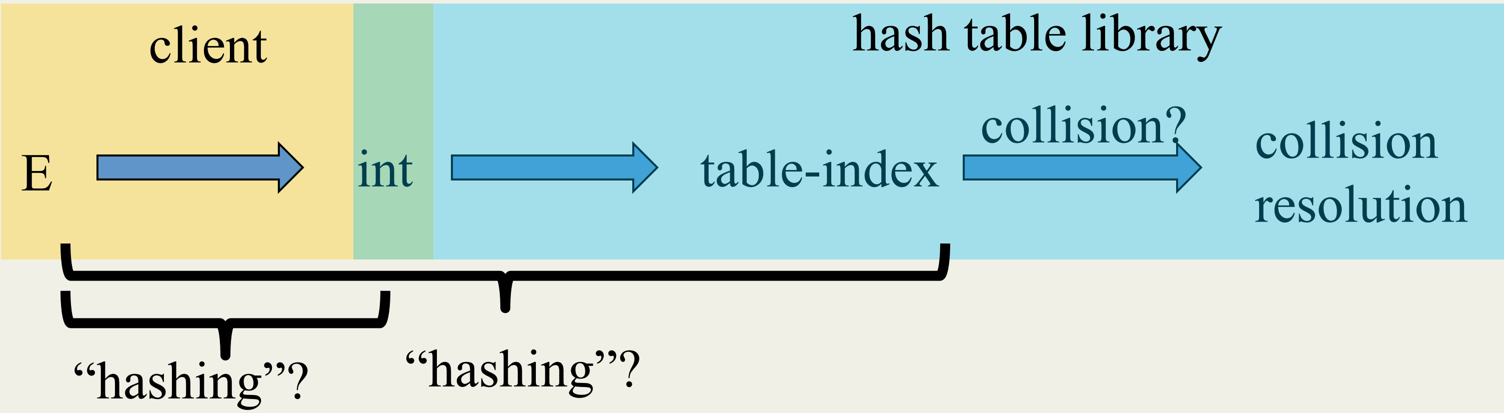
- When hash tables are a reusable library, the division of responsibility generally breaks down into two roles:



- We will learn both roles, but most programmers "in the real world" spend more time as clients while understanding the library

More on roles

Some ambiguity in terminology on which parts are "hashing"



Two roles must both contribute to minimizing collisions (heuristically)

- Client should aim for different ints for expected items
  - Avoid "wasting" any part of **`E`** or the 32 bits of the **`int`**
- Library should aim for putting "similar" **`int`s** in different indices
  - Conversion to index is almost always "mod table-size"
  - Using prime numbers for table-size is common

What to hash?

We will focus on the two most common things to hash: *ints* and *strings*

– For objects with several fields, usually best to have most of the "identifying fields" contribute to the hash to avoid collisions

– Example:

```
class Person {
   String first; String middle; String last;
   Date birthdate;
}
```

– An inherent trade-off: hashing-time vs. collision-avoidance
  - Bad idea(?): Use only first name
  - Good idea(?): Use only middle initial
  - Admittedly, what-to-hash-with is often unprincipled ☹

Hashing integers

- key space = integers
- Simple hash function:
  - Client: `g(x) = x`
  - Library: `f(x) = g(x) % TableSize`
  - Fairly fast and natural
- Example:
  - `TableSize` = 10
  - Insert 7, 18, 41, 34, 10
  - Insert 44?
  - (As usual, only looking at keys, not values)

0	10
1	41
2	
3	
4	34
5	
6	
7	7
8	18
9	

Collision-avoidance

- With "`x % TableSize`" the number of collisions depends on
  - the ints inserted (obviously)
  - **`TableSize`**
- Larger table-size tends to help, but not always
  - Example: 70, 17, 14, 9, 10
  - What's a table size that would work well? Poorly?

    `TableSize` = 9 and **`TableSize`** = 60
- Technique: Pick table size to be prime. Why?
  - Real-life data tends to have a pattern
  - "Multiples of 61" are probably less likely than "multiples of 60"
  - Next lecture shows one collision-handling strategy does *provably* well with prime table size

Okay, back to the client

- If keys aren't **`int`s**, the client must convert to an **`int`**
  - Why can't the library do this for us?
  - Trade-off: speed versus distinct keys hashing to distinct **`int`s**
- Very important example: Strings
  - Key space K = $s_0 s_1 s_2 \ldots s_{m-1}$
    - (where s_i are chars: $s_i \in [0,52]$ or $s_i \in [0,256]$ or $s_i \in [0,2^{16}]$)
  - Some choices: Which avoid collisions best?

1. h(K) = s_0 % TableSize
2. h(K) = $\left(\sum_{i=0}^{m-1} s_i \right)$ % TableSize
3. h(K) = $\left(\sum_{i=0}^{k-1} s_i \cdot 37^i \right)$ % TableSize

Specializing hash functions

How might you hash differently if all your strings were web addresses (URLs)?

Combining hash functions

A few rules of thumb / tricks:

1. Use all 32 bits (careful, that includes negative numbers)
2. Use different overlapping bits for different parts of the hash
   – This is why a factor of 37^i works better than 256^i
   – Example: “abcde” and “ebcda”
3. When smashing two hashes into one hash, use bitwise-xor
   – bitwise-and produces too many 0 bits
   – bitwise-or produces too many 1 bits
4. Rely on expertise of others; consult books and other resources
5. If keys are known ahead of time, choose a *perfect hash*

Combining Hashes

h1 = 10110011: (unicode for the int “3”)
h2 = 01100101: (unicode for the char “e”)

h1 AND h2	h1 OR h2	h1 XOR h2
10110011	10110011	10110011
01100101	01100101	01100101
00100001	11110111	11010110

One expert suggestion


```
int result = 17;
foreach field f
    int fieldHashcode =
        boolean: (f ? 1: 0)
        byte, char, short, int: (int) f
        long: (int) (f ^ (f >>> 32))
        float: Float.floatToIntBits(f)
        double: Double.doubleToLongBits(f), then above
        Object: object.hashCode( )
     result = 31 * result + fieldHashcode
```

Hashing and comparing

- Need to emphasize a critical detail:
  - We initially *hash* key **`E`** to get a table index
  - To check an item is what we are looking for, *compareTo* **`E`**
    - Does it have an equal key?

- So a hash table needs a hash function and a comparator
  - The Java library uses a more object-oriented approach: each object has methods **`equals`** and **`hashCode`**

```
class Object {
  boolean equals(Object o) {…}
  int hashCode() {…}
  …
}
```

Equal Objects Must Hash the Same

- The Java library make a crucial assumption clients must satisfy
  - And all hash tables make analogous assumptions
- Object-oriented way of saying it:

  If **`a.equals(b)`**, then **`a.hashCode()==b.hashCode()`**

- Why is this essential?
- Why is this up to the client?
- So *always* override **`hashCode`** *correctly* if you override **`equals`**
  - Many libraries use hash tables on your objects

By the way: comparison has rules too

We have not emphasized important “rules” about comparison for:

- Dictionaries
- Sorting (future major topic)

Comparison must impose a consistent, total ordering:

For all **`a`**, **`b`**, and **`c`**,

`(reflexivity)`: **`a.compareTo(a) == 0`**

`(transitivity)`: If **`a.compareTo(b) < 0`** and **`b.compareTo(c)<0`**, then **`a.compareTo(c) < 0`**

`(symmetry)`: If **`a.compareTo(b) < 0`**, then **`b.compareTo(a) > 0`**
If **`a.compareTo(b)== 0`**, then **`b.compareTo(a)==0`**

This is surprisingly awkward because of subclassing…

Example

```
class MyDate {
  int month;
  int year;
  int day;

  boolean equals(Object otherObject) {
     if(this==otherObject) return true; // common?
     if(otherObject==null) return false;
     if(getClass()!=other.getClass()) return false;
     return month = otherObject.month
            && year = otherObject.year
            && day = otherObject.day;
  }
  // wrong: must also override hashCode!
}
```

Tougher example

- Suppose you had a **`Fraction`** class where **`equals`** returned **`true`** for 1/2 and 3/6, etc.
- Then must override **`hashCode`** and cannot hash just based on the numerator and denominator
  - Need 1/2 and 3/6 to hash to the same int
- If you write software for a living, you are less likely to implement hash tables from scratch than you are likely to encounter this issue

Conclusions and notes on hashing

- The hash table is one of the most important data structures
  - Supports only **`find`**, **`insert`**, and **`delete`** efficiently
  - Have to search entire table for other operations

- Important to use a good hash function

- Important to keep hash table at a good size

- Side-comment: hash functions have uses beyond hash tables
  - Examples: Cryptography, check-sums

- Big remaining topic: Handling collisions