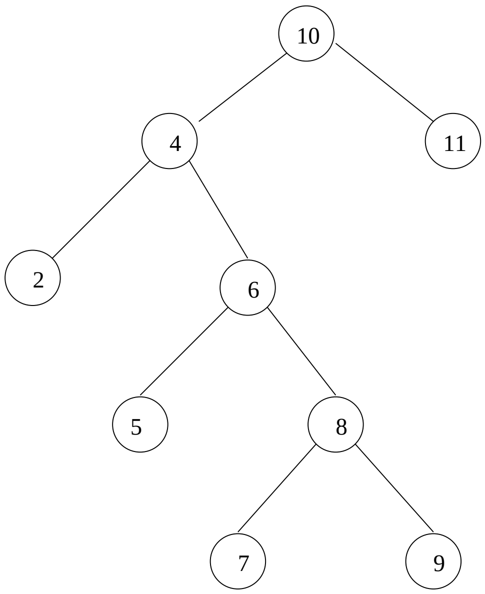
CSE 373 Second Midterm May 22, 1998

All questions count equally. Answer all questions.

1. Insert the successive keys $2, 1, 4, 5, 9, 3$ into an initially empty AVL tree. Show the tree after each insertion.

2. (a) Give the result of performing the operation $Find(9)$ on the splay tree given below.

   (b) Give the result of performing the operation $Delete(9)$ from the splay tree given below.



3. Let $b(N)$ denote the number of 1's in the binary representation of N. For example, the binary representation of 13 is 1101, so $b(13) = 3$ Give a formula in terms of N and $b(N)$ for the minimum number of comparisons between keys required to build a binomial queue of size N from N given keys Hint: It takes only one comparison to merge two binomial trees of the same rank.

4. Here are two algorithms for finding the kth-smallest key in an array A of size n, where k and n are given positive integers.

   Algorithm 1 Use the Build-Heap algorithm to convert the array to a min-heap. Then perform Delete-Min $k-1$ times and return the key at the root of the heap.

   Algorithm 2 Insert the first k keys of the array into an initially empty max-heap. For each of the remaining keys, insert the key into the max-heap and then execute one Delete-Max operation. Return the key at the root of the max-heap.

   (a) Briefly explain why Algorithm 2 works.

   (b) Using Big O notation, give the worst-case execution time of Algorithm 1.

   (c) Using Big O notation, give the worst-case execution time of Algorithm 2.