CSE 373: Introduction

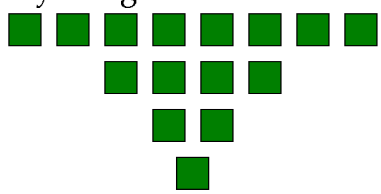
Chapter 1

Math Review

- Things to review on your own (§1.2.1–1.2.5)
  - exponents
  - logarithms
  - series
  - modular arithmetic
  - proof techniques

Brad's Take on Logarithms

- Understanding $\log_b x$
  - textbook definition: $\log_b x = y \;\Rightarrow\; b^{\,y} = x$
    ($\log_b x$ is the power to which b must be taken to get x)
  - more useful: $\log_b x$ is the number of times you must divide x by b to get 1



2^3: $\log_2 8 = 3$
2^2: $\log_2 4 = 2$
2^1: $\log_2 2 = 1$
2^0: $\log_2 1 = 0$

- b is almost always 2 and omitted by default

Brad's Take on Series I

$1 + 2 + 3 + 4 + \ldots + n = ?$

Mathematically:

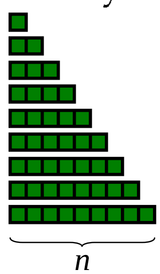
$=$

Brad's Take on Series I (cont'd)

$1 + 2 + 3 + 4 + \ldots + n = ?$

Geometrically:

=



Brad's Take on Series II

$1 + 2 + 4 + 8 + \ldots + n/2 + n = ?$

Geometrically:

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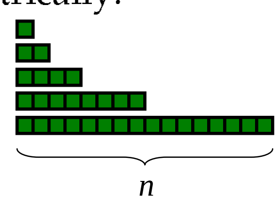


C++ Review

- Classes:
  - constructors/destructors
  - separation of interface and implementation
  - `vector` and `string` classes
- Pointers
- Dynamic memory allocation: `new`, `delete`
- Parameter passing, return values
- Templates (we'll cover briefly in class)

Recursion

Recursive function: A function that calls itself

- Analogous to recurrence relations in math:

$0! = 1$	$fact(0) = 1$
$x! = x \cdot (x-1)!$	$fact(x) = x \cdot fact(x-1)$

- Recursively in C++:

Disadvantages of Recursion

- Function calls are *expensive*:
  - take more time than standard operations
  - require memory proportional to the call depth
- Simple cases can be rewritten with loops:

```
int fact(int x) {
  if (x == 0) {
    return 1;
  } else {
    return x * fact(x-1);
  }
}
```

```
int fact(int x) {
  int product;
  product = 1;
  while (x > 0) {
    product *= x;
    x--;
  }
  return product;
}
```


Recursion II

Fibonnaci Numbers:

$fib_0 = 1$

$fib_1 = 1$

$fib_x = fib_{x-1} + fib_{x-2}$

- Recursively in C++:

```
int fib(int x) {


}
```

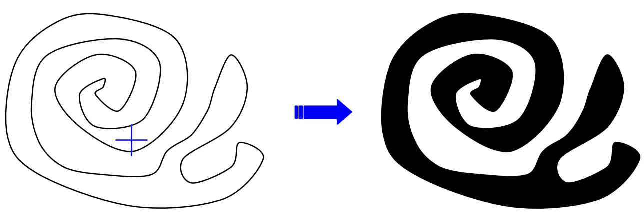
Disadvantages of Recursion II

- Elegance disguises redundant computation
- What is the call chain like for `fib(5)`? `fib(10)`?
- Does `fib()` have a simple iterative rewrite?

Recursion III

```
void PaintFill(int pixel[][],int x,int y);
```

- pixels are either black (1) or white (0)
- starting at pixel (**x**,**y**) change white pixels to black, stopping at boundaries



Recursion III (continued)

```
void PaintFill(int pixel[][],int x,int y){



}
```

Does `PaintFill()` have an iterative rewrite?

Recursion Summary

- Recursive routines must:
  - have a base case
  - always make progress towards the base case
- Be sure to keep an eye out for:
  - recursive calls that have simple iterative rewrites
  - redundant computation

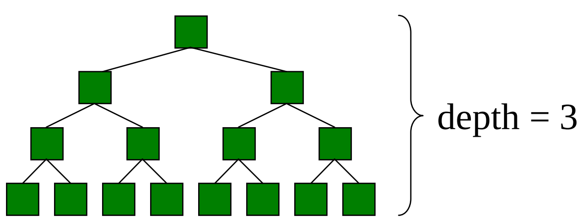
Inductive Proofs

Inductive proof – A way to prove a property true for an infinite number of (enumerable) cases

- prove property true for base case(s)
- assume it's true for the first k-1 instances, and use them to prove it's true for the k^{th} instance

Simple Inductive Proof

Prove: Every complete binary tree of depth d contains $2^{d+1} - 1$ nodes



Simple Inductive Proof (cont'd)

Proof (by induction):

- Let P(i) = "A complete binary tree of depth i contains 2^{i+1} - 1 nodes"
- We must prove P(i) true for all $i \geq 0$
- *base case:* Prove P(0) is true

Simple Inductive Proof (cont'd)

Proof (continued):

- *inductive step:* Assuming P(0), P(1), …, P(k-1) are true, prove P(k) is true

- Therefore, for all $i \geq 0$, P(i) is true

Induction and Recursion

Induction and Recursion are analogous concepts

- both use base cases
- both solve "big" problems based on the assumption that "smaller" problems are solved in a similar way
- both require that you assume the recursive/ inductive step works without checking every case
- both have similar pitfalls
  - determining the number of base cases
  - handling the base case(s) correctly
  - getting the inductive step to work for all non-base cases

An Incorrect Inductive Proof

Prove: When h horses are within a fenced area, they are all the same color

Proof (by induction):

- Let P(i) = "when i horses are within a fenced area, they are all the same color"
- *base case:* when 1 horse is in a fenced in area, it is the same color as itself. Therefore, P(1) is true.
- *inductive step:* Assume P(1), P(2), …, P(k-1) are true.
  - Consider k horses in a fenced-in area.
  - Lead one of the horses, a, out of the area such that k-1 horses remain. Since P(k-1) is true, the remaining horses must all be the same color.
  - Now lead a back in and lead a different horse, b, out, once again leaving k-1 horses within the fence. Since P(k-1) is true, these horses must also all be the same color.
  - Since both subsets of k-1 horses were the same color, a and b must be the same color, and therefore all k horses must be the same color
  - Therefore P(1), …, P(k-1) $\Rightarrow$ P(k) is true

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