CSE 373: Self-Adjusting Trees
("Cookbook Data Structures")

Chapter 4

(*and Section 12.2*)

Motivation

Most Binary Search Tree Operations are O(d)

- d can range from O(log n) to O(n)
- generally, d is O(log n)
- statistically, d is O(log n) on average
- **but**, for "common" insertion orders, d can be O(n)
  - *e.g.*, inserting sorted lists in order

A Solution

Self-Adjusting Binary Search Trees: BST's that automatically rearrange themselves to keep operations O(log *n*)

- AVL Trees
- Splay Trees
- Red-Black Trees

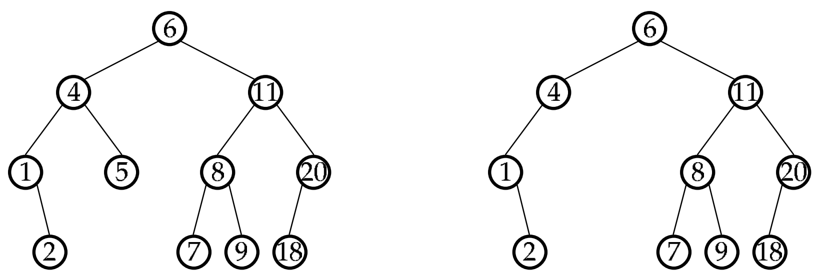
AVL Trees

The idea: A *balance condition* is placed on the tree. Whenever an **`insert()`** breaks the condition, we rearrange the tree to fix it.

What should the balance condition be?

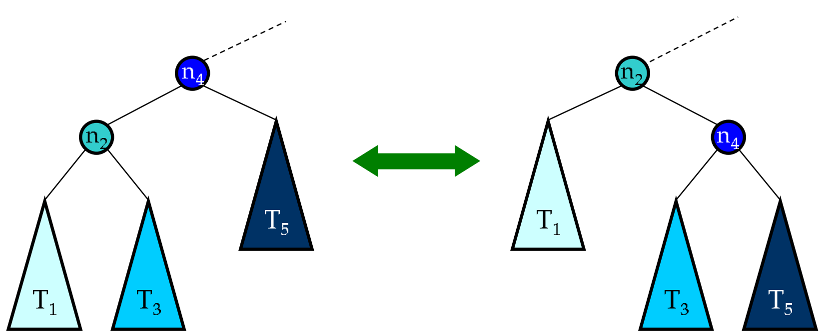
AVL Tree Strategy

Balance Condition: Every node's left and right subtrees must have a height difference of no more than one



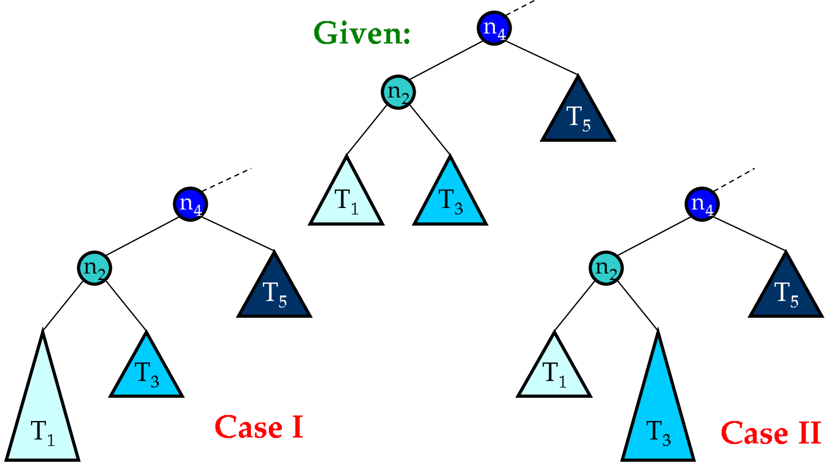
Rotations

Rotation: a simple way of rearranging a tree without breaking the binary search property

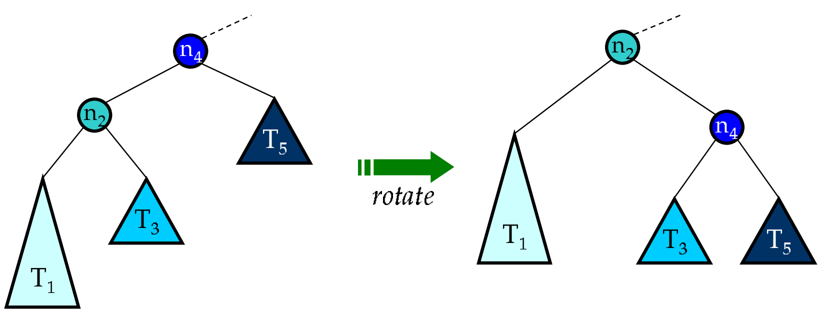


UW, Autumn 1999 **CSE 373 – Data Structures and Algorithms** Brad Chamberlain

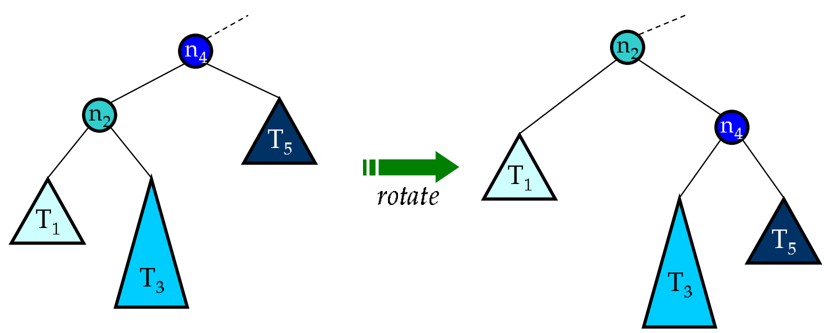
Two Cases of Bad Inserts



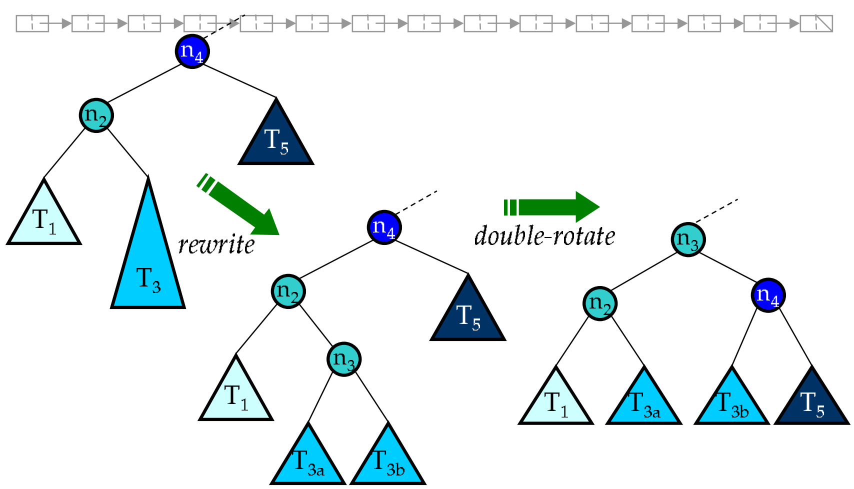
Fixing Case I



Trying to Fix Case II



Fixing Case II



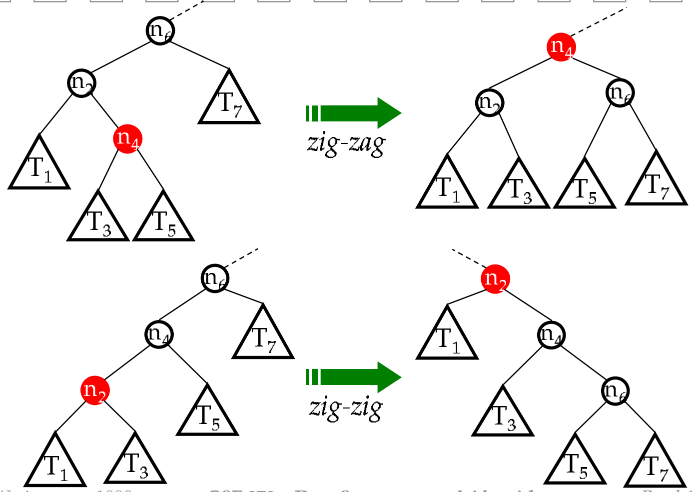
AVL Tree Summary

- Keep every node's subtrees "almost balanced"
- When insertions break the "almost balanced" condition, use rotations to fix things up
- Use lazy deletion to keep things simple
- All operations are O(log n)
- Implementation Cost:
  - must store depth of each node's child subtrees
  - must implement 4 cases for bad insertion ($2 \times$ L,R)

Splay Trees

- Every time a node is accessed, rotate it to the top of the tree no matter what
- Over time, trees tend to get shallower since all accessed nodes get rotated towards the top
- Result: Although any one operation may require O(n) time, a series of k operations is guaranteed to be O(k log n) – *amortized analysis*
- Benefits:
  - no need to store depths of nodes' subtrees

Splaying

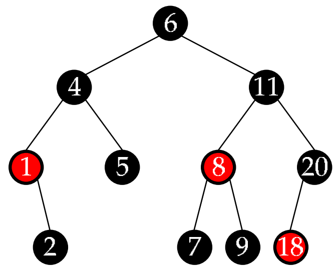


Red-Black Trees

Red -Black Trees: Binary Search Trees with the following properties:

- every node is colored either red or black
- the root is always black
- if a node is red, its children must be black
- every path from a node to a NULL pointer must contain the same number of black nodes

Example Red-Black Tree



Intuitively…

- every path from root to leaf has same number of black nodes
- though they may have different # red nodes, alternate at worst
- Thus, worst-case $d \approx 2\log(n) = O(\log n)$

Inserting into Red-Black Trees

```
Insert(T,0);
Insert(T,3);
Insert(T,22);
Insert(T,23);
Insert(T,24);
```