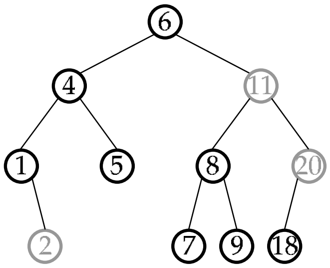
Design Decision: Lazy Deletion

Lazy Deletion: Rather than deleting a node from a tree, merely *mark* it as being deleted

- operate around it as usual
- (just don’t return it as the result of a `Find()` op)

```
Delete(T,2);
Delete(T,20);
Delete(T,11);
```



CSE 373: Self-Adjusting Trees
(*“Cookbook Data Structures”*)

Chapter 4

(*and Section* 12.2)

Motivation

All Binary Search Tree Operations are O(*d*)

- *d* can range from log *N* to *N*
- generally, *d* is O(log *N*)
- statistically, *d* is O(log *N*) on average
- **but**, for common(?) insertion orders, *d* will be O(*N*)
  - *i.e.,* inserting sorted lists in order

A Solution

Self-Adjusting Binary Search Trees: BST's that automatically rearrange themselves to keep operations O(log *N*)

- AVL Trees
- Splay Trees
- Red-Black Trees

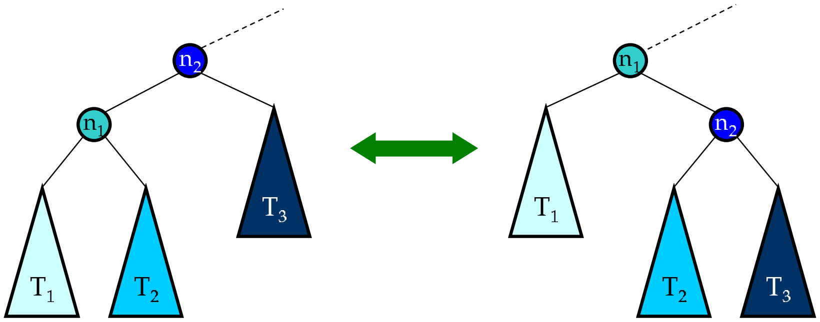
AVL Trees

The idea: A *balance condition* is placed on the tree. Whenever an `insert()` breaks the condition, we rearrange the tree to fix it.

What should the balance condition be?

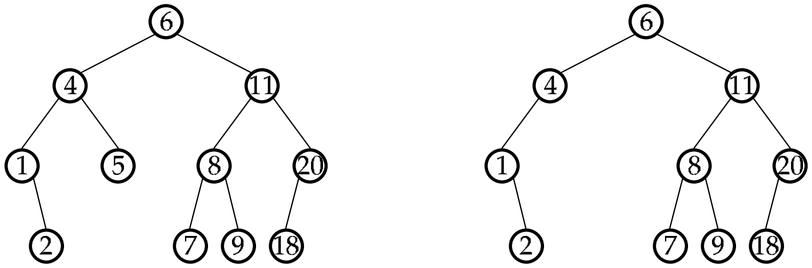
Rotations

Rotation: a simple way of rearranging a tree without breaking the binary search property

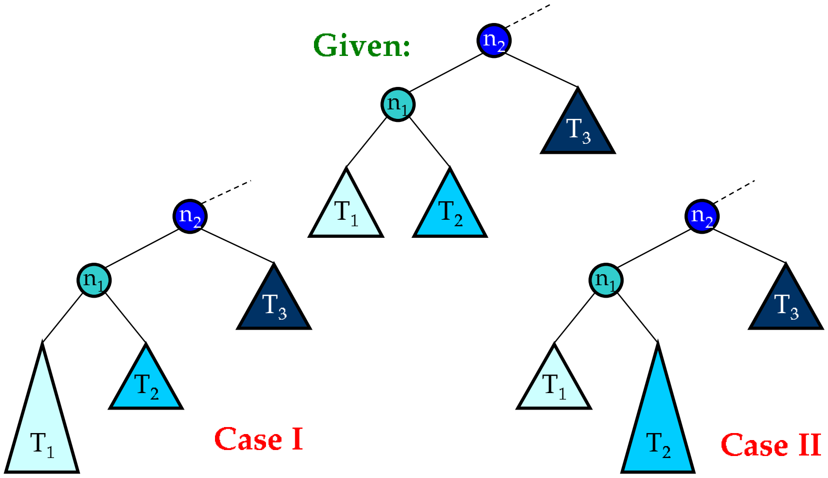


AVL Tree Strategy

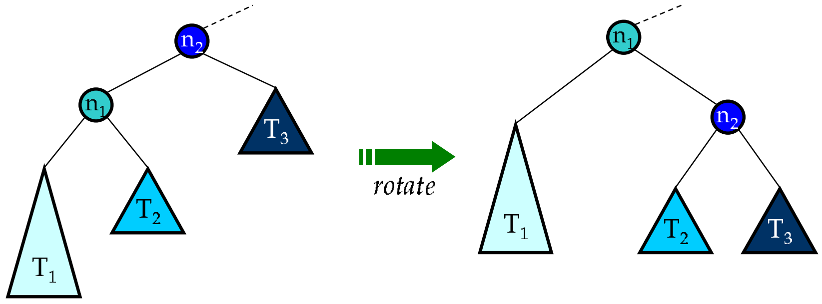
Balance Condition: Every node's left and right subtrees must have a height difference of no more than one



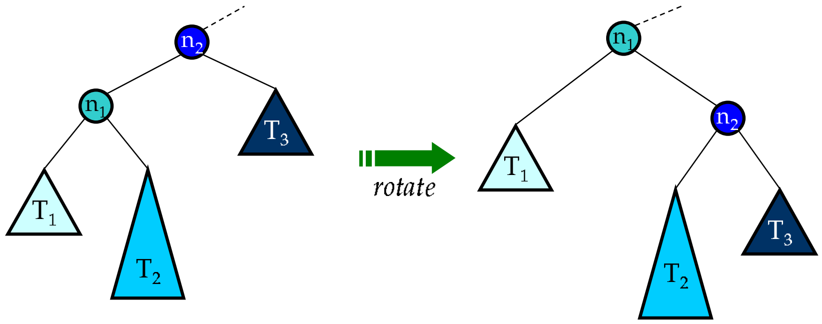
Two Cases of Bad Inserts



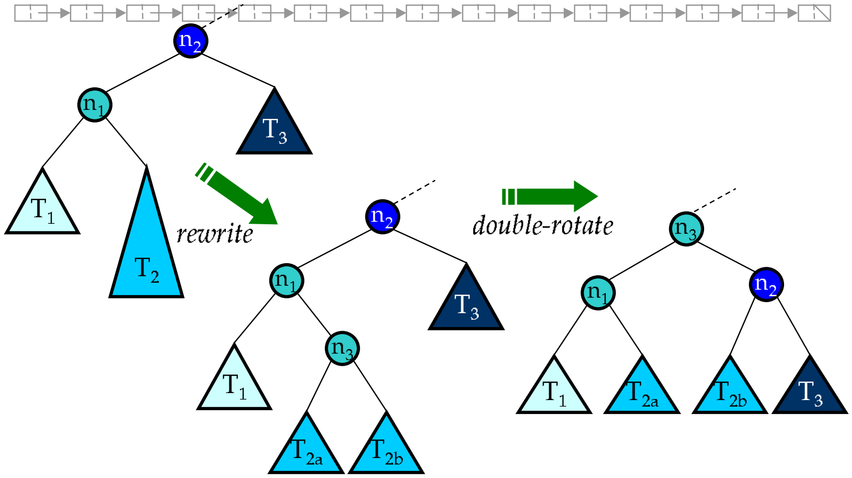
Fixing Case I



Trying to Fix Case II



Fixing Case II



AVL Tree Summary

- Keep every node’s subtrees “almost balanced”
- When insertions break the “almost balanced” condition, use rotations to fix things up
- Use lazy deletion to keep things simple
- All operations are O(log *N*)
- Implementation Cost:
  - must store depth of each node’s child subtrees
  - must implement 4 cases for bad insertion

Splay Trees

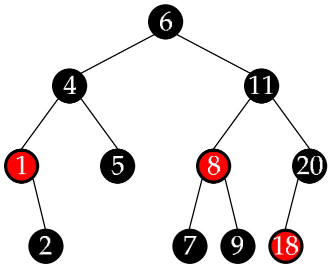
- Every time a node is accessed, rotate it to the top of the tree no matter what
- Over time, trees tend to get shallower because rotations don't make the tree any deeper
- Result: Although any one operation may require O(*N*) time, a series of *k* operations is guaranteed to be O(*k* log *N*) – *amortized analysis*
- Benefits:
  - no need to store depths of nodes' subtrees

Red-Black Trees

Red -Black Trees: Binary Search Trees with the following properties:

- every node is colored either red or black
- the root is always black
- if a node is red, its children must be black
- every path from a node to a NULL pointer must contain the same number of black nodes

Example Red-Black Tree



Intuitively…

- every path from root to leaf has same number of black nodes
- though they may have different # red nodes, alternate at worst
- Thus, $d = 2\log(N+1) = O(\log N)$

Inserting into Red-Black Trees

```
Insert(T,0);
Insert(T,3);
Insert(T,22);
Insert(T,23);
Insert(T,24);
```